Statistical Methods in Simulation

Metode statistice in simulare

Computer simulation

▫ A **computer simulation**, a **computer model**, or a **computational model** is a computer program, or network of computers, that attempts to simulate an abstract model of a particular system.

▫ Computer simulations have become a useful part of mathematical modeling of many natural systems in physics (computational physics), astrophysics, chemistry and biology, human systems in economics, psychology, social science, and engineering. Simulations can be used to explore and gain new insights into new technology, and to estimate the performance of systems too complex for analytical solutions.[1]

▫ Computer simulations vary from computer programs that run a few minutes, to network-based groups of computers running for hours, to ongoing simulations that run for days. The scale of events being simulated by computer simulations has far exceeded anything possible (or perhaps even imaginable) using the traditional paper-and-pencil mathematical modeling: over 10 years ago, a desert-battle simulation, of one force invading another, involved the modeling of 66,239 tanks, trucks and other vehicles on simulated terrain around Kuwait, using multiple supercomputers in the DoD High Performance Computer Modernization Program;[2] a 1-billion-atom model of material deformation (2002); a 2.64-million-atom model of the complex maker of protein in all organisms, a ribosome, in 2005;[3] and the Blue Brain project at EPFL(Switzerland), began in May 2005, to create the first computer simulation of the entire human brain, right down to the molecular level.[4]

Simulation versus modeling

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Types

▫ Computer models can be classified according to several independent pairs of attributes, including:

▫ Stochastic or deterministic (and as a special case of deterministic, chaotic) - see External links below for examples of stochastic vs. deterministic simulations

▫ Steady-state or dynamic

▫ Continuous or discrete (and as an important special case of discrete, discrete event or DE models)

▫ Local or distributed.

▫ Equations define the relationships between elements of the modeled system and attempt to find a state in which the system is in equilibrium. Such models are often used in simulating physical systems, as a simpler modeling case before dynamic simulation is attempted.

▫ Dynamic simulations model changes in a system in response to (usually changing) input signals.

▫ *Stochastic* models use *random number generators* to model chance or random events;

▫ A *discrete event simulation* (DES) manages events in time. Most computer, logic-test and fault-tree simulations are of this type. In this type of simulation, the simulator maintains a queue of events sorted by the simulated time they should occur. The simulator reads the queue and triggers new events as each event is processed. It is not important to execute the simulation in real time. It's often more important to be able to access the data produced by the simulation, to discover logic defects in the design, or the sequence of events.

▫ A *continuous dynamic simulation* performs numerical solution of differential-algebraic equations or differential equations (either partial or ordinary). Periodically, the simulation program solves all the equations, and uses the numbers to change the state and output of the simulation. Applications include flight simulators, construction and management simulation games, chemical process modeling, and simulations of electrical circuits. Originally, these kinds of simulations were actually implemented on analog computers, where the differential equations could be represented directly by various electrical components such as op-amps. By the late 1980s, however, most "analog" simulations were run on conventional digital computers that emulate the behavior of an analog computer.

▫ A special type of discrete simulation which does not rely on a model with an underlying equation, but can nonetheless be represented formally, is *agent-based simulation.* In agent-based simulation, the individual entities (such as molecules, cells, trees or consumers) in the model are represented directly (rather than by their density or concentration) and possess an internal *state* and set of behaviors or *rules* which determine how the agent's state is updated from one time-step to the next.

▫ Distributed models run on a network of interconnected computers, possibly through the Internet. Simulations dispersed across multiple host computers like this are often referred to as "distributed simulations". There are several standards for distributed simulation, including Aggregate Level Simulation Protocol (ALSP), Distributed Interactive Simulation (DIS), the High Level Architecture (simulation) (HLA) and the Test and Training Enabling Architecture (TENA).

