Calibration

and

Sensitivity Analysis

5.04.2017

Calibration

- Calibration = adjustment parameters for a model closer to reality

- What factors are important in the simulation model?
  - of 390 only 23 were considered major (136 samples)
  - of 400 - only 15 -

- Examples of factors that can be investigated by Sensitivity Analysis:
  - parameter values;
  - choice of distributions;
  - system entities;
  - the details of the subsystems;
  - crucial data in the simulation.

- *Sensitivity Analysis:*
  - Let *I* = input,
  - *O* = output.

Sensitivity is:

$$S = dO / dI$$

- A great value for S indicates a high sensitivity of O from I.

- If O is an important result in the simulation and S is high then pay attention to this aspect of modeling.

- If S is small then the model must remain simple.

- **Calibration - Sensitivity Analysis**

 Sensitivity Analysis will generate an automated report indicating the Most Sensitive Parameters from the complete chosen list

 Decide upon Most Important Processing Parameters, say the top 3 to 10 of them

 Decide upon most Important Calibration Parameters, say the top 3 or 10 of them

 These numbers depend upon available computer power

- *Definition*:

Calibrating is the process of tuning a model to fit detailed real data.

or

Calibration - a test of the model with known *input* and *output* information that is used to adjust or estimate factors for which data are not available.

This is a multi-step, often iterative, process in which the model’s processes are altered so that the model’s predictions come to fit, with reasonable tolerance, a set of detailed real data. This approach is generally used for establishing the feasibility of the computational model; i.e., for showing that it is possible for the model to generate results that match the real data. This approach is more often used with emulation than with intellective models.

Calibrating a model may require the researcher to both set and reset parameters and to alter the fundamental programming, procedures, algorithms, or rules in the computational model.

Calibrating establishes, to an extent the validity of the internal workings of the model and its results (at least in a single case). The researcher may choose to halt calibration after achieving either a parameter or process level of validation.

To calibrate a model the researcher begins with the uncalibrated model. Then a trace of the model's predictions and the processes that generated them is generated. This information is then checked against real data. If the simulated predictions of the dependent variable(s) matches the real dependent variable the model is considered to be calibrated. Otherwise, first the parameters and then the processes are checked for accuracy.

This check may involve going back and talking to experts at doing the task the model seeks to simulate or collecting new observational detail to fill in details or to check the accuracy of the original real data. Once both parameters and processes are accurate, if the model predictions are still not matching the real data, the modeler typically moves to adding additional lower level or auxiliary processes that were originally thought to be less important.

Calibration occurs at two levels.

- At one level, the models predictions are compared against real data. This can be characterized as analysis of the dependent variable(s).
- At another level, the processes and parameters within the model are compared with data about the processes and parameters that produced the behavior of concern. This can be characterized as analysis of the independent (and control) variable(s). To calibrate a model it is important to have access to detailed data on one or more cases. Participant observation or other ethnographic data is often the best possible data for calibrating as typically only such data provides the level of detail needed by the modeler at both the process and outcome level. Calibrating models of subject matter experts typically requires interacting with an expert and discussing whether or not the model matches in its reasons and its results the behavior of the expert, and if not, why not.

In calibrating a model, the level of match required between the model and the real data depends in part on the **research goals**. The level of match also depends on the quality of the real data and the degree to which that data does not represent a pathologic or extreme data point. How should the cases for calibrating the computational model be chosen? The ideal is to use a set of cases that span the key categories across which the model is expected to operate. The next best option is to choose two to four cases that represent typical behavior and one to two that represent important extremes.

The basic idea here is that by looking at both the typical and the extreme the boundaries on processes, parameters and outcomes can be set with some degree of confidence. In practice, however, the researcher who wishes to calibrate a model is often lucky to even have one case with sufficient detail.

That case, moreover, is often more a matter of opportunity than plan. Critics of calibration often argue that any model with sufficient parameters can always be adjusted so that some combination of parameters generates the observed data. Thus, the argument proceeds, calibration does not establish the validity of a model in a meaningful way. At one level, this criticism has some truth in it for some models. In particular, large multiparameter models often run the risk of having so many parameters that there is no guarantee that the model is doing anything more than curve fitting.

For many computational models this criticism is less appropriate. In particular, for models where the process is represented not by parameterized equations but by rules, interactive processes, or a combination of procedures and heuristics there are often few if any parameters. There is no guarantee that a sufficiently large set of procedure and heuristics, that often interact in complex and non-linear ways, can be altered so that they will generate the observed data. For procedural models, calibration becomes a process of altering ‘‘how things are done’’ rather than ‘‘how things are weighted.’’ This distinction is critical as it separates process matching from curve fitting.

Problem: We have *real values:*

$$r_1, r_2, \ldots, r_n$$

and the *simulation values:*

$$s_1, s_2, \ldots, s_m$$

What do we mean by, or how do we decide if they "match"?

What if we do not have real values?

(Expert opinions).

- *Regression*

$$y = f(x_1, x_2, \ldots, x_n) =$$

$$= c_1 * x_1 + c_2 * x_2 + \ldots + c_n * x_n$$

k samples

$$\sum_{i=1,k} (c_1 * x_{1,i} + c_2 * x_{2,i} + \ldots + c_n * x_{n,i} - y_i)^2 = min$$

- *Correlation*(y, x_i) =?
- *Factorial Analysis*

$$\overline{X} = \frac{\overline{X} - \mu}{\sigma / \sqrt{n}}$$

References

Keith C. Clarke, Stacy Hoppen, and Leonard J. Gaydos

Methods And Techniques for Rigorous Calibration of a Cellular Automaton Model of Urban Growth

Abstract

Several lessons about the process of calibration were learned during development of a self-modifying cellular automaton model to predict urban growth. This model, part of a global change research project on human-induced land transformations, was used to predict the spatial extent of urban growth 100 years into the future. The context of the prediction was to evaluate urban environmental disturbances such as land use conversion, urban heat island intensification, and greenhouse gas generation. Using data for the San Francisco Bay area as a test case, methods were developed, including interactive and statistical versions of the model, animation and visualization tools, automated testing methods, and Monte Carlo simulations. This presentation will enumerate, analyze, and discuss the lessons learned during the extensive process of model calibration. Experience with the methods developed may have broader use in assisting the rigorous calibration for other CA models, and perhaps those coupled environmental models with an extensive spatial data component. These methods are now under test as the project moves to a new data set for the Washington, D.C.-Baltimore area.

