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A NOTE ON NON-MONOTONIC LOGICS

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Dedicated to Professor Emil Muntean on his 60[th] anniversary

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Rezumat: Notă asupra logicilor nemonotone. Raţionamentul aproximativ e deosebit de interesant pentru că modelează mai exact reprezentarea şi tratarea cunoştinţelor în cazul informaţiilor incomplete Această lucrare introduce o modalitate de a obţine teoreme pornind de la astfel de cunostinţe (knowledge) incomplete, similar cu deducţiile în cazul clasic al logicii de ordinul întâi Pentru cazul teoriilor normale, se demonstrează că problema e complet reductibilă la cazul clasic

1. Introduction The classical logics are inadequate to capture the tentative nature of human reasoning Since people's knowledge about the world is necessarily incomplete, there will be times when we could be forced to draw conclusions based on an incomplete specification of pertinent details of the situations Under such circumstances, assumptions are made (implicitly or explicitly) about the state of the unknown factors Because these assumptions are not irrefutable, they may have to be withdrawn at some later time,if new evidence prove them invalid If this happens, the new evidence will prevent some assumptions from being made, hence all conclusions which can be arrived at only in conjunction with those assumptions will no longer be derivable

In common-sense reasoning, assumptions are often based on both supporting evidence and the absence of contradictory evidence Traditional logics cannot emulate this form of

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reasoning, because they lack any tools for considering the absence of knowledge

Non-monotonic logic has been developed to deal with reasoning about incomplete informations There are four major formalizations of non-monotonic reasoning

- McCarty's circumscription [1]
- Moore's autoepistemic logic [4]
- Reiter's default logic [5]
- McDermott and Doyle non-monotonic logic [2],[3]

Reiter's default logic [5] is one of the most proeminent formalizations of non-monotonic reasoning One of the reasons for its attractiveness is the simplicity and naturalness of its underlying idea This logic represents defaults as certain type of inference rules whose applicability does not only depend on the derivability, but also on the underivability of some formulas

Classical logic deals with the formalization of absolutely correct forms of reasoning The aim of this note is to prove that,

in the normal context, the problem is completely reducible to classical case The deductive systems of logic allow us to formalize reasoning of rigurous proof of theorem and to infer conclusions from premises It defines a deduction relation between formulas, denoted by $\vdash$ This relation has the following properties [6]

- reflexivity

  $U_1, U_2, \ldots, U_n, V \vdash V$

- monotonicity

  if $U_1, U_2, \ldots, U_n \vdash V$ then $U_1, U_2, \ldots, U_n, Z \vdash V$

- transitivity

$$\text{if } U_1, U_2, \ \ ,U_n \vdash V \text{ and } U_1, U_2, \ \ ,U_n, V \vdash Z$$

$$\text{then} \quad U_1, U_2, \ \ ,U_n \vdash Z$$

where $U_1, U_2, \ \ ,U_n, V, Z$ are the formulas in first-order logic

2. Default logic The property of monotonicity tell us that a derived result cannot be invalidated by further results Also, the inference rules in deductive systems of classical logic are permissive They are always of the form $U_1, U_2, \ \ ,U_n \vdash_{r_k} V$ with the significance "If $U_1, U_2, \ \ ,U_k$ are theorems, then by rule r_k (of anty k) it results that V is a theorem "

A system which should be able to model non-monotonic resoning should also contain restrictive rules, of the form

" V is a theorem if $U_1, U_2, \ \ ,U_k$ are not theorems "

Default logic allows formalizing default reasoning by means of particular inference rules, called **defaults A default** has the form $\frac{\alpha \, M\beta}{\gamma}$ and is interpreted as follows "if one belives α and if is consistent to belive β, then one can also belives γ"

A default theory will comprise, besides the default rules, a set of closed formulas of predicate logic which represent the basic knowledge and are treated as axioms

<u>Definition 1</u> A **default theory** T is a pair (D,F) where

(i) D is a set of defaults (***d***) $\frac{\alpha \, M\beta_1, \ \ ,M\beta_m}{\gamma}$, and $\alpha, \beta_1, \ \ ,\beta_m, \gamma$ are closed formulas in first-order logic

(ii) F is a set of closed formulas in first-order logic

- α is called the **prerequisite** of default

- γ is called the **consequent** of default

We denote by *Pre(d)* the prerequisite α of the default $d \in D$,and by *Cons(d)* the consequent γ of the same d Similary,we introduce $Pre = \bigcup_{d \in D} Pre(d)$

<u>Definition 2</u> An **extension of default theory** T is any set of all formulas that can be infered by means of the classical inference rules or by means of the defaults We will denote this set by *Th(D,F)* and we will call them the **set of theorems** of $T=(D,F)$