CGI computer simulation

▫ Formerly, the output data from a computer simulation was sometimes presented in a table, or a matrix, showing how data was affected by numerous changes in the simulation parameters. The use of the matrix format was related to traditional use of the matrix concept in mathematical models; however, psychologists and others noted that humans could quickly perceive trends by looking at graphs or even moving-images or motion-pictures generated from the data, as displayed by **computer-generated-imagery** (CGI) animation. Although observers couldn't necessarily read out numbers, or spout math formulas, from observing a moving weather chart, they might be able to predict events (and "see that rain was headed their way"), much faster than scanning tables of rain-cloud coordinates. Such intense graphical displays, which transcended the world of numbers and formulae, sometimes also led to output that lacked a coordinate grid or omitted timestamps, as if straying too far from numeric data displays. Today, weather forecasting models tend to balance the view of moving rain/snow clouds against a map that uses numeric coordinates and numeric timestamps of events.

▫ Similarly, CGI computer simulations of CAT scans can simulate how a tumor might shrink or change, during an extended period of medical treatment, presenting the passage of time as a spinning view of the visible human head, as the tumor changes.

▫ Other applications of CGI computer simulations are being developed to graphically display large amounts of data, in motion, as changes occur during a simulation run.

Computer simulation in practical contexts

- Computer simulations are used in a wide variety of practical contexts, such as:
- analysis of air pollutant dispersion using atmospheric dispersion modeling
- design of complex systems such as aircraft and also logistics systems.
- design of Noise barriers to effect roadway noise mitigation
- flight simulators to train pilots
- weather forecasting
- Simulation of other computers is emulation.
- forecasting of prices on financial markets (for example Adaptive Modeler)
- behavior of structures (such as buildings and industrial parts) under stress and other conditions
- design of industrial processes, such as chemical processing plants
- Strategic Management and Organizational Studies
- Reservoir simulation for the petroleum engineering to model the subsurface reservoir
- Process Engineering Simulation tools.
- Robot simulators for the design of robots and robot control algorithms
- Urban Simulation Models that simulate dynamic patterns of urban development and responses to urban land use and transportation policies. See a more detailed article on Urban Environment Simulation.
- Traffic engineering to plan or redesign parts of the street network from single junctions over cities to a national highway network, for transportation system planning, design and operations. See a more detailed article on Simulation in Transportation.
- modeling car crashes to test safety mechanisms in new vehicle models

... Computer simulation in practical contexts

▫ The reliability and the trust people put in computer simulations depends on the validity of the simulation model, therefore verification and validation are of crucial importance in the development of computer simulations. Another important aspect of computer simulations is that of reproducibility of the results, meaning that a simulation model should not provide a different answer for each execution. Although this might seem obvious, this is a special point of attention in stochastic simulations, where random numbers should actually be semi-random numbers. An exception to reproducibility are human in the loop simulations such as flight simulations and computer games. Here a human is part of the simulation and thus influences the outcome in a way that is hard if not impossible to reproduce exactly.

▫ Vehicle manufacturers make use of computer simulation to test safety features in new designs. By building a copy of the car in a physics simulation environment, they can save the hundreds of thousands of dollars that would otherwise be required to build a unique prototype and test it. Engineers can step through the simulation milliseconds at a time to determine the exact stresses being put upon each section of the prototype.[7]

▫ Computer graphics can be used to display the results of a computer simulation. Animations can be used to experience a simulation in real-time e.g. in training simulations. In some cases animations may also be useful in faster than real-time or even slower than real-time modes. For example, faster than real-time animations can be useful in visualizing the buildup of queues in the simulation of humans evacuating a building. Furthermore, simulation results are often aggregated into static images using various ways of scientific visualization.

▫ In debugging, simulating a program execution under test (rather than executing natively) can detect far more errors than the hardware itself can detect and, at the same time, log useful debugging information such as instruction trace, memory alterations and instruction counts. This technique can also detect buffer overflow and similar "hard to detect" errors as well as produce performance information and tuning data.

Statistical methods

A common goal for a statistical research project is to investigate causality, and in particular to draw a conclusion on the effect of changes in the values of predictors or independent variables on dependent variables or response. There are two major types of causal statistical studies: experimental studies and observational studies. In both types of studies, the effect of differences of an independent variable (or variables) on the behavior of the dependent variable are observed. The difference between the two types lies in how the study is actually conducted. Each can be very effective. An experimental study involves taking measurements of the system under study, manipulating the system, and then taking additional measurements using the same procedure to determine if the manipulation has modified the values of the measurements. In contrast, an observational study does not involve experimental manipulation. Instead, data are gathered and correlations between predictors and response are investigated.