- Introduction
- The Urban Transition Model
- Calibration of the Model
- Evaluation of Calibration
- Predictions
- Conclusion

EMPIRICAL CALIBRATION OF SIMULATION MODELS

Claudia Werker and Thomas Brenner*

Version: 13.05.2004 (Pdf)

Table of Content

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THE APPLICATION OF BUILDING ENERGY SIMULATION AND CALIBRATION
IN TWO HIGH-RISE COMMERCIAL BUILDINGS IN SHANGHAI

Yiqun Pan1, Zhizhong Huang2, Gang Wu3, Chen Chen4

ABSTRACT

The method of calibrated computer simulation is summarized and introduced based on related literatures and guidelines, which is used to analyze the energy consumption of two high-rise commercial buildings in Shanghai, China. The detailed data of the buildings and systems are collected and input to build up models with DOE-2, then the output of simulation is compared to the measured energy consumption data to refine and calibrate the models. Several energy conservation measures (ECMs) are analyzed based on the calibrated models, including using variable speed chilled water pumps instead of constant variable speed ones, using free cooling during winter and mild seasons, replacing old low efficiency cooling towers with new high efficiency ones, decreasing lighting power densities. Energy saving performance is simulated and calculated to find out which ECM is the best option for each building.

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Automatic Preparation, Calibration, and Simulation of Deformable Objects

Dan Morris

ABSTRACT

Many simulation environments – particularly those intended for medical simulation – require solid objects to deform at interactive rates, with deformation properties that correspond to real materials. Furthermore, new objects may be created frequently (for example, each time a new patient's data is processed), prohibiting manual intervention in the model preparation process. This paper provides a pipeline for rapid preparation of deformable objects with no manual intervention, specifically focusing on mesh generation (preparing solid meshes from surface models), automated calibration of models to finite element reference analyses (including a novel approach to reducing the complexity of calibrating nonhomogeneous objects), and automated skinning of meshes for interactive simulation.

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An Improved Procedure for Developing a Calibrated Hourly Simulation Model of an Electrically Heated and Cooled Commercial Building.

A Thesis by *TAREK EDMOND BOU-SAADA*

ABSTRACT

With the increased use of building energy simulation programs, calibration of simulated data to measured data has been recognized as an important factor in substantiating how well the model fits a real building. Model calibration to measured monthly utility data has been utilized for many years. Recently, efforts have reported calibrated models at the hourly level. Most of the previous methods have relied on very simple comparisons including bar charts, monthly percent difference time-series graphs, and x-y scatter plots. A few advanced methods have been proposed as well which include carpet plots and comparative 3-D time-series plots. Unfortunately, at hourly levels of calibration, many of the traditional graphical calibration techniques become overwhelmed with data and suffer from data overlap.

Submitted to the Office of Graduate Studies of Texas A&M University in partial fulfillment of the requirements for the degree of MASTER OF SCIENCE

A Calibration Procedure for Microscopic Traffic Simulation

Lianyu Chu, Henry X. Liu, Jun-Seok Oh, Will Recker

ABSTRACT

Simulation modeling is an increasingly popular and effective tool for analyzing transportation problems that are not amendable to study by other means. For any simulation study, model calibration is a crucial step to obtaining any results from analysis. This paper presents a systematic, multi-stage procedure for the calibration and validation of PARAMCIS simulation models. The procedure is demonstrated in a calibration study with a corridor network in the southern California. The model validation results for the study network are also summarized.

Submitted to 2004 TRB Annual Meeting

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WATERSHED MODEL CALIBRATION AND VALIDATION: THE HSPF EXPERIENCE

Abstract

Model calibration and validation are necessary and critical steps in any model application. For most all watershed models, calibration is an iterative procedure of parameter evaluation and refinement, as a result of comparing simulated and observed values of interest. Model validation is in reality an extension of the calibration process. Its purpose is to assure that the calibrated model properly assesses all the variables and conditions which can affect model results, and demonstrate the ability to predict field observations for periods separate from the calibration effort.

- Calibration and validation have been defined by the American Society of Testing and Materials, as follows (ASTM, 1984):
- Calibration - a test of the model with known input and output information that is used to adjust or estimate factors for which data are not available.
- Validation - comparison of model results with numerical data independently derived from experiments or observations of the environment.

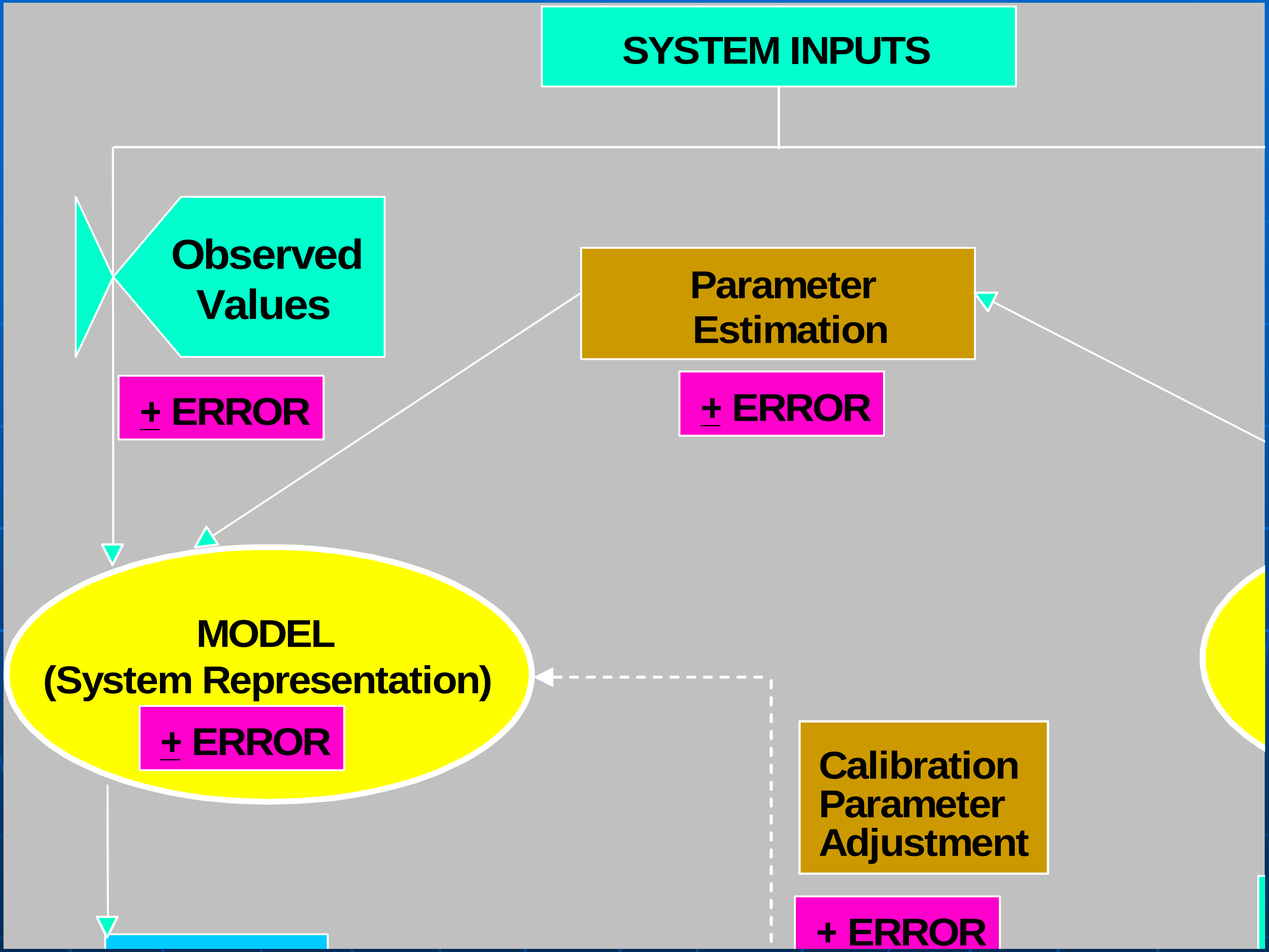
Model validation is in reality an extension of the calibration process. Its purpose is to assure that the calibrated model properly assesses all the variables and conditions which can affect model results. While there are several approaches to validating a model, perhaps the most effective procedure is to use only a portion of the available record of observed values for calibration; once the final parameter values are developed through calibration, simulation is performed for the remaining period of observed values and goodness-of-fit between recorded and simulated values is reassessed. This type of split-sample calibration/validation procedure is commonly used, and recommended, for many watershed modeling studies. Model credibility is based on the ability of a single set of parameters to represent the entire range of observed data. If a single parameter set can reasonably represent a wide range of events, then this is a form of validation.

