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ON THE EXACTNESS OF A DATA DEPENDENCE ANALYSIS METHOD

ALEXANDRU VANCEA AND FLORIAN MIRCEA BOIAN

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1. Preliminaries

Parallelizing compilers [Wolfe96] are based upon subscript analysis to detect data dependences between pairs of array references inside loop nests. The most widely used approximate subscript analysis tests are the GCD test and the Banerjee test [Bane97]. These tests are approximate in the sense that neither is perfectly accurate, nor is the combination of them. Both tests approximate on the conservative side, focusing on assuming dependence, so their use never results in unsafe parallelization.

This paper studies the exactness of the Banerjee test and provides a set of conditions which can be tested together with the Banerjee's inequalities and the GCD test for obtaining a more exact data dependence information. Section 2 introduces the necessary background and states a sufficiency condition for the exactness of the Banerjee test, as studied in [Psa91]. Section 3 claims to be the main contribution of this paper, showing that in some circumstances, this sufficiency condition is also necessary for the exactness of the Banerjee test.

2. A sufficient condition for the exactness of the Banerjee test

Definition 1. *Let a_0, a_1, ... , a_n be integers. For every k, $1 \le k \le n$, let L_k and U_k be integers such that $L_k \le U_k$. The equation*

$$a_1 I_1 + a_2 I_2 + \ldots + a_n I_n = a_0$$

is said to be $(L_1,U_1;\ L_2,U_2;\ \ldots;\ L_n,U_n)$ - solvable (*or* interval solvable *or* I-solvable) *if there exist integers i_1, i_2, ... , i_n such that*

- $a_1 i_1 + a_2 i_2 + \ldots + a_n i_n = a_0$, *and*

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- *for every k, $1 \le k \le n$ we have $L_k \le i_k \le U_k$.*

Definition 2. a). *For every k, $1 \le k \le m$, let $a_{k0}, a_{k1}, a_{k2}, \ldots, a_{kn}$ be integers. For every k', $1 \le k' \le n$ let $L_{k'}$ şi $U_{k'}$ be integers such that $L_{k'} \le U_{k'}$. The equations set*

$$\begin{aligned} a_{11}I_1 + a_{12}I_2 + \ldots + a_{1n}I_n &= a_{10} \\ a_{21}I_1 + a_{22}I_2 + \ldots + a_{21n}I_n &= a_{20} \\ &\ldots\ldots \\ a_1 I_1 + a_{m2}I_2 + \ldots + a_{mn}I_n &= a_{m0} \end{aligned}$$

is said to be simultaneous *$(L_1, U_1; L_2, U_2; \ldots; L_n, U_n)$ - solvable if there exist integers $i_1, i_2, \ldots, i_n$ such that*

$$\begin{aligned} a_{11}i_1 + a_{12}i_2 + \ldots + a_{1n}i_n &= a_{10} \\ a_{21}i_1 + a_{22}i_2 + \ldots + a_{2n}i_n &= a_{20} \\ &\ldots\ldots \\ a_{m1}i_1 + a_{m2}i_2 + \ldots + a_{mn}i_n &= a_{m0} \end{aligned}$$

and for every k', $1 \le k' \le n$ we have $L_{k'} \le i_{k'} \le U_{k'}$.

b). Let $a_1, \ldots, a_n$, L and U be integers. An equation of the form

$$a_1I_1 + a_2I_2 + \ldots + a_nI_n = [L, U]$$

is called an interval equation *and it has solution if exists $a_0 \in [L, U]$ so that $a_1I_1 + a_2I_2 + \ldots + a_nI_n = a_0$ be I-solvable.*

Let's consider the perfect loop nest below in which the two references at the m-dimensional array A cause a potential data dependence.

```
for j1 := inf1 to sup1 do
    for j2 := inf2 to sup2 do
        . .   .
        for jr := infr to supr do
        . .   .
B := A[f1(j1,...jr),f2(j1,...jr),...,fm(j1,...jr)] ... A[g1(j1,...jr),g2(j1,...jr),...,gm(j1,...jr)] := ...
        .   .   .
        end for
            .   .   .
    end for
end for
```

We assume for simplicity that :

- $\forall k, 1 \le k \le r$, inf_k and sup_k are integers and that $inf_k \le sup_k$;
- $\forall k, 1 \le k \le r$, f_k and j_k are linear functions of the form:

(1) $$f_k(j_1, \ldots, j_r) = c_{k1}j_1 + c_{k2}j_2 + \ldots + c_{kr}j_r + c_{k0}, \text{ and}$$
$$g_k(j_1, \ldots, j_r) = d_{k1}j_1 + g_{k2}j_2 + \ldots + d_{kr}j_r + d_{k0}$$

respectively, where $c_{ki}, d_{ki} \in Z, 0 \le i \le r$.