A default theory can have an empty extension However, it can be proved [5] that a non-empty extension exists for so called normal default theories, which all defaults have the form $\frac{\alpha\ M\beta}{\beta}$

By analogy with the definition of a deduction for a formula U, and in accordance with definition 1 and definition 2, we can introduce the

<u>Definition 3</u> Let $T=(D,F)$ be a default theory, and U and V two set of formulas in the first-order logic We denote $U \vdash V$ (and we call this V is non-monotonic deductible from U) if V is obtained from U either by application of a classical inference rule (like modus ponens, for example) or by a default rule In this last case, U contains α and V contains β, if the normal default applied is. (d) $\frac{\alpha\ M\beta}{\beta}$

We can specify that the default d is applied by denoting

$U \vdash_d V$ or $U \vdash V$ by rule d Now, we are ready to define the concept of a proof for a formula U according to a default theory $T=(D,F)$

<u>Definition 4</u> A formula U is a **theorem** in a default theory $T=(D,F)$ (or, $U \in Th(D,F)$) if it exists a finite sequence of set of formulas $U_0, U_1, \ldots, U_n$, such that

$U_0 = F$, $U_1 = F \cup \{\alpha\}$, $\alpha \in Pre$, $U \in U_n$ and

a) $U_i \vdash U_{i+1}$, i=1,2, ,n-1

b) U_i is consistent,i=1,2, ,n (therefore U_i does not contain a formula V and his logical negation $\neg V$)

Observation: The sequence $U_0, U_1, \ldots, U_n$ has the property

$U_0 \subseteq U_1 \subseteq \quad \subseteq U_n$

3. The main result Example Let *T=(D,F)* be the normal default theory having the following set of premises

(i) F={ C -> D , A $\wedge$ B -> E , E $\vee$ D , D -> G } and

(ii) D={ d_1, d_2, d_3, d_4 } as

$(d_1)\quad \dfrac{E \vee G \quad M(A \wedge G)}{A \wedge G}$

$(d_2)\quad \dfrac{A \quad MB}{B}$

$(d_3)\quad \dfrac{A \wedge E \quad MC}{C}$

$(d_4)\quad \dfrac{M\rceil E}{\rceil E}$

According to definition 4,a proof for U=D may be the following

1) $U_0 = F$,

2) $U_1 = F \cup \{ E \vee G \}$,

3) $U_2 = U_1 \cup \{ A \wedge G \}$, $U_1 \vdash U_2$ by rule d_1,

4) $U_3 = U_2 \cup \{ A , G \}$, $U_2 \vdash U_3$ by rule $\dfrac{A \wedge G}{A, G}$,

5) $U_4 = U_3 \cup \{ B \}$, $U_3 \vdash U_4$ by rule d_2,

6) $U_5 = U_4 \cup \{ A \wedge B \}$, $U_4 \vdash U_5$ by rule $\frac{A,B}{A \wedge B}$,

7) $U_6 = U_5 \cup \{ E \}$, $U_5 \vdash U_6$ by rule $\frac{A \wedge B, A \wedge B \rightarrow E}{E}$,

8) $U_7 = U_6 \cup \{ A \wedge E \}$, $U_6 \vdash U_7$ by rule $\frac{A,E}{A \wedge E}$,

9) $U_8 = U_7 \cup \{ C \}$, $U_7 \vdash U_8$ by rule d_3,

10) $U_9 = U_8 \cup \{ D \}$, $U_8 \vdash U_9$ by rule $\frac{C, C \rightarrow D}{D}$

As $D \in U_9$, $U_0, U_1,$, U_9 is a proof for D

The following theorem emphasizes a conection between the relation $\vdash$ and the classical relation $\models$ of deductibility in the first-order logic

<u>Theorem.</u> If $T=(D,F)$ is a normal default theory then $U \in Th(D,F)$ iff $F,P \models U$ where P is the set of formulas defined as

$$"\alpha\text{->}\beta \in P \text{ iff } \frac{\alpha \quad M\beta}{\beta} \in D"$$

Proof: The direct implication results by induction about the number k of utilised defaults

If k=0, then we have $F \models U$ and thus $F,P \models U$

Let $U \in Th(D,F)$ such that for U are applied k+1 defaults If the last default is

(d) $\frac{\alpha \quad M\beta}{\beta}$, then $U(=\beta) \in U_n$,

$U_{n-1} \vdash_d U_n$, and $\alpha \in U_{n-1}$ By induction hypotesis, as for α are applied k defaults, $F,P \models \alpha$ As $\alpha\text{->}\beta \in P$, we obtain

$F,P \models \beta(=U)$

By analogy,the converse implication can be proved

Observation: If a default theory is normal, then a deduction in this theory can be simulated as usual way in first-order theory

A similar theorem can be proved for the seminormal default theories [5]

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