... Statistical methods

- The basic steps of a statistical experiment are:

   1. Planning the research, including finding the number of replicates of the study, using the following information: preliminary estimates regarding the size of treatment effects, alternative hypotheses, and the estimated experimental variability. Consideration of the selection of experimental subjects and the ethics of research is necessary. Statisticians recommend that experiments compare (at least) one new treatment with a standard treatment or control, to allow an unbiased estimate of the difference in treatment effects.

   2. Design of experiments, using blocking to reduce the influence of confounding variables, and randomized assignment of treatments to subjects to allow unbiased estimates of treatment effects and experimental error. At this stage, the experimenters and statisticians write the *experimental protocol* that shall guide the performance of the experiment and that specifies the *primary analysis* of the experimental data.

   3. Performing the experiment following the experimental protocol and analyzing the data following the experimental protocol.

   4. Further examining the data set in secondary analyses, to suggest new hypotheses for future study.

   5. Documenting and presenting the results of the study.

▫ Experiments on human behavior have special concerns. The famous Hawthorne study examined changes to the working environment at the Hawthorne plant of the Western Electric Company. The researchers were interested in determining whether increased illumination would increase the productivity of the assembly line workers. The researchers first measured the productivity in the plant, then modified the illumination in an area of the plant and checked if the changes in illumination affected productivity. It turned out that productivity indeed improved (under the experimental conditions). However, the study is heavily criticized today for errors in experimental procedures, specifically for the lack of a control group and blindness. The Hawthorne effect refers to finding that an outcome (in this case, worker productivity) changed due to observation itself. Those in the Hawthorne study became more productive not because the lighting was changed but because they were being observed

Key terms used in statistics

Null hypothesis

▫ Interpretation of statistical information can often involve the development of a null hypothesis in that the assumption is that whatever is proposed as a cause has no effect on the variable being measured.

▫ The best illustration for a novice is the predicament encountered by a jury trial. The null hypothesis, H0, asserts that the defendant is innocent, whereas the alternative hypothesis, H1, asserts that the defendant is guilty. The indictment comes because of suspicion of the guilt. The H0 (status quo) stands in opposition to H1 and is maintained unless H1 is supported by evidence"beyond a reasonable doubt". However,"failure to reject H0" in this case does not imply innocence, but merely that the evidence was insufficient to convict. So the jury does not necessarily *accept* H0 but *fails to reject* H0. While one can not"prove" a null hypothesis one can test how close it is to being true with a power test, which tests for type II errors.

Error

▫ Working from a <u>null hypothesis</u> two basic forms of error are recognized:

▫ <u>Type I</u> errors where the null hypothesis is falsely rejected giving a "false positive".

▫ Type II errors where the null hypothesis fails to be rejected and an actual difference between populations is missed.

▫ Error also refers to the extent to which individual observations in a sample differ from a central value, such as the sample or population mean. Many statistical methods seek to minimize the mean-squared error, and these are called "<u>methods of least squares</u>."

▫ Measurement processes that generate statistical data are also subject to error. Many of these errors are classified as <u>random</u> (noise) or <u>systematic</u> (<u>bias</u>), but other important types of errors (e.g., blunder, such as when an analyst reports incorrect units) can also be important.

... Key terms used in statistics

Confidence intervals

▫ Most studies will only sample part of a population and then the result is used to interpret the null hypothesis in the context of the whole population. Any estimates obtained from the sample only approximate the population value. Confidence intervals allow statisticians to express how closely the sample estimate matches the true value in the whole population. Often they are expressed as 95% confidence intervals. Formally, a 95% confidence interval of a procedure is a range where, if the sampling and analysis were repeated under the same conditions, the interval would include the true (population) value 95% of the time. This does *not* imply that the probability that the true value is in the confidence interval is 95%. (From the frequentist perspective, such a claim does not even make sense, as the true value is not a random variable. Either the true value is or is not within the given interval.) One quantity that is in fact a probability for an estimated value is the credible interval from Bayesian statistics.