Phase I

- Data collection
- Model input prepar
- Parameter evaluati

Phase II

- Calibration
- Validation


SYSTEM INPUTS
Observed Values
+ ERROR
Parameter Estimation
+ ERROR
MODEL (System Representation)
+ ERROR
Calibration Parameter Adjustment
+ ERROR

- Hydrologic Calibration
- Snow Calibration Calibration of the Model
- Hydraulic Calibration
- Sediment Erosion Calibration
- Instream Sediment Transport Calibration
- Nonpoint Source Loading and Water Quality Calibration

Model Performance Criteria

- Models are approximations of reality; they can not precisely represent natural systems.
- There is no single, accepted statistic or test that determines whether or not a model is validated
- Both graphical comparisons and statistical tests are required in model calibration and validation.
- Models cannot be expected to be more accurate than the errors (confidence intervals) in the input and observed data.

Graphical Comparisons:

- Timeseries plots of observed and simulated values for fluxes (e.g. flow) or state variables (e.g. stage, sediment concentration, biomass concentration)
- Observed vs. simulated scatter plots, with a 45o linear regression line displayed, for fluxes or state variablesGraphical Comparisons:
- Timeseries plots of observed and simulated values for fluxes (e.g. flow) or state variables (e.g. stage, sediment concentration, biomass concentration)
- Observed vs. simulated scatter plots, with a 45o linear regression line displayed, for fluxes or state variables
- Cumulative frequency distributions of observed and simulated fluxes or state variable (e.g. flow duration curves)
- Cumulative frequency distributions of observed and simulated fluxes or state variable (e.g. flow duration curves)

Statistical Tests:

- Error statistics, e.g. mean error, absolute mean error, relative error, relative bias, standard error of estimate, etc.
- Correlation tests, e.g. linear correlation coefficient, coefficient of model-fit efficiency, etc.
- Cumulative Distribution tests, e.g. Kolmogorov-Smirnov (KS) test

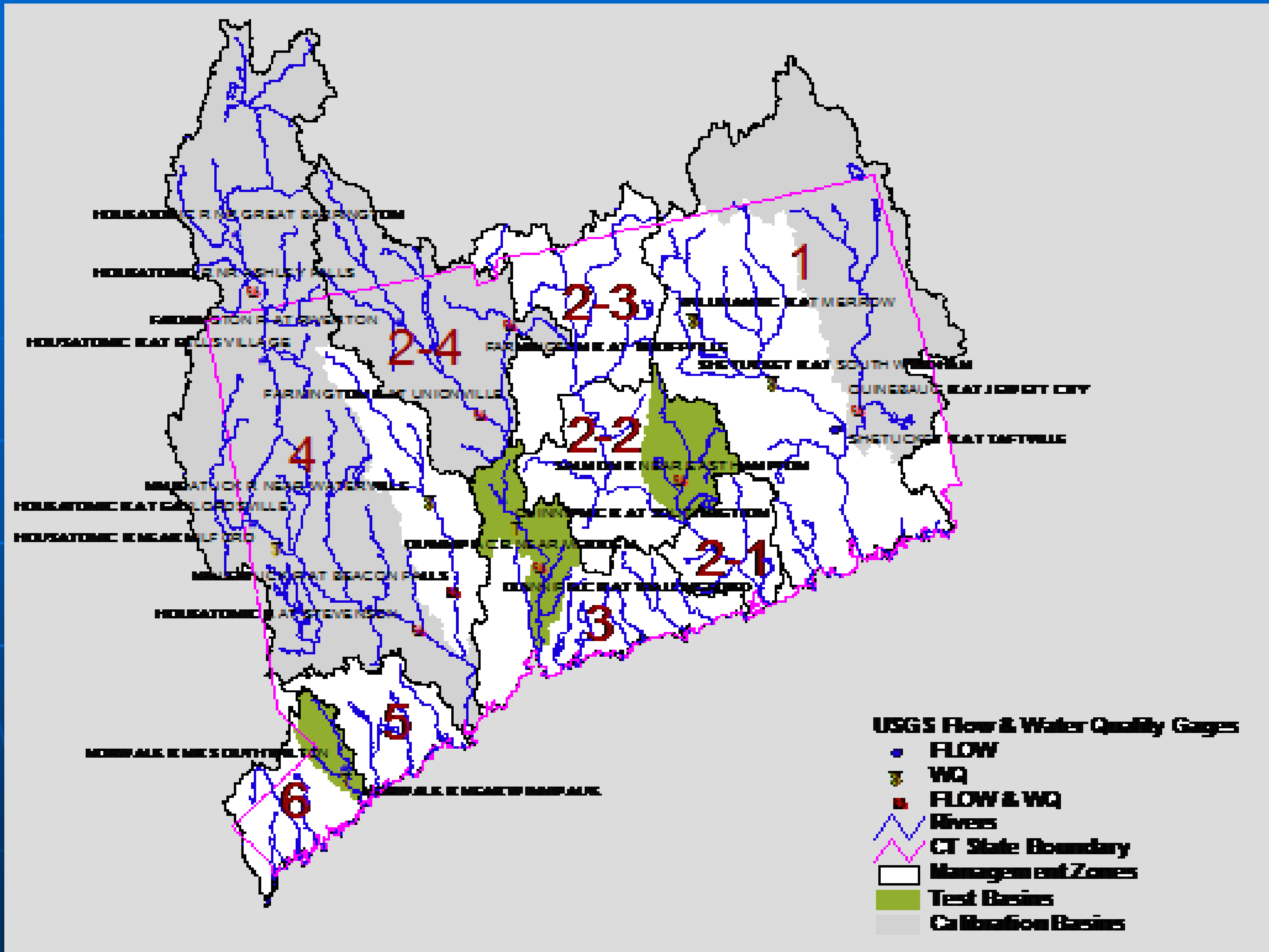
The values in the table attempt to provide some general guidance, in terms of the percent mean errors or differences between simulated and observed values, so that users can gage what level of agreement or accuracy (i.e. very good, good, fair) may be expected from the model application.

