Decision Making Methods

- The ELECTRE methods
- The PROMETHEE methods
- Group decision making
- **Sensitivity analysis**

Introduction to Decision Making Methods, **János Fülöp**
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http://academic.evergreen.edu/projects/bdei/documents/decisionmakingmethods.pdf

1. Sensitivity analysis

Some ***values*** of the multiattribute decision models are often **subjective**. ***The weights of the criteria*** and the scoring values of the alternatives against the subjective (judgmental) criteria **contain always some *uncertainties***.

It is therefore **an important question** how the ***final ranking*** or ***the ranking values*** of the alternatives **are *sensitive* to the *changes* of some *input parameters*** of the *decision model*.

The simplest case is when **the value of the weight of a single criterion** is allowed to vary. For additive *multiattribute models*, the ranking values of the alternatives are *simple linear functions* of this *single variable* and attractive graphical tools can be applied to present a *simple* ***sensitivity analysis*** to a user.

(Forman and Selly, 2001).

For ***a wide class of multiattribute decision models*** Mareschal, 1988 showed how to determine ***the stability intervals or regions*** for the weights of different criteria. These consist of ***the values that the weights*** of one or more criteria can take ***without altering the results*** given by the initial set of weights, all other weights being kept constant.

Wolters and Mareschal, 1995 proposed ***a linear programming model*** to find ***the minimum modification of the weights*** required ***to make a certain alternative ranked first***.

Triantaphyllou and Sanchez, 1997 presented an approach of a ***more complex sensitivity analysis with the change of the scores*** of the alternatives against the criteria, as well.

A methodology was presented by Mészáros and Rapcsák, 1996 for ***a wide class of MAUT models*** where the aggregation is based on generalized means, including so the additive and multiplicative models as well. In this approach *the weights and the scores* of the alternatives against the criteria **can change simultaneously** in given intervals. The following questions were addressed:

– *What are the intervals of the final ranking values* of the alternatives with the restriction that the *intervals of the weights and scores are given*?

– *What are the intervals of the weights and scores* with the restriction that *the final ranking of the alternatives does not change*?

– Consider a *subset of alternatives* whose ranking values *are allowed to change in an interval. In what intervals are the weights and scores* allowed to vary, and how will these modifications effect *the ranking values of the entire set of alternatives*?

Mészáros and Rapcsák, 1996 pointed out that these questions lead to the optimization of linear fractional functions over rectangles and proposed an efficient technique to solve these problems. Some of the results of Mészáros and Rapcsák, 1996 were recently extended by Ekárt and Németh, 2005 for more general decision functions.

Sensitivity Analysis is:

A technique used to determine how different values of an independent variable will impact a particular dependent variable under a given set of assumptions. This technique is used within specific boundaries that will depend on one or more input variables, such as the effect that changes in interest rates will have on a bond's price.

Sensitivity analysis is a way to predict the outcome of a decision if a situation turns out to be different compared to the key prediction(s).

Investopedia explains ***Sensitivity Analysis :*** (http://www.investopedia.com/terms/s/sensitivityanalysis.asp)

Sensitivity analysis is very useful when attempting to determine the impact the actual outcome of a particular variable will have if it differs from what was previously assumed. By creating a given set of scenarios, the analyst can determine how changes in one variable(s) will impact the target variable.

For example, an analyst might create a financial model that will value a company's equity (the dependent variable) given the amount of earnings per share (an independent variable) the company reports at the end of the year and the company's price-to-earnings multiple (another independent variable) at that time. The analyst can create a table of predicted price-to-earnings multiples and a corresponding value of the company's equity based on different values for each of the independent variables.

Sensitivity analysis (**SA**) is the study of how the variation (uncertainty) in the output of a mathematical model can be apportioned, qualitatively or quantitatively, to different sources of variation in the input of a model [1].

1. a b Saltelli, A., Ratto, M., Andres, T., Campolongo, F., Cariboni, J., Gatelli, D. Saisana, M., and Tarantola, S., 2008, Global Sensitivity Analysis. The Primer, John Wiley & Sons.

In more general terms uncertainty and ***sensitivity analyses*** investigate the robustness of a study when the study includes some form of mathematical modelling. While uncertainty analysis studies the overall uncertainty in the conclusions of the study, sensitivity analysis tries to identify what source of uncertainty weights more on the study's conclusions. For example, several guidelines for modelling (see e.g. one from the US EPA) or for impact assessment (see one from the European Commission) prescribe sensitivity analysis as a tool to ensure the quality of the modelling / assessment.