Significance (*Statistical significance***)**

▫ Statistics rarely give a simple Yes/No type answer to the question asked of them. Interpretation often comes down to the level of statistical significance applied to the numbers and often refer to the probability of a value accurately rejecting the null hypothesis (sometimes referred to as the p-value).

▫ Referring to statistical significance does not necessarily mean that the overall result is significant in real world terms. For example, in a large study of a drug it may be shown that the drug has a statistically significant but very small beneficial effect, such that the drug will be unlikely to help the patient in a noticeable way.

Examples

Some well-known statistical tests and procedures are:

- Analysis of variance (ANOVA)
- Chi-square test
- Correlation
- Factor analysis
- Mann–Whitney U
- Mean square weighted deviation (MSWD)
- Pearson product-moment correlation coefficient
- Regression analysis
- Spearman's rank correlation coefficient
- Student's t-test
- Time series analysis

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- *STATISTICAL METHODS FOR RESEARCH WORKERS,* By Ronald A. Fisher (1925)
  http://psychclassics.yorku.ca/Fisher/Methods/chap6.htm

1. Studiul unei variabile aleatoare

- media şi dispersia (interval de încredere)

- repartiţia (histograma)   (teorema limită centrală)

T1: media de selecţie este o estimaţie absolute corectă a mediei teoretice

T2: dispersia de selecţie este o estimaţie corectă a dispersiei teoretice

T3: Oricare ar fi v.a. X variabila x barat normată converge in repartiţie la N(0,1)

Deci: $Z_n \rightarrow N(0,1)$

unde: $Z_n = (S_n - M(S_n)) / radical(D(S_n))$

iar: $S_n = X_1 + X_2 + ... + X_n$

2. Verificarea ipotezelor statice

- $M(X) = m$

       sel vol.mare

       sel.vol.mic

- $m_1 = m_2$

- despre repartiţie – testul Hi^2

$Hi^2 = $ suma $\{ (fo_i – ft_i)^2 \mid i=1,n\}$ are repartiţia hi pătrat

3. Dependenţa a două variabile

$Y = a X + b$ a,b=?

      (M.C.M.M.P.)

Corelatie

Raport de corelatie

4. Numere aleatoare

- *uniforme*
- *normale*
- *exponentiale*
- *lognormale*
- *rep. discretă*

Numere aleatoare

1. Numere aleatoare Uniforme: *Random*

- RND   [0,1]   - genereaza valori reale intre [0,1]
                        (pe scurt U[0,1])

- Random(n); - genereaza nr intregi intre 0 si n
                        (pe scurt U(n))

- In general: U([a,b])= a+(b-a)*U([0,1])

2. Numere aleatoare Normale N(0,1) şi N(a,σ);

- legea *limita centrala*
- caz particular : $Z_n = (X_1+X_2+...+X_n - n/2)/abatere_n$
  pt. n=12, $abatere_n =1$;

      Functia Norm; { N(0,1) }

         s:=-6;

         Pentru i:=1,12 executa

            s:=s+Random;

         sfpt
         Norm:=s

      SfNorm

  N(a, σ) = a + σ*Norm

Ex.2. Generarea unui sir de nr. aleatoare normale si analiza lor (gruparea pe intervale)

3. Numere aleatoare Lognormale

- Definitie: Variabila aleatore X are repartitia *lognormala* daca Z=Ln(X) are repartitia normala!

- Dacă mx şi sx² sunt media, respectiv dispersia lui X, atunci

    $$s_Z^2 = \ln (sx^2/mx^2 + 1)$$

    $$m_Z = \ln mx - 0.5 * s_Z^2$$

- unde m_Z şi s_Z^2 sunt media, respectiv dispersia lui Z.