	% Difference Between Simulated and Recorded Values		
	Very Good	Good	Fair
Hydrology/Flow	< 10	10 - 15	15 - 25
Sediment	< 20	20 - 30	30 - 45
Water Temperature	< 7	8 - 12	13 - 18
Water Quality/Nutrients	< 15	15 - 25	25 - 35
Pesticides/Toxics	< 20	20 - 30	30 - 40

The Connecticut Watershed Model (CTWM), based on HSPF, was developed to evaluate nutrient sources and loadings within each of six nutrient management zones that lie primarily within the state of Connecticut, and assess their delivery efficiency to Long Island Sound (LIS). The CTWM evolved by first performing calibration and validation on three small test basins across the state (Norwalk, Quinnipiac, and Salmon) representing a range of land uses, including urban, forest, and agricultural. The model was then extended to three major river calibration basins (Farmington, Housatonic, and Quinebaug) and subsequently expanded to a statewide model by using the most spatially applicable set of calibrated watershed parameters in non-calibrated areas.

Criteria

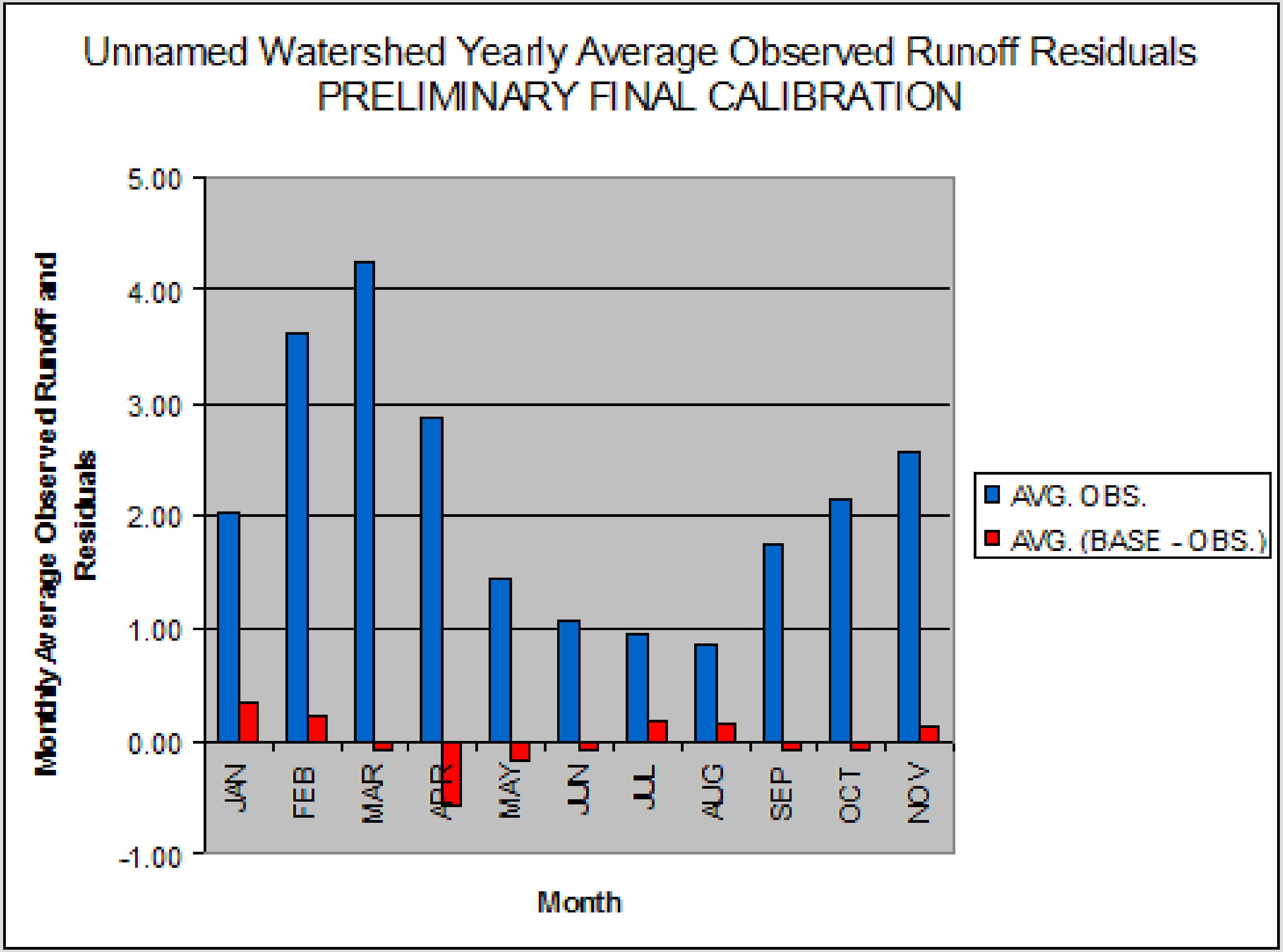
R	← 0.75	0.80	0.85	0.90	0.95 →
R^2	← 0.6	0.7	0.8	0.9 →	
Daily Flows	Poor	Fair	Good	Very Good	
Monthly Flows	Poor	Fair	Good	Very Good	


1
2-3
2-4
2-2
4
2-1
3
5
6
USGS Flow & Water Quality Gages
FLOW
WQ
FLOW & WQ
Rivers
CT State Boundary
Management Zones
Test Basins
Calibration Basins

Summary of CTWM hydrologic calibration/validation - annual flow and correlation coefficients

		Calibration Period (1991-1995)				Validation Period (1986-1990)			
Station Name	Station Number	Mean Observed Annual Flow (inches)	Mean Simulated Annual Flow (inches)	R Average Daily	R Average Monthly	Mean Observed Annual Flow (inches)	Mean Simulated Annual Flow (inches)	R Average Daily	R Average Monthly
<u>Test Watershed Gages</u>									
Salmon River nr East Hampton	01193500	23.6	24.4	0.83	0.92	26.3	25.8	0.79	0.92
Quinnipiac River at Wallingford	01196500	26.3	26.4	0.82	0.94	29.0	28.3	0.71	0.91
Norwalk River at South Wilton	01209700	21.4	21.7	0.84	0.93	25.9	25.2	0.75	0.91
<u>Major Basin Gages</u>									
Quinebaug River at Jewett City	01127000	23.8	23.6	0.82	0.93	27.2	24.7	0.86	0.95
Farmington River at Tariffville	01189995	26.2	26.0	0.85	0.92	26.2	29.1	0.87	0.94
Housatonic River at Stevenson	01205500	31.7	31.9	0.88	0.98	34.6	31.5	0.87	0.96

