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CONSEQUENCES OF THEOREMS CONCERNING THE CONVERGENCE OF CHORD METHOD

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Dedicated to Professor Emil Muntean on his 60[th] anniversary

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REZUMAT. - Consecințe ale teoremelor privind convergența metodei coardei. Lucrarea își propune de a pune în evidență câteva consecințe ale unei teoreme de convergență ale metode coardei

$$x_{n+1} = x_n - \Lambda_n P(x_n)$$

metodă folosită în rezolvarea ecuației $P(x) = \theta$, unde $P: X \to Y$, X, Y fiind spații Fréchet

1. In this paper some consequences of the convergence of Chord method are given

Let be the equation

$$P(x) = \theta \tag{1}$$

where $P: X \to Y$ is a continuous nonlinear mapping, X and Y Fréchet spaces [3], $\theta \in Y$ the null element of the space

Let be any $x_0, x_{-1} \in D \subset X$ and $\Lambda_n = \left[x_n, x_{n-1}, P\right]^{-1}$ the generalized divided quotient [2] of P

Starting from the initial approximation x_0, x_{-1} and using the algorithm

$$x_{n+1} = x_n - \Lambda_n P(x_n) \tag{2}$$

known as "the Chord method", the sequence (x_n) is generated, each term of it being an

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approximate of the solution of (1)

Obviously, the Chord method cannot be applied in the following two situations

a) applying the algorithm (2), x_n terms of sequence which are not in D are generated,

b) the mapping $[x_n, x_{n-1}, P]^{-1}$ does not exist

To apply the iterative method (2) at each step, the mapping $[x_k, x_{k-1}, P]^{-1}$ is needed To avoid this inconvenient, a "modified" method may be applied

$$x_{n+1} = x_n - [x_0, x_{-1}; P]^{-1} P(x_n) \tag{2'}$$

which, to generate the (x_n) approximations, uses only the mapping

$$\Lambda_0 = [x_0, x_{-1}, P]^{-1}$$

Although it gives a "weaker" approximation that (2), it is often use in practice

We mention that both the Chord method (2) and the modified one (2') applied to the approximative solving of equation (1) are identical with the succesive approximations method

$$x_{n+1} = A(x_n) \qquad (n = 0, 1, \ ) \tag{3'}$$

applied to the equations equivalent with (1), respectively

$$x = x - [x^{(1)}, x^{(2)}, P]^{-1} P(x) \tag{3_1}$$

and

$$x = x - [x_0, x_{-1}, P]^{-1} P(x) \tag{3_2}$$

Concerning the convergence of Chord method, in [1] the following theorem is proved

THEOREM A *If the following conditions are satisfied for initial approximates* $x_0, x_{-1} \in X$

1 $\Lambda_0 = [x_0, x_{-1}, P]^{-1}$ *exists*,

2 $\| \Lambda_0 P(x_i) \| \le \eta_i, \ i = 0, -1$ *and* $\eta_0 < 1/4 \eta_{-1}$;

3 $\| \Lambda_0 [u,v,w,P] \| \leq \tilde{K}, \quad \forall\ u,v,w \in S(x_0, 5/4\eta_{-1}),$

4 $\tilde{h}_0 = \tilde{K}\eta_{-1} \leq 1/4,$

then the equation (1) *has at least one solution* $x^* \in S$, *which is the limit of sequence* (x_n) *generated by* (2), *the order of convergence being*

$$\| x^* - x_n \| \leq \frac{1}{2^{s_n-1}} q^{s_{n-1}} (4h_0)^{s} \eta_0 \tag{4}$$

where $0 < q < 1$, *and* s_n *is the general term of the sequence of partial summas of Fibonacci sequence, with* $u_1 = u_2 = 1$

2. In the following, we modify the hypothesis concerning the existence of mapping $\Lambda_0 = \left[x_0, x_{-1}, P\right]^{-1}$, using another mapping, connected with it

We probe the following

THEOREM 1 *Supposing the existence of any continuous linear mapping* $\Lambda \in (Y,X)^{\#}$ *which has an inverse and the following conditions fulfilled for initial approximates* $x_0, x_{-1} \in S \subset X$

1^0 $\| \Lambda P(x_i) \| \leq \bar{\eta}_i, \quad i = 0, -1 \quad$ *and* $\quad \bar{\eta}_0 \leq 1/4\, \bar{\eta}_{-1},$

2^0 $\| \Lambda\left[x_0, x_{-1}\ P\right] - I \| \leq a < 1$, *I beeing the identical mapping,*

3^0 $\| \Lambda [u,v,w,P] \| \leq \bar{K}, \forall\ u,v,w \in S\left(x_0, 5/4\, \bar{\eta}_{-1}\right),$

4^0 $\bar{h}_0 = \dfrac{\bar{K}\eta_{-1}}{(1-a)^2} \leq 1/4$

then the equation (1) *has a solution* $x^* \in S$, *which is the limit of sequence* (x_n) *generated by* (2), *the order of convergence being*

$$\| x^* - x_n \| < \frac{1}{2^{s_n-1}} q^{s_n} (4\bar{h}_0)^{s_n} \bar{\eta}_0 \tag{5}$$

where s_n *and* q *has the significance given bellow*

Proof We show that, from the hypothesis of theorem 1, the conditions of theorem A follow

Hypothesis 2^0 of theorem 1 implies, based on Banach's theorem, the existence of mapping

$$H = (\Lambda [x_0, x_{-1}, P])^{-1} \tag{6}$$

which for

$$)|H|(\le \frac{1}{1-a}$$

It follows the existence of

$$H\Lambda = \Lambda_0 = [x_0, x_{-1}, P]^{-1}$$

so the condition 1^0 of theorem A is verified

To fulfill the condition 2^0 of the same theorem, we consider

$$)|\Lambda_0 P(x_i)|(=)|H \Lambda P(x_i)|(\le$$
$$\le)|H|(\cdot)|\Lambda P(x_i)|(\le \frac{\bar{\eta}_i}{1-a}, \quad i = 0, -1$$

Changing η_i respectively with $\dfrac{\eta_i}{1-a}$, $i = 0, -1$, we obtain te condition 1^0 of theorem A

In order to obtain the condition 3^0 of theorem A, we have

$$)|\Lambda_0 [x^{(1)}, x^{(2)}, x^{(3)}; P]|(\le)|H \Lambda [x^{(1)}, x^{(2)}, x^{(3)}, P]|(\le$$
$$\le \frac{\bar{K}}{1-a}$$

so $\tilde{K}$ corresponds to $\dfrac{\bar{K}}{1-a}$

According with the expressions for K and η_{-1} we may evaluate $\bar{h}_0$, so the condition 4 of theorem A

Then due to theorem A, it results the existence of solution for equation (1), which is the limit of sequence generated by (x_n), the rapidity of convergence being given by (5)

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