Ex.3. Generarea unui sir de nr. aleatoare lognormale si analiza lor (gruparea pe intervale)

4. Numere aleatoare Poisson

- Repartitie Poisson:

$$P(X=k)=(\lambda^k/(k!))*e^{-\lambda} \quad , \quad k \geq 0$$

- Fie $L:=e^{-\lambda}$. Alg. Pt. generarea lui X duce la funcţia:

```
function fPoisson:integer;      var  j : integer;      P,u : real;
begin
    j:=0;  P:=random;
    If L>P then      Repeat

                        u:=random;
                        P:=P*u;      j:=j+1
                until P<L

    fPoisson:=j
end;
```

5. Numere aleatoare Exponentiala

- Daca X are rep. Poisson atunci Timpul T dintre 2 aparitii ale lui X are repartitie Exponentiala
- Densitatea la rep. exponentiala este: $\varphi(x)=\lambda e^{-\lambda x}$, x>0.
- Generator:

function exponen:real;

var z:real;

begin

  z:=random;

  exponen:=-ln(z)/lambda;

end;

6. Numere aleatoare Repartitia Finita

$$| \; v1 \; v2 \; \dots \; vn \; |$$
$$| \; p1 \; p2 \; \dots \; pn \; |$$

■ Generator: notăm prin c_i frecvenţele cumulate:

$$c_1 = p_1, \quad c_i = c_{i-1} + p_i, \quad i \geq 2$$

Functia Repfinit;
i:=1;
Cit timp u<c_i executa
i:=i+1;
sfcât
RepFinit:=vi
sfRepfinit

Exemple: 1.

- **Determinarea capacităţii portante a elementelor de rezistenţă ale unei clădiri**

C = [(b*h2*Rb + Aa*Ra*(h-a')] / (h-a) = f(Ra,Rb,a,b,a',h,A)

unde

C = capacitatea portantă
Ra = rezistenţa armăturii
Rb= rezistenţa betonului
a = distanţa între axa armăturii întinse şi faţa de beton
b = lăţimea secţiunii de beton
a'= distanţa între axa armăturii comprimate şi faţa de beton
h = înălţimea secţiunii de beton
Aa = cantitatea de armătură.

Se mai ştie că Aa are repartiţie lognormală, iar celelalte 6 variabile au repartiţie normală, cu parametri cunoscuţi. Determinaţi repartiţia lui C.

legătura dintr repartiţia normală şi cea lognormală:

$Y = \ln X$

cu

$sy2 = \ln (sx2/mx2+1)$
$my = \ln mx - 0.5 * sy2$

La noi: $mx = 3.99$, iar $sx2=0.416$

Generalizare:
$R = f(X1, X2, ..., Xn)$

Exemple: 2.

Simularea producerii cutremurelor de pământ

Elemente ce caracterizează un cutremur:

(data,magnitudinea)

Se ştie că:

- magnitudinea cutremurelor are o lege lognormală cu media 3.99 şi dispersia 0.416,

- intervalul de timp dintre două cutremure are o repartiţie exponenţială cu media 0.6.

Simulaţi producerea cutremurelor pe următorii n ani.

(n=100)

legătura dintr repartiţia normală şi cea lognormală:

$$Y = \ln X, \quad cu \quad sy2 = \ln (sx2/mx2+1)$$

$$my = \ln mx - 0.5 * sy2$$

La noi: mx = 3.99, iar sx2=0.416

Algoritmul SimCutremur este:

```
Fie Tc:=2006;  nc:=0;
Citeste Tf;
Câttimp Tc < Tf  executa
Fie d:= expo;
Tc:= Tc+d;
m := lognormal;
nc:=nc+1
Tipăreşte nc, Tc, m
```

sfalgoritm

Exemple: 3.

Simularea fluxului de calatori într-o statie CFR

Într-o staţie CFR (ex. Cluj) sosesc trenuri conform mersului trenurilor (cunoscut, caz ideal fara intirzieri, dar real cu întârzieri aleatoare). Mersul trenurilor furnizează pe lângă ora sosirii trenului şi numărul vagoanelor de persoane. Se ştie că într-un vagon sosesc un număr de calatori generati de UNIFORM[10,100]. Un tren poate avea întârziere cu probabilitatea p=0.20 iar durata întârzierii are repartiţia exponenţială de valoare medie 15 minute.