Let $\gamma = (inf_1,sup_1;\ inf_2,sup_2;\ \ldots\ ;inf_r,sup_r;\ inf_1,sup_1;\ inf_2,sup_2;\ \ldots\ ;inf_r,sup_r)$. Between the two references at the m-dimensional array A exists a data dependence if and only if the following equations set is γ-solvable:

$$(2)\qquad \begin{array}{l} f_1(j_1', j_2', \ldots, j_r') - g_1(j_1'', j_2'', \ldots, j_r'') = 0 \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ f_m(j_1', j_2', \ldots, j_r') - g_m(j_1'', j_2'', \ldots, j_r'') = 0 \end{array}$$

The practical method of testing for a dependence assumes conservatively the dependence until at least one of the system's equation proves to be I-unsolvable, at which moment the independence is proved. Such an approach is called *subscript-by-subscript testing* [Bane88].

So, we choose an arbitrary equation from the set (2) which taken into account (1), we write as:

$$c_{k1}j_1 + \ldots + c_{kr}j_r + c_{k0} - d_{k1}j_1 - \ldots - d_{kr}j_r - d_{k0} = 0$$

Eliminating the possible 0 coefficient terms and simplifying the notation (factorizing on the index values), subscript by subscript testing means to determine if an equation of the form

$$(3)\qquad a_1 I_1 + a_2 I_2 + \ldots + a_n I_n = a_0$$

is $(L_1,U_1;\ L_2,U_2;\ \ldots\ ,L_n U_n)$-solvable, where $n \le 2r$ and $a_k \neq 0,\ 1 \le k \le n$.
The generalization of the GCD test [Bane97] for interval equations is given below without proof, which is immediately.

Theorem 1. *Let $a_1, a_2, \ldots, a_n$ be nonzero integers and let U, L be integers. Let*

$$g = gcd(a_1, a_2, \ldots, a_n)$$

The interval equation $a_1x_1 + a_2x_2 + \ldots + a_nx_n = [L, U]$ is I-solvable iff $\lceil L/g \rceil \le \lfloor U/g \rfloor$.
In [Bane76] it is shown that in the particular case in which all the coefficients of the dependence equation have an absolute value of 1, the Banerjee test is exact, in the sense that it solves the problem of the integer *solutions for the considered interval. Li et al. arrived at the same conclusion [Li90] in the situation in which a coefficient of the dependence equation has an absolute value of 1 and*

$$\max_{1 \le i \le n}(|a_i|) \le U_k - L_k + 1$$

In [Psa91] it is given a sufficiency condition for the exactness of the Banerjee test, condition which proves to be far less restrictive than that of Li et al.

The positive part a^+ and *the negative part* a^- of a real number a are defined as simplifying notations in [Bane88]:

$$a^+ = max(a, 0)$$
$$a^- = max(-a, 0)$$

The lower and upper bounds computed by the Banerjee test [Bane97] are in this context

$$(4) \qquad l_{inf} = \sum_{i=1}^{n} (a_i^+ L_i - a_i^- U_i)$$
$$l_{sup} = \sum_{i=1}^{n} (a_i^+ U_i - a_i^- L_i)$$

The exact formulation of the Banerjee test as in [Bane97] tells us that if

$$a_0 \in [l_{inf}, l_{sup}]$$

then exist ***real*** numbers $r_1, r_2, \ldots, r_n$ such that

- $a_1 r_1 + a_2 r_2 + \ldots + a_n r_n = a_0$ and
- $\forall i \in N,\ 1 \le i \le n,\ L_i \le r_i \le U_i$.

The sufficiency condition will refer to the integer values assumed by the expression $a_1 l_1 + \ldots + a_n l_n$.

Theorem 2. *Let $a_1, a_2, \ldots, a_n$ be nonzero integers. For each k, $1 \le k \le n$, let L_k and U_k be integers with $L_k < U_k$. If there exists a permutation π of the set $\{1, 2, \ldots, n\}$ such that*

- $|a_{\pi(1)}| = 1$ *and*
- *for each j,* $2 \le j \le n$,

$$|a_{\pi(j)}| \le 1 + \sum_{k=1}^{j-1} |a_{\pi(k)}| (U_{\pi(k)} - L_{\pi(k)})$$

then for each integer x from the interval

$$[\sum_{i=1}^{n} (a_i^+ L_i - a_i^- U_i),\ \sum_{i=1}^{n} (a_i^+ U_i - a_i^- L_i)]$$

there exist integers $x_1, x_2, \ldots, x_n$ such that

- $a_1 x_1 + a_2 x_2 + \ldots + a_n x_n = x$, *and*
- *for each i,* $1 \le i \le n$, $L_i \le x_i \le U_i$.