Annual Simulated and Observed Runoff (inches)				
	Unnamed Watershed			
	Precipitation	Simulated Flow	Observed Flow	Percent Error
1990	58.9	35.1	35.6	-1.4%
1991	47.0	23.3	22.8	2.1%
1992	45.7	23.7	20.1	15.2%
1993	47.6	27.6	26.0	5.8%
1994	46.3	25.9	25.5	1.5%
1995	44.0	20.7	21.0	-1.4%
1996	62.0	39.4	41.5	-5.3%
1997	42.2	21.4	23.2	-8.4%
1998	42.2	22	23.9	-8.6%
1999	46.9	21.6	24.8	-14.8%
Total	482.7	260.7	264.4	-1.4%
Average	48.3	26.1	26.4	-1.4%



Unnamed Watershed Yearly Average Observed Runoff Residuals
PRELIMINARY FINAL CALIBRATION
Monthly Average Observed Runoff and Residuals
5.00
4.00
3.00
2.00
1.00
0.00
-1.00
JAN
FEB
MAR
APR
MAY
JUN
JUL
AUG
SEP
OCT
NOV
Month
AVG. OBS.
AVG. (BASE - OBS.)

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Sensitivity Analysis

Sensitivity analysis (SA) is " the study of how the variation in the output of a model (numerical or otherwise) can be apportioned, qualitatively or quantitatively, to different sources of variation".[1] However, when the assumptions are uncertain, and/or there are alternative sets of assumptions to chose from, the inference will also be also uncertain. Investigating the uncertainty in the inference (regardless of its source) goes under the name of **Uncertainty analysis**.

***S**ensitivity **A**nalysis* tries to identify those assumptions which weight the most in determining the uncertainty in the inference ('screening' sensitivity analysis). 'Quantitative' sensitivity analysis tries not only to identify but also to quantify the relative importance the influential assumptions. In the preceding discussion the term 'factor' if often used instead of 'assumption' - implying that assumptions have been translated into factors entering the model, e.g. with defined numerical values possibly drawn from factor-value distributions - while 'model output' can be used instead of inference.

Overview

- A mathematical model is defined by a series of equations, input factors, parameters, and variables aimed to characterize the process being investigated.
- Input is subject to many sources of uncertainty including errors of measurement, absence of information and poor or partial understanding of the driving forces and mechanisms. This uncertainty imposes a limit on our confidence in the response or output of the model. Further, models may have to cope with the natural intrinsic variability of the system, such as the occurrence of stochastic events.

- Good modeling practice requires that the modeler provides an evaluation of the confidence in the model, possibly assessing the uncertainties associated with the modeling process and with the outcome of the model itself. Uncertainty and Sensitivity Analysis offer valid tools for characterizing the uncertainty associated with a model.
- In models involving many input variables sensitivity analysis is an essential ingredient of model building and quality assurance. National and international agencies involved in impact assessment studies have included section devoted to sensitivity analysis in their guidelines. Examples are the European Commission, the White House Office for Budget and Management, the Intergovernmental Panel on Climate Change and the US Environmental Protection Agency.

Methodology

- There are several possible procedures to perform uncertainty (UA) and sensitivity analysis (SA). The most common sensitivity analysis is sampling-based. A sampling-based sensitivity is one in which the model is executed repeatedly for combinations of values sampled from the distribution (assumed known) of the input factors. Sampling based methods can also be used to decompose the variance of the model output (see references).

… Methodology

- In general, UA and SA are performed jointly by executing the model repeatedly for combination of factor values sampled with some probability distribution. The following steps can be listed:

1. Specify the target function and select the input of interest
2. Assign a distribution function to the selected factors
3. Generate a matrix of inputs with that distribution(s) through an appropriate design
4. Evaluate the model and compute the distribution of the target function
5. Select a method for assessing the influence or relative importance of each input factor on the target function.

Applications

- Sensitivity Analysis can be used to determine:

1. The model resemblance with the process under study
2. The quality of model definition
3. Factors that mostly contribute to the output variability
4. The region in the space of input factors for which the model variation is maximum
5. Optimal - or instability - regions within the space of factors for use in a subsequent calibration study
6. Interactions between factors

Sensitivity Analysis is popular in financial applications, risk analysis, signal processing, neural networks and any area where models are developed. SA can also be used in model-based policy assessment studies see e.g. [1].

• Environmental

- Computer environmental models are increasingly used in a wide variety of studies and applications. For example global climate model are used for both short term weather forecasts and long term climate change.
- Moreover, computer models are increasingly used for environmental decision making at a local scale, for example for assessing the impact of a waste water treatment plant on a river flow, or for assessing the behavior and life length of bio-filters for contaminated waste water.

- In both cases sensitivity analysis may help understanding the contribution of the various sources of uncertainty to the model output uncertainty and system performance in general. In these cases, depending on model complexity, different sampling strategies may be advisable and traditional sensitivity indexes have to be generalized to cover multivariate sensitivity analysis, heteroskedastic effects and correlated inputs.

- **Business**

  - In a decision problem, the analyst may want to identify cost drivers as well as other quantities for which we need to acquire better knowledge in order to make an informed decision. On the other hand, some quantities have no influence on the predictions, so that we can save resources at no loss in accuracy by relaxing some of the conditions. See Corporate finance: Quantifying uncertainty.

Sensitivity analysis can help in a variety of other circumstances which can be handled by the settings illustrated below:

- to identify critical assumptions or compare alternative model structures
- guide future data collections
- detect important criteria
- optimize the tolerance of manufactured parts in terms of the uncertainty in the parameters
- optimize resources allocation
- model simplification or model lumping, etc.

However there are also some problems associated with sensitivity analysis in the business context:

- Variables are often interdependent, which makes examining them each individually unrealistic, e.g.: changing one factor such as sales volume, will most likely affect other factors such as the selling price.
- Often the assumptions upon which the analysis is based are made by using past experience/data which may not hold in the future.
- Assigning a maximum and minimum (or optimistic and pessimistic) value is open to subjective interpretation. For instance one persons 'optimistic' forecast may be more conservative than that of another person performing a different part of the analysis. This sort of subjectivity can adversely affect the accuracy and overall objectivity of the analysis.