- Simulaţi sosirea trenurilor pe durata unei zile, tipărind numărul călătorilor sositi în gară.

- Dacă aceşti călători pleacă în oraş cu autobuze, tramvaie, sau troleibuze, determinaţi pentru fiecare staţie s∈{Sautobuz, Stroleibuz, Stramvai}:

- - durata medie de aşteptare a unui călător în staţia s;

- - lungimea medie a cozii din staţia s.

- Pentru aceasta se ştie procentul călătorilor care pleacă cu fiecare mijloc de transport în comun, care e capacitatea acestui mijloc şi durata dintre 2 sosiri consecutive:

mijloc de transport	procentul călătorilor	capacitatea	durata dintre 2 sosiri consecutive
autobuz	20%	U[10,50]	15' + Expo(0.3)
troleibuz	25%	U[20,80]	8'+ Expo(0.2)
tramvai	35%	U[20,120]	6' + Expo(0.5)

...

Procedure OZi; { }
  Init;
 Câttimp (nu s-a terminat ziua) execută {Tc<Tfinal}
    Dacă soseşte tren atunci
        *gen. nr.calatorilor sositi
        *gen. sosirea noului tren (extr. din mersul trenurilor)
        *dacă va avea intârziere, generează întârzierea
    sfdacă
    Pentru i:=1;3 execută
    "i" *generează sosirea şi urcarea în mijlocul de transport

    sfp
    Tc:=Tc+1
  sfcât
  *calculează rezultatele dorite
sfOZI

Exemple: 4.

Simularea stocarii intr-un depozit

Cimentul necesar unui şantier de construcţii este păstrat într-un depozit. Se ştie că:

- necesarul zilnic de ciment este $N(a,\sigma)$, a=30 tone, $\sigma2=2$;
- preţul de depozitare pe o zi a unei tone de ciment este d=1 (unitatea în mii lei);
- lipsa cimentului din depozit costa c lei/ tonă pe zi, c=40;
- reaprovizionarea depozitului se face printr-un transport de la o fabrică de ciment, cu o cantitate fixă = 400 tone şi costă v mii lei (v=75). Comanda se face când în depozit cantitatea scade sub o cantitate critică Q, iar cererea este satisfăcută după h zile; h are repartiţia

$$\begin{pmatrix} 2 & 3 & 4 \\ 0.25 & 0.4 & 0.35 \end{pmatrix}$$

a) Simulați funcţionarea depozitului pe durata unui an de zile (240 zile lucratoare), precizând valoarea cheltuielilor de depozitare, penalizare şi reaprovizionare pentru Q=100.

b) Determinați valoarea lui Q pentru care costul total este minim.

Modelare:

Cd - cost de depozitare: -provine din imobilizarea unor sume de bani:

-pret cost al produselor (blocare sume)

-pret de functionare a depozitului

Cp – Cost penalizare pentru lipsa de stoc (oamenii nu lucrează)

Cr - Cost de reaprovizionare (posta, transport ...)

Cost total functionare depozit: Ct = Cd+Cp+Cr

Presupunem ca depozitul are capacitate nelimitata si are cantitate suficienta de materiale.

Interpretarea rezultatelor

Ct este o variabilă aleatoare !

Estimarea valorii medii ! – Interval de încredere $\rightarrow$ (Ctb-r, Ctb+r)

Optimizarea funcţionării:

Min=999999.0

Pentru Q:=90 ; 180 ; 5 execută

    *simularea de mai sus =Ctb cu eroarea r.

    Dacă Ctb<Min atunci

    Min:=Ctb;   Qoptim:=Q

    sfdacă

sfpentru

INTERVAL DE INCREDERE pt. valoarea medie

IntInc ::=   (x-r, x+r) unde

x = media de selectie, iar raza

$r = \alpha_q$ * radical(dx/n)

    la nivelul de semnificaţie q! Avem:

q	α_q
5%	1.96
1%	2.56
0.1%	3

Exemple: 5.