Proof. The proof can be found in [Psa91].

Theorem 3, which follows immediately from theorem 2, shows that the hypothesis of theorem 2 is a sufficient condition for the Banerjee test to be exact, that is for checking **integer** solutions between the loop limits and not just the real ones.

Theorem 3. *Let a_0 be an integer and $a_1, a_2, \ldots, a_n$ nonzero integers. For each k, $1 \le k \le n$, let L_k and U_k be integers such that $L_k < U_k$. If there exists a permutation π of the numbers $\{1, 2, \ldots, n\}$ such that*

- $|a_{\pi(1)}| = 1$ *and*
- *for each j,* $2 \le j \le n$,

$$|a_{\pi(j)}| \le 1 + \sum_{k=1}^{j-1} |a_{\pi(k)}| (U_{\pi(k)} - L_{\pi(k)})$$

then

$$a_0 \in [\sum_{i=1}^{n} (a_i^+ L_i - a_i^- U_i), \sum_{i=1}^{n} (a_i^+ U_i - a_i^- L_i)]$$

if and only if

$$a_1 I_1 + a_2 I_2 + \ldots + a_n I_n = a_0$$

is $(L_1, U_1; L_2, U_2; \ldots; L_n, U_n)$ *– solvable, that is, the Banerjee test looks for* ***integer*** *solutions for the equation* $a_1 I_1 + a_2 I_2 + \ldots + a_n I_n = a_0$ *between the loop limits.*

Empirical results reported in [Shen90] shows that the number of iterations for a cycle is relatively high, but the coefficients of the dependence equations have in general small values, frequently 1. These empirical results combined with the formal result of theorem 3 demonstrates that the Banerjee test proves to be exact in practice.

The sufficiency theorem suggests that the practical exact application of the Banerjee test has a complexity which is at least exponential (factorial of the number of a_i values) because in the worst case we have to consider all the possible permutations of the coefficients values. But, if the conditions of theorem 3 are satisfied by an arbitrary permutation of the a_i values, these conditions will be satisfied also by the permutation which has these values sorted in *ascending order*. Thus, we have the following theorem as a consequence which shows that once the coefficients are properly sorted, the actual testing could be done in linear time with respect to the number of the coefficients.

Theorem 4. *Let* a_0 *be an integer and* $a_1 \leq a_2 \leq \ldots \leq a_n$ *nonzero integers. For every* k, $1 \leq k \leq n$, *let* L_k *and* U_k *be integers such that* $L_k < U_k$. *If*

- $|a_1| = 1$, *and*
- *for each* j, $2 \leq j \leq n$,

$$|a_j| \leq 1 + \sum_{k=1}^{j-1} |a_k| (U_k - L_k)$$

then

$$a_0 \in [\sum_{i=1}^{n} (a_i^+ L_i - a_i^- U_i), \sum_{i=1}^{n} (a_i^+ U_i - a_i^- L_i)]$$

if and only if

$$a_1 I_1 + a_2 I_2 + \ldots + a_n I_n = a_0$$

is $(L_1, U_1; L_2, U_2; \ldots; L_n, U_n)$ *– solvable, that is, the Banerjee test looks for integer solutions for the equation* $a_1 I_1 + a_2 I_2 + \ldots + a_n I_n = a_0$ *between the loop limits.*

3. The sufficiency condition is also necessary

Definition 3. *If S and S' are integer sets we define addition of integer sets as follows:*

$$S + S' = \{\, s+s' \mid s\in S \text{ şi } s'\in S' \,\}$$

Let's notice that if $S = [L,U]$ *and* $S' = \{s_1, s_2, \ldots, s_n\}$ *then we will have*

$$[L,U] + S' = \bigcup_{i=1}^{n} [L + s_i,\ U + s_i]$$

The result of the next lemma is evident.

Lemma 1. *If* $M \le x \le N$ *then* $a^+M - a^-N \le ax \le a^+N - a^-M$ *These limits are the extreme values of the function* $f(x) = ax$ *in the region specified by the inequalities* $M \le x \le N$.