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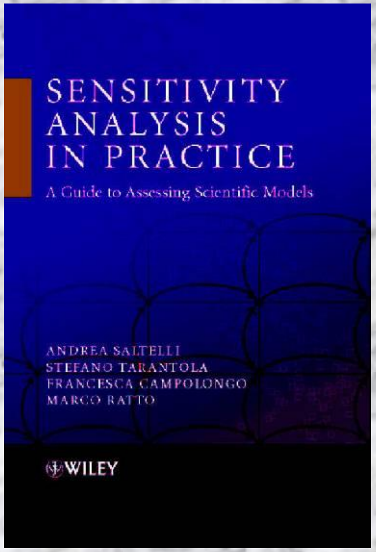
Tutorial on *Sensitivity Analysis*

To start with, we propose the following material :

- Cookbook
- Tutorial 1
- Tutorial 2
- Tutorial 3
- Book on Sensitivity Analysis (Preface)



A REAL TEST CASE

We also suggest some bibliographic material, where the reader will find the grater part of sensitivity analysis studies, together with applications and reviews:

- Sensitivity Analysis for Chemical Models
- Composite Indicators
- Archer, G.; Saltelli, A.; Sobol', I. M. *Journal of Statistical Computation and Simulation* **1997**, *58*, 99

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An Introduction to Sensitivity Analysis

Prepared for the

MIT System Dynamics in Education Project

Under the Supervision of

Dr. Jay W. Forrester

by

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September 6, 1996

Vensim Examples added October 2001

Lemonade Stand

In the first exploration, let’s look at a lemonade stand located on a college campus.

As usual, we are particularly interested in the behavior of the stock, the number of cups of

lemonade that are ready to be sold to customers. The stand is open eight hours every day.

Howard, the owner, is the only person working in the stand.

Epidemics

In the second exploration we look at an epidemics model. The model was already

used in a previous chapter in Road Maps, so it is possible that you have already built it.

Coffeehouse

We now return to Howard, the owner of the lemonade stand on a college campus.

Howard realized that it could be more profitable for him to sell coffee because students

tend to drink more coffee than lemonade, and they drink it at any time of the day and

night. Therefore, he closed his lemonade stand and opened a 24-hour Coffeehouse.

Howard bases the Coffeehouse model on the model he used in his lemonade stand to

model the number of cups of “Coffee ready.” We will run the simulation over a period of

two days, or 48 hours.

Specific parameter values can change the appearance of the graphs representing the behavior of the system. But significant changes in behavior do not occur for all parameters. System dynamics models are in general insensitive to many parameter changes. It is the structure of the system, and not the parameter values, that has most influence on the behavior of the system.

Sensitivity analysis is an important tool in the model building process. By showing that the system does not react greatly to a change in a parameter value, it reduces the modeler’s uncertainty in the behavior. In addition, it gives an opportunity for a better understanding of the dynamic behavior of the system.

We encourage you to experiment with the three models from this paper (as well as any other models that you have built) on your own. For example, try to change several parameters at the same time, observe the behavior produced, and compare it to the conclusions in this paper. Can you suggest any parameter values that would produce the “optimal,” or most desirable behavior? The use of sensitivity analysis in such policy analysis will be explored in a later paper in this series.

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Sensitivity analysis

Sensitivity analysis (SA) is the study of how the variation (uncertainty) in the output of a mathematical model can be apportioned, qualitatively or quantitatively, to different sources of variation in the input of a model [1]. Put another way, it is a technique for systematically changing parameters in a model to determine the effects of such changes.

In more general terms uncertainty and sensitivity analyses investigate the robustness of a study when the study includes some form ofmathematical modelling. Sensitivity analysis can be useful to computer modellers for a range of purposes[2], including:

- support decision making or the development of recommendations for decision makers (e.g. testing the robustness of a result);
- enhancing communication from modellers to decision makers (e.g. by making recommendations more credible, understandable, compelling or persuasive);
- increased understanding or quantification of the system (e.g. understanding relationships between input and output variables); and
- model development (e.g. searching for errors in the model).

While uncertainty analysis studies the overall uncertainty in the conclusions of the study, sensitivity analysis tries to identify what source of uncertainty weights more on the study's conclusions. For example, several guidelines for modelling (see e.g. one from the US EPA) or forimpact assessment (see one from the European Commission) prescribe sensitivity analysis as a tool to ensure the quality of the modelling/assessment.

… Sensitivity analysis

The problem setting in sensitivity analysis has strong similarities with design of experiments. In design of experiments one studies the effect of some process or intervention (the 'treatment') on some objects (the 'experimental units'). In sensitivity analysis one looks at the effect of varying the inputs of a mathematical model on the output of the model itself. In both disciplines one strives to obtain information from the system with a minimum of physical or numerical experiments.

In uncertainty and sensitivity analysis there is a crucial trade off between how scrupulous an analyst is in exploring the input assumptions and how wide the resulting inference may be. The point is well illustrated by the econometrician Edward E. Leamer (1990) [3]:

I have proposed a form of organized sensitivity analysis that I call ‘global sensitivity analysis’ in which a neighborhood of alternative assumptions is selected and the corresponding interval of inferences is identified. Conclusions are judged to be sturdy only if the neighborhood of assumptions is wide enough to be credible and the corresponding interval of inferences is narrow enough to be useful.

Note Leamer’s emphasis is on the need for 'credibility' in the selection of assumptions. The easiest way to invalidate a model is to demonstrate that it is fragile with respect to the uncertainty in the assumptions or to show that its assumptions have not been taken 'wide enough'. The same concept is expressed by Jerome R. Ravetz, for whom bad modeling is when *uncertainties in inputs must be suppressed lest outputs become indeterminate.*[4]

In modern econometrics the use of sensitivity analysis to anticipate criticism is the subject of one of the ten commandments of applied econometrics (from Kennedy, 2007[5]):