Proces de naştere şi moarte

N(t) = numărul clienţilor existenţi la momentul t (din coadă!)

Sosire în coadă (naştere):

$P(N(t+\Delta t)=n+1 | N(t)=n) = \lambda_n . \Delta t + o(\Delta t)$

Plecare din coadă (moarte):

$P(N(t+\Delta t)=n-1 | N(t)=n) = \mu_n . \Delta t + o(\Delta t)$

Mai multe schimbari într-un interval de timp Δt au probabilitatea $= o(\Delta t)$

$\lambda_n =$ intensitatea de natalitate

$\mu_n . =$ intensitatea de mortalitate

Dorim să calculăm

$$P_n(t) = P(N(t) = n)$$

Observăm că

$P_n(t + \Delta t) = P_n(t) * [1 - \lambda_n \Delta t + o(\Delta t)] * [1 - \mu_n \Delta t + o(\Delta t)] +$

$\quad + P_{n-1}(t)) * [\lambda_{n-1} \Delta t + o(\Delta t)] * [1 - \mu_{n-1} \Delta t + o(\Delta t)] +$

$\quad + P_{n+1}(t) * [1 - \lambda_{n+1} \Delta t + o(\Delta t)] * [\mu_{n+1} \Delta t + o(\Delta t)]$

$\quad + \sum \{ [P_{n-i}(t) + P_{n+i}(t)] * [1 - o(\Delta t)] \mid i > 1 \}$

de unde

$$[P_n(t + \Delta t) - P_n(t)] / \Delta t = \lambda_{n-1} P_{n-1}(t) + \mu_{n+1} P_{n+1}(t) - (\lambda_n + \mu_n) P_n(t) + o(\Delta t) / \Delta t$$

sau

$$P'_n(t) = \lambda_{n-1} P_{n-1}(t) + \mu_{n+1} P_{n+1}(t) - (\lambda_n + \mu_n) P_n(t)$$

şi

$$P'_0(t) = \mu_1 P_1(t) - \lambda_0 P_0(t)$$

In plus

$$\sum \{P_n(t)|\ n \geq 0\} = 1.$$

Presupunem că avem un **proces staţionar**, deci

$$P_n(t) = p_n$$

In acest caz

$$\lambda_0\ p_0 = \mu_1 p_1$$

şi

$$\lambda_n\ p_n - \mu_{n+1} p_{n+1} = \lambda_{n-1}\ p_{n-1} - \mu_n p_n\ \ \ = \ldots = 0$$

deci

$$\lambda_n\ p_n = \mu_{n+1} p_{n+1}$$

...

Rezultă că

$$p_{n+1} = \lambda_n \, p_n / \, \mu_{n+1} = \ldots = \Pi \{\lambda_i / \, \mu_{i+1} \mid i \geq 0\}.p_o$$

şi

$$p_o = 1 / [1 + \Pi \{\lambda_i / \, \mu_{i+1} \mid i \geq 0\}].$$

In cazul unui regim permanent avem

$$\lambda_n = \lambda$$

$$\mu_n = \mu$$

şi $\quad p_o = 1 - f$

unde $f = \lambda / \mu$ se numeşte intensitate de trafic.

Putem calcula nr.mediu de unităţi din coadă:

$$\text{barat} = \sum \{n p_n \mid n \geq 0\} = (1-f) \sum \{n.f^n \mid n \geq 0\} = f/(1-f)$$

Exemple: 6.

Simularea unei cozi de asteptare

Variabile:

  V= intervalul dintre două veniri (sosiri în coadă)

  S = durata unui serviciu

  TT= timpul total de aşteptare al tuturor clienţilor

  NS= numărul clienţilor

  TL= timpul total de "şomaj" al statiei

Se cere:

  - timpul mediu de aşteptare al unui client

  - numărul mediu de clienţi în sistem

Model: (A | S | C : L, d)

unde:

- A= repartiţia timpului între două sosiri;
- S= repartiţia duratei unui serviciu;
- C= numărul satţiilor de servire;
- L= lungimea maximă a cozii;
- d= disciplina servirii (de obicei FIFO).