The problem setting in ***sensitivity analysis*** has strong similarities with design of experiments. In design of experiments one studies the effect of some process or intervention (the 'treatment') on some objects (the 'experimental units'). In sensitivity analysis one looks at the effect of varying the inputs of a mathematical model on the output of the model itself. In both disciplines one strives to obtain information from the system with a minimum of physical or numerical experiments.

In uncertainty and ***sensitivity analysis*** there is a crucial trade off between how scrupulous an analyst is in exploring the input assumptions and how wide the resulting inference may be. The point is well illustrated by the econometrician Edward E. Leamer (1990) [2]:

I have proposed a form of organized sensitivity analysis that I call 'global sensitivity analysis' in which a neighborhood of alternative assumptions is selected and the corresponding interval of inferences is identified. Conclusions are judged to be sturdy only if the neighborhood of assumptions is wide enough to be credible and the corresponding interval of inferences is narrow enough to be useful.

2. Leamer, E., (1990) Let's take the con out of econometrics, and Sensitivity analysis would help. In C. Granger (ed.), Modelling Economic Series. Oxford: Clarendon Press 1990.

Note Leamer's emphasis is on the need for 'credibility' in the selection of assumptions. The easiest way to invalidate a model is to demonstrate that it is fragile with respect to the uncertainty in the assumptions or to show that its assumptions have not been taken 'wide enough'. The same concept is expressed by Jerome R. Ravetz, for whom bad modeling is when uncertainties in inputs must be suppressed lest outputs become indeterminate.[3]

3. Ravetz, J.R., 2007, No-Nonsense Guide to Science, New Internationalist Publications Ltd..

In modern econometrics the use of ***sensitivity analysis*** to anticipate criticism is the subject of one of the ten commandments of applied econometrics (from Kennedy, 2007[4]):

Thou shall confess in the presence of sensitivity. Corollary: Thou shall anticipate criticism [···] When reporting a sensitivity analysis, researchers should explain fully their specification search so that the readers can judge for themselves how the results may have been affected. This is basically an ‘honesty is the best policy’ approach, advocated by Leamer, (1978[5]).

4. Ravetz, J.R., 2007, No-Nonsense Guide to Science, New Internationalist Publications Ltd.
5. Kennedy, P. (2007). A guide to econometrics, Fifth edition. Blackwell Publishing..

The use of mathematical modelling can be the subject of controversies, see Nassim Nicholas Taleb[6] in Economics, and Orrin H. Pilkey and Linda Pilkey Jarvis[7] in Environmental Sciences. As noted by the latter Authors, this increases the relevance of ***sensitivity analysis*** in today's modelling practice[1] .

6. Taleb, N. N., (2007) The Black Swan: The Impact of the Highly Improbable, Random House.
7. Pilkey, O. H. and L. Pilkey-Jarvis (2007), Useless Arithmetic. Why Environmental Scientists Can't Predict the Future. New York: Columbia University Press..

There are a number of questions that could be asked concerning the sensitivity of an optimal solution to changes in the data.

Every commercial linear-programming system provides this elementary sensitivity analysis, since the calculations are easy to perform using the tableau associated with an optimal solution. There are two variations in the data that invariably are reported: objective function and right-hand-side ranges. The objective-function ranges refer to the range over which an individual coefficient of the objective function can vary, without changing the basis associated with an optimal solution. In essence, these are the ranges on the objective-function coefficients over which we can be sure the values of the decision variables in an optimal solution will remain unchanged. The right-hand-side ranges refer to the range over which an individual right-hand-side value can vary, again without changing the basis associated with an optimal solution. These are the ranges on the right-hand-side values over which we can be sure the values of the shadow prices and reduced costs will remain unchanged.

Further, associated with each range is information concerning how the basis would change if the range were exceeded. These concepts will become clear if we deal with a specific example.

We will consider for concreteness the *custom-molder* example in order to increase the complexity somewhat, let us add a third alternative to the production possibilities.

Suppose that, besides the six-ounce juice glasses x1 and the ten-ounce cocktail glasses x2, our molder is approached by a new customer to produce a champagne glass.

The champagne glass is not difficult to produce except that it must be molded in two separate pieces—the bowl with stem and then base.

As a result, the production time for the champagne glass is 8 hours per hundred cases, which is greater than either of the other products.

The storage space required for the champagne glasses is 1000 cubic feet per hundred cases; and the contribution is $6.00 per case, which is higher than either of the other products.

There is no limit on the demand for champagne glasses. Now what is the optimal product mix among the three alternatives?