Lemma 2. *Let* $[L,U]$ *be an integer interval with integer limits and let* a, M *and* N *be integers such that* $M < N$ *and we let* $S = \{\, ax \mid x \in Z,\ M \le x \le N \,\}$. *Then*

$$[L,U] + S = [L + a^+M - a^-N,\ U + a^+N - a^-M]$$

if and only if $\quad |a| \le U - L + 1.$

Proof. For a = 0 lemma is trivially verified. Let $a > 0$. Then, taking into account the definitions of positive and negative parts of a number we have

$$[L + a^+M - a^-N,\ U + a^+N - a^-M] = [L + aM,\ U + aN]$$

The general form for an element from S is $aM + ka$, with $k = 0, \ldots, N - M$. Based on definition 3 we have

$$[L,U] + S = \bigcup_{k=0}^{N-M} [L + aM + ka,\ U + aM + ka] \tag{5}$$

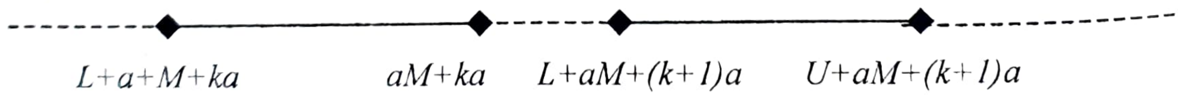
So, we have to prove that

$$[L,U] + S = [L + aM,\ U + aN] \Leftrightarrow \quad a \le U - L + 1.$$

Let's notice that $L + aM = L + aM + ka,\ for\ k = 0$

$$U + aN = L + aM + ka, \text{ for } k = N - M$$

which means we have to determine a necessary and sufficient condition for the reunion (5) to be an interval. This happens if and only if every two succesive intervals are not disjoint or, in the worst case, are disjoint but adiacent, like in the illustration below



which brings us to the necessary and sufficient condition

$$L + aM + (k+1)a \le U + aM + ka + 1 \quad \Leftrightarrow \quad a \le U - L + 1$$

The case $a < 0$ is analogous and we obtain $-a \le U - L + 1$. Combining the results we obtain

$$[L,U] + S = [L + a^+M - a^-N,\ U + a^+N - a^-M]$$

if and only if $\quad |a| \le U - L + 1$, *q.e.d.*

Using the result of lemma 2 we show further that the sufficiency condition of theorem 4 is also a necessity condition for the exactness of the Banerjee test.

Theorem 5. *Let $a_1, a_2, \dots, a_n$ be nonzero integers and $\forall\, k \in N,\ 1 \le k \le n$, let L_k and U_k be integers such that $L_k < U_k$. If for every x from the interval*

$$[\sum_{i=1}^{n}(a_i^+L_i - a_i^-U_i), \sum_{i=1}^{n}(a_i^+U_i - a_i^-L_i)]$$

there exist integers $x_1, x_2, \dots, x_n$ such that

- $a_1x_1 + a_2x_2 + \dots + a_nx_n = x$ *and*
- $\forall\, i \in N,\ 1 \le i \le n,\ L_i \le x_i \le U_i$

then there exists a permutation π of the set $\{1, 2, \dots, n\}$ such that

(i) $|a_{\pi(1)}| = 1$ *and*

(ii) $\forall\, j \in N,\ 2 \le j \le n \quad |a_{\pi(j)}| \le 1 + \sum_{k=1}^{j-1} |a_{\pi(k)}|(U_{\pi(k)} - L_{\pi(k)})$

Proof. We will demonstrate by induction upon n. Let $n = 1$. By hypothesis we have that for every x from the interval

$$[a_1^+L_1 - a_1^-U_1, a_1^+U_1 - a_1^-L_1]$$

there exists an integer x_1, $L_1 \le x_1 \le U_1$, such that $a_1x_1 = x$, from where we have

$$[a_1^+L_1 - a_1^-U_1, a_1^+U_1 - a_1^-L_1] \subset \bigcup_{L_1 \le x_1 \le U_1}[a_1x_1, a_1x_1]$$

On the other hand, by lemma 1 it follows that

$$\bigcup_{L_1 \le x_1 \le U_1}[a_1x_1, a_1x_1] \subset [a_1^+L_1 - a_1^-U_1, a_1^+U_1 - a_1^-L_1]$$

so we will have the equality:

$$\bigcup_{L_1 \le x_1 \le U_1}[a_1x_1, a_1x_1] = [a_1^+L_1 - a_1^-U_1, a_1^+U_1 - a_1^-L_1]$$

from where applying lemma 2 in which $L = U = 0$ we have that $|a_1| = 1$, so for $n = 1$ the conclusion is verified. We may assume now that the theorem is true for $n\text{-}1$ and we will get the conclusion for n. Let