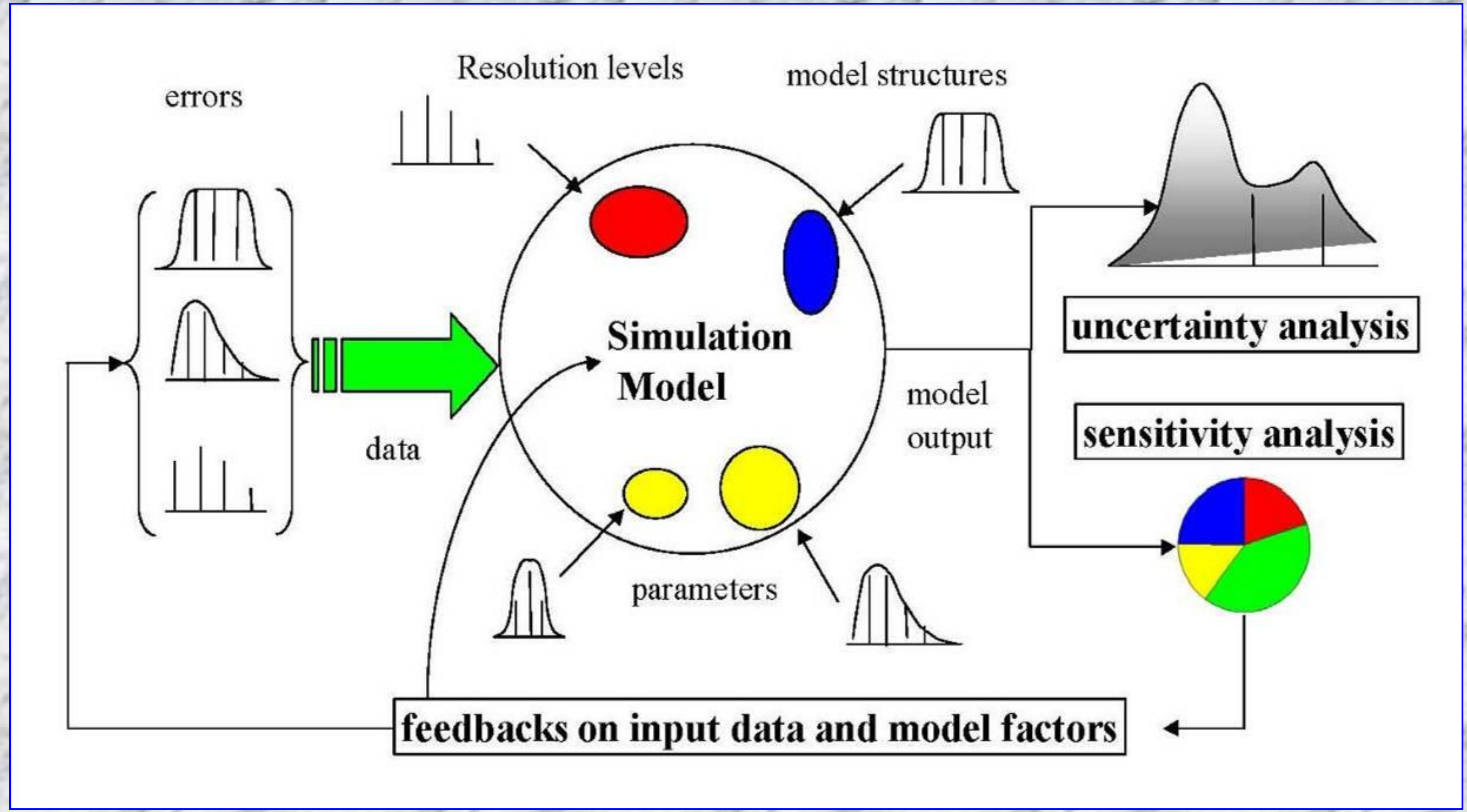
… Sensitivity analysis

Thou shall confess in the presence of sensitivity. Corollary: Thou shall anticipate criticism [···] When reporting a sensitivity analysis, researchers should explain fully their specification search so that the readers can judge for themselves how the results may have been affected. This is basically an 'honesty is the best policy' approach, advocated by Leamer, (1978[6]).

The use of mathematical modelling can be the subject of controversies, see Nassim Nicholas Taleb[7] in Economics, and Orrin H. Pilkey and Linda Pilkey Jarvis[8] in Environmental Sciences. As noted by the latter Authors, this increases the relevance of sensitivity analysis in today's modelling practice[1] .

Mathematical problems met in social, economic or natural sciences may entail the use of mathematical models, which generally do not lend themselves to a straightforward understanding of the relationship between input factors (what goes into the model) and output (the model's dependent variables). Such an appreciation, i.e. the understanding of how the model behaves in response to changes in its inputs, is of fundamental importance to ensure a correct use of the models.

- A mathematical model is defined by a series of equations, input factors, parameters, and variables aimed to characterize the process being investigated.



Ideal scheme of a possibly sampling-based sensitivity analysis. Uncertainty arising from different sources — errors in the data, parameter estimation procedure, alternative model structures — are propagated through the model for uncertainty analysis and their relative importance is quantified via sensitivity analysis.

Errors

In sensitivity analysis Type I error is assessing as important a non important factor, and Type II error assessing as non important an important factor. Type III error corresponds to analyzing the wrong problem, e.g. via an incorrect specification of the input uncertainties. Possible pitfalls in sensitivity analysis are:

•Unclear purpose of the analysis. Different statistical tests and measures are applied to the problem and different factors rankings are obtained. The test should instead be tailored to the purpose of the analysis, e.g. one uses Monte Carlo filtering if one is interested in which factors are most responsible for generating high/low values of the output.

•Too many model outputs are considered. This may be acceptable for quality assurance of sub-models but should be avoided when presenting the results of the overall analysis.

•Piecewise sensitivity. This is when one performs sensitivity analysis on one sub-model at a time. This approach is non conservative as it might overlook interactions among factors in different sub-models (Type II error).

Applications

Sensitivity analysis can be used

- To simplify models
- To investigate the robustness of the model predictions
- To play what-if analysis exploring the impact of varying input assumptions and scenarios
- As an element of quality assurance (unexpected factors sensitivities may be associated to coding errors or misspecifications).

It provides as well information on:

- Factors that mostly contribute to the output variability
- The region in the space of input factors for which the model output is either maximum or minimum or within pre-defined bounds (see Monte Carlo filtering above)
- Optimal — or instability — regions within the space of factors for use in a subsequent calibration study
- Interaction between factors

Sensitivity Analysis is common in physics and chemistry[26], in financial applications, risk analysis, signal processing, neural networks and any area where models are developed. Sensitivity analysis can also be used in model-based policy assessment studies . Sensitivity analysis can be used to assess the robustness of composite indicators [27], also known as indices, such as the Environmental Pressure Index.

Environmental

Computer environmental models are increasingly used in a wide variety of studies and applications. For example global climate model are used for both short term weather forecasts and long term climate change.

Moreover, computer models are increasingly used for environmental decision making at a local scale, for example for assessing the impact of a waste water treatment plant on a river flow, or for assessing the behavior and life length of bio-filters for contaminated waste water.

In both cases sensitivity analysis may help understanding the contribution of the various sources of uncertainty to the model output uncertainty and system performance in general. In these cases, depending on model complexity, different sampling strategies may be advisable and traditional sensitivity indexes have to be generalized to cover multivariate sensitivity analysis, heteroskedastic effects and correlated inputs.

Business

In a decision problem, the analyst may want to identify cost drivers as well as other quantities for which we need to acquire better knowledge in order to make an informed decision. On the other hand, some quantities have no influence on the predictions, so that we can save resources at no loss in accuracy by relaxing some of the conditions. See Corporate finance: Quantifying uncertainty. Sensitivity analysis can help in a variety of other circumstances which can be handled by the settings illustrated below:

- to identify critical assumptions or compare alternative model structures
- guide future data collections
- detect important criteria
- optimize the tolerance of manufactured parts in terms of the uncertainty in the parameters
- optimize resources allocation
- model simplification or model lumping, etc.

However there are also some problems associated with sensitivity analysis in the business context:

- Variables are often interdependent, which makes examining them each individually unrealistic, e.g.: changing one factor such as sales volume, will most likely affect other factors such as the selling price.
- Often the assumptions upon which the analysis is based are made by using past experience/data which may not hold in the future.
- Assigning a maximum and minimum (or optimistic and pessimistic) value is open to subjective interpretation. For instance one persons 'optimistic' forecast may be more conservative than that of another person performing a different part of the analysis. This sort of subjectivity can adversely affect the accuracy and overall objectivity of the analysis.