The formulation of the custom-molding example, including the new activity of producing champagne glasses, is straightforward. We have exactly the same capacity limitations—hours of production capacity, cubic feet of warehouse capacity, and limit on six-ounce juice-glass demand—and one additional decision variable for the production of champagne glasses. Letting

$x1$ = Number of cases of six-ounce juice glasses, in hundreds;

$x2$ = Number of cases of ten-ounce cocktail glasses, in hundreds;

$x3$ = Number of cases of champagne glasses, in hundreds;

and measuring the contribution in hundreds of dollars, we have the following formulation of our custommolder example:

Maximize $z = 5x1 + 4.5x2 + 6x3$, (hundreds of dollars)

subject to:

$$
\begin{aligned}
6x1 + 5x2 + 8x3 &\le 60, && \text{(production capacity; hours)} \\
10x1 + 20x2 + 10x3 &\le 150, && \text{(warehouse capacity; hundreds of sq. ft.)} \qquad (1) \\
x1 &\le 8, && \text{(demand for 6 oz. glasses; hundreds of cases)} \\
x1 \ge 0,\ x2 \ge 0,\ x3 &\ge 0.
\end{aligned}
$$

If we add one slack variable in each of the less-than-or-equal-to constraints, the problem will be in the following canonical form for performing the simplex method:

$$6x1 + 5x2 + 8x3 + x4 = 60, \quad (2)$$

$$10x1 + 20x2 + 10x3 + x5 = 150, \quad (3)$$

$$x1 + x6 = 8, \quad (4)$$

$$5x1 + 4.5x2 + 6x3 - z = 0. \quad (5)$$

The corresponding initial table:

BV	CV	x1	x2	x3	x4	x5	x6
x4	60	6	5	8	1		
x5	150	10	20	10	0	1	
x6	8	1	0	0			1
-z	**0**	**5**	**4.5**	**6**			

The final table (computed by formula):

BV	CV	x1	x2	x3	x4	x5	x6
x2	4 2/7	0	1	- 2/7	- 1/7	3/35	0
x6	1 4/7	0	0	-1 4/7	- 2/7	1/14	1
x1	6 3/7	1	0	1 4/7	2/7	- 1/14	0
-z	**-51 3/7**	**0**	**0**	**- 4/7**	**- 11/14**	**- 1/35**	**0**

We wish to analyze the effect on the optimal solution of changing various elements of the problem data without re-solving the linear program or having to remember any of the intermediate tableaus generated in solving the problem by the simplex method. The type of results that can be derived in this way are conservative, in the sense that they **provide sensitivity analysis** for changes in the problem data small enough so that the same decision variables remain basic, but not for larger changes in the data.

In the sensitivity analysis discussed thus far, we have restricted our presentation to changes in the problem data that can be made without changing the optimal basis. Consequently, what we have been able to say is fairly conservative. We did go so far as to indicate the variable that would enter the basis and the variable that would leave the basis when a boundary of a range was encountered. Further, in the case of alternative optimal solutions and alternative optimal shadow prices, the indicated pivot was completed at least far enough to exhibit the particular alternative. One important point in these discussions was the ease with which we could determine the pivot to be performed at a boundary of a range. This seems to indicate that it is relatively easy to make systematic calculations beyond the indicated objective-function or righthand-side ranges. This, in fact, is the case; and the procedure by which these systematic calculations are made is called **parametric programming**.

Glass Problem ~ Formulation

	Prod.	Time (hrs)	Storage	Contribution	Limit
Juice	100	6	10	$5	<800
Cocktail	100	5	20	$4.50	N/A
Champagne	100	8	10	$6	N/A

Number of Glasses
Hundreds of cases per week

Juice	6.428571
Cocktail	4.285714
Champagne	1.571429

60 Hours production/week
15,000 cubic feet storage space

Constraints

72.57142857	<=	60	Production capacity must not exceed 60 hours
165.7142857	<=	150	Warehouse capacity must not exceed 150
6.428571429	<=	8	Demand for 6oz glasses must be less than 8

Total Contribution

60.85714286

Glass Problem ~ Optimum

	Prod.	Time (hrs)	Storage	Contribution	Limit
Juice	100	6	10	$5	<800
Cocktail	100	5	20	$4.50	N/A
Champagne	100	8	10	$6	N/A

Number of Glasses	
Hundreds of cases per week	
Juice	6.428571
Cocktail	4.285714
Champagn	0

60 Hours production/week
15,000 cubic feet storage space

Constraints

60	<=	60	Production capacity must not exceed 60 hours
150	<=	150	Warehouse capacity must not exceed 150
6.428571429	<=	8	Demand for 6oz glasses must be less than 8