(6)
$$L = \sum_{i=1}^{n-1}(a_i^+L_i - a_i^-U_i)$$
$$U = \sum_{i=1}^{n-1}(a_i^+U_i - a_i^-L_i)$$

By the induction hypothesis it results that a permutation π' of the set $\{1, 2, \ldots, n-1\}$ exists such that

$$|a_{\pi'(1)}| = 1$$

and

$$\forall j \in N,\ 2 \le j \le n-1 \qquad |a_{\pi'(j)}| \le 1 + \sum_{k=1}^{j-1} |a_{\pi'(k)}| (U_{\pi'(k)} - L_{\pi'(k)})$$

We define a permutation π of the set $\{1, 2, \ldots, n\}$ as

$$\pi(i) = \begin{cases} \pi'(i), & \text{if } \ 1 \le i \le n-1 \\ n, & \text{if } \ i = n \end{cases}$$

and we have to show (for the rest of the values the relation is true by the induction hypothesis) that

$$|a_{\pi(n)}| \le 1 + \sum_{k=1}^{n-1} |a_{\pi(k)}| (U_{\pi(k)} - L_{\pi(k)})$$

but since $\pi(n) = n$, this brings us to show that

(7)
$$|a_n| \le 1 + \sum_{i=1}^{n-1} |a_i| (U_i - L_i)$$

The theorem hypothesis says that for every x from the interval

$$[L + a_n^+ L_n - a_n^- U_n, U + a_n^+ U_n - a_n^- L_n]$$

there exist the integers $x_1, x_2, \ldots, x_n$ such that

$$a_1 x_1 + a_2 x_2 + \ldots + a_n x_n = x, \text{ and}$$
$$\forall i \in N,\ 1 \le i \le n,\ L_i \le x_i \le U_i.$$

This hypothesis combined with lemma 2 and with the Banerjee's equations (4) for computing the limits for a sum shows that for every x from

$$[L + a_n^+ L_n - a_n^- U_n, U + a_n^+ U_n - a_n^- L_n]$$

there exists integers $w = \sum_{i=1}^{n-1} a_i x_i$ and x_n, such that

$$x = w + a_n x_n;$$
$$L \le w \le U;$$

$$L_n \le x_n \le U_n;$$

It follows then that we have the relation

$$[L + a_n^+ L_n - a_n^- U_n, U + a_n^+ U_n - a_n^- L_n] \subset \bigcup_{x_n = L_n}^{U_n} [L + a_n x_n, U + a_n x_n]$$

On lemma 1 we obtain immediately also that

$$\bigcup_{L_n \le x_n \le U_n} [L + a_n x_n, U + a_n x_n] \subset [L + a_n^+ L_n - a_n^- U_n, U + a_n^+ U_n - a_n^- L_n]$$

so it follows the equality

$$\bigcup_{x_n = L_n}^{U_n} [L + a_n x_n, U + a_n x_n] = [L + a_n^+ L_n - a_n^- U_n, U + a_n^+ U_n - a_n^- L_n]$$

and now we can apply lemma 2, from where it results

$$|a_n| \le U - L + 1$$

which means

$$|a_n| \le 1 + \sum_{i=1}^{n-1} |a_i| (U_i - L_i)$$

Now the relation is verified for every a_j with $j = 2,\ldots,n$ and so the theorem is proved.

4. Conclusions

The GCD test and the Banerjee test are the two tests commonly used in order to determine whether a dependence equation has an integer solution subject to the constraints imposed by the loop limits. The GCD test computes the greatest common divisor of the coefficients of the dependence equation. If the gcd does not divide the RHS then the conclusion of the test is that there is ***no dependence***, otherwise there ***may be a dependence.*** The Banerjee test computes the extreme values assumed by the LHS expression under the constraints imposed by the loop limits. If $\min \le a_0 \le \max$ is not true then we conclude ***no dependence***, otherwise there ***may be a dependence***. So, it is obvious that neither the GCD test nor the Banerjee test provide any definite data dependence information.

In lemma 2 and theorems 4 and 5 we have presented sufficient and necessary conditions for the Banerjee test to be exact. Taking also into account the empirical practical evaluations of the forms of the data dependence equations we can conclude that in practice the application by a parallelizing compiler of the Banerjee test together with the conditions stated in this paper proves to be an exact method.

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