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What Does *Sensitivity Analysis* Mean?

A technique used to determine how different values of an independent variable will impact a particular dependent variable under a given set of assumptions. This technique is used within specific boundaries that will depend on one or more input variables, such as the effect that changes in interest rates will have on a bond's price.

Sensitivity analysis is a way to predict the outcome of a decision if a situation turns out to be different compared to the key prediction(s).

Investopedia explains *Sensitivity Analysis*

Sensitivity analysis is very useful when attempting to determine the impact the actual outcome of a particular variable will have if it differs from what was previously assumed. By creating a given set of scenarios, the analyst can determine how changes in one variable(s) will impact the target variable.

For example, an analyst might create a financial model that will value a company's equity (the dependent variable) given the amount of earnings per share (an independent variable) the company reports at the end of the year and the company's price-to-earnings multiple (another independent variable) at that time. The analyst can create a table of predicted price-to-earnings multiples and a corresponding value of the company's equity based on different values for each of the independent variables.

Sensitivity ~ http://www.investopedia.com/terms/s/sensitivity.asp

What Does *Sensitivity* Mean?

The magnitude of a financial instrument's reaction to changes in underlying factors. Financial instruments, such as stocks and bonds, are constantly impacted by many factors. Sensitivity accounts for all factors that impact a given instrument in a negative or positive way in an attempt to learn how much a certain factor will impact the value of a particular instrument.

Investopedia explains *Sensitivity*

Interest rates are one of the most important underlying factors in the movement of bond prices and are closely watched by bond investors. These investors get a better idea of how their bonds will be affected by interest rate movements by incorporating sensitivity into their analyses.

Spreadsheet Sensitivity Analysis

Spreadsheets and the Case Projects

The Dynamic Strategic Planning workbook is accompanied by a number of spreadsheet-based tools for data analysis. We have supplied these tools so that the users of this workbook can concentrate upon the use and implementation of decision analysis and strategic planning, rather than focusing upon the mechanics of the mathematics underlying their use.

The current form of the spreadsheets is a consequence of a combination of factors: academic research, pedagogical design, and in-class experiences. Based upon new developments, they are being routinely improved.

However, no amount of care in tool design can substitute for expertise on the part of the user.

The case projects have been designed assuming that these tools will be used effectively. The purpose of this document is to assure that you, the user of these tools, are prepared to exploit them to their fullest - specifically, that you are able to make use of spreadsheet sensitivity analysis tools.

http://msl1.mit.edu/rdn/d_table.pdf

Sensitivity Analysis Using Excel

The main goal of sensitivity analysis is to gain insight into which assumptions are critical, i.e., which assumptions affect choice. The process involves various ways of changing input values of the model to see the effect on the output value. In some decision situations you can use a single model to investigate several alternatives. In other cases, you may use a separate spreadsheet model for each alternative.

MANUAL WHAT-IF ANALYSIS

Using this approach, you enter values into cells C4:C6 and see what the effect is on net cash flow.

For example, with the predetermined price of $29, you may think that Units Sold will be in the range between 500 and 900 units. Keeping other input assumptions at base case, the corresponding Net Cash Flows are $–1,500 and $6,900. When we vary a single input assumption, keeping all other input assumptions at their base case values, we say we are doing "one at a time" or "singlefactor“ sensitivity analysis.

	A	B	C
1	Controllable Input		
2		Unit Price	$29
3	Uncontrollable Inputs		
4		Units Sold	700
5		Unit Variable Cost	$8
6		Fixed Costs	$12,000
7	Performance Measure		
8		Net Cash Flow	$2,700

http://www.treeplan.com/chapters/02_decan_20071029_1042.pdf

Chapter 6: **Sensitivity Analysis**

Chapter 6: Sensitivity Analysis

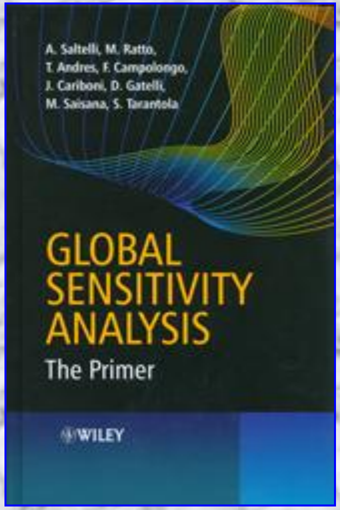
Suppose that you have just completed a linear programming solution which will have a major impact on your company, such as determining how much to increase the overall production capacity, and are about to present the results to the board of directors. How confident are you in the results? How much will the results change if your basic data (e.g. profit per item produced, or availability of a component) is slightly wrong? Will that have a minor impact on your results? Will it give a completely different outcome, or change the outcome only slightly?

These are the kinds of questions addressed by sensitivity analysis. Formally, the question is this: is my optimum solution (both the values of the variables and the value of the objective function) sensitive to a small change in one of the original problem coefficients (e.g. coefficients of the variables in the objective function or constraints, or the right hand side constants in the constraints)? If Z or the x_i change when an original coefficient is changed, then we say that the LP is *sensitive*. We could ask, for example, if the Acme Bicycle Company solution is sensitive to a reduction in the availability of the metal finishing machine from 4 hours per day to only 3 (i.e. a change in the third constraint from $x_1+x_2 \leq 4$ to $x_1+x_2 \leq 3$).

http://www.sce.carleton.ca/faculty/chinneck/po/Chapter6.pdf

Sensitivity Analysis

Global Sensitivity AnalysisThe Primer
Saltelli, A., Ratto, M., Andres, T., Campolongo, F., Cariboni, J., Gatelli, D. Saisana, M., and Tarantola, S., 2008, John Wiley & Sons (ISBN: 978-0-470-05997-5)**Who needs** Sensitivity Analysis



Tutorial on Sensitivity Analysis

SimLab Software for Sensitivity Analysis

What's New

•Sixth **International Conference**on **Sensitivity Analysis of Model Output**, Bocconi University of Milan, **19-22 July 2010**

•Sixth **Summer School** on**Sensitivity Analysis of Model Output**, Villa La Stella, Fiesole - Florence, **14-17 September 2010**

http://sensitivity-analysis.jrc.ec.europa.eu/

Sensitivity Analysis

Multi-disciplinary Design Optimization

Centre for Aerospace Systems Design & Engineering
Department of Aerospace Engineering
Indian Institute of Technology
Mumbai 400 076

sensitivity.ppt (67 slides)

Sensitivity Analysis – Other References

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- *SENSITIVITY AND RISK ANALYSES* ~ *http://www.adb.org/documents/handbooks/water_supply_projects/Chap7-r6.PDF*
- *Sensitivity Analysis of LP* ~ http://www.youtube.com/watch?v=rACFwIt2szk
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