Total Contribution

51.42857143

An example for analysis ~ http://web.mit.edu/15.053/www/AMP-Chapter-03.pdf

Glass Problem ~ Sensitivity Report

Microsoft Excel 9.0 Sensitivity Report
Worksheet: [Glass, pg 110.xls]Optimum
Report Created: 7/18/01 5:58:51 PM

Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
I6	Juice	6.428571429	0	5	0.4	0.363636364
I7	Cocktail	4.285714286	0	4.5	2	0.333333333
I8	Champagne	0	-0.571428571	6	0.571428571	1E+30

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
A14	Constraints	60	0.785714286	60	5.5	22.5
A15	Constraints	150	0.028571429	150	90	22
A16	Constraints	6.428571429	0	8	1E+30	1.571428571

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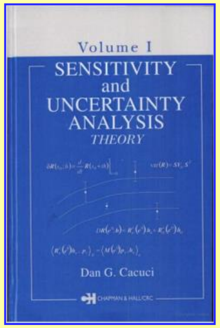
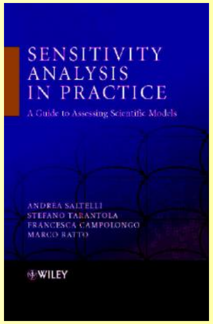
- Andrea Saltelli, Marco Ratto, Stefano Tarantola and Francesca Campolongo, SENSITIVITY ANALYSIS PRACTICES. STRATEGIES FOR MODEL-BASED INFERENCE - European Commission, Joint Research Centre of Ispra (I).
- Stephen L Nelson, Jr M.D., PH.D., Stephen L. Nelson, - Excel 2007 Data Analysis For Dummies - The sensitivity Report - Page 11-27

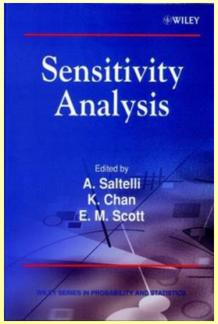
- Decision Making Techniques, How to Make Good Decisions, http://www.mindtools.com/pages/main/newMN_TED.htm
- Investopedia explains ***Sensitivity Analysis***, http://www.investopedia.com/terms/s/sensitivityanalysis.asp
- Sensitivity analysis, http://en.wikipedia.org/wiki/Sensitivity_analysis
- Lucia Breierova, Mark Choudhari, ***An Introduction to Sensitivity Analysis***, http://sysdyn.clexchange.org/sdep/Roadmaps/RM8/D-4526-2.pdf
- David J. Pannell , ***Sensitivity analysis: strategies, methods, concepts, examples*** , http://cyllene.uwa.edu.au/~dpannell/dpap971f.htm



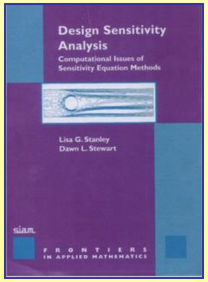
- D. G. Cacuci, ***Sensitivity and Uncertainty Analysis: Theory***



- Andrea Saltelli, ***Sensitivity analysis in practice: a guide to assessing scientific models***



- Andrea Saltelli, ***Sensitivity analysis***



- Lisa Gayle Davis Stanley, Dawn L. Stewart, ***Design sensitivity analysis: computational issues of sensitivity equation methods***

Definition

Sensitivity analysis (SA) is " the study of how the variation in the output of a model (numerical or otherwise) can be apportioned, qualitatively or quantitatively, to different sources of variation".[1] However, when the assumptions are uncertain, and/or there are alternative sets of assumptions to chose from, the inference will also be also uncertain. Investigating the uncertainty in the inference (regardless of its source) goes under the name of **Uncertainty analysis**.

Sensitivity Analysis tries to identify those assumptions which weight the most in determining the uncertainty in the inference ('screening' sensitivity analysis). 'Quantitative' sensitivity analysis tries not only to identify but also to quantify the relative importance the influential assumptions. In the preceding discussion the term 'factor' if often used instead of 'assumption' - implying that assumptions have been translated into factors entering the model, e.g. with defined numerical values possibly drawn from factor-value distributions - while 'model output' can be used instead of inference.

Overview

- A mathematical model is defined by a series of equations, input factors, parameters, and variables aimed to characterize the process being investigated.
- Input is subject to many sources of uncertainty including errors of measurement, absence of information and poor or partial understanding of the driving forces and mechanisms. This uncertainty imposes a limit on our confidence in the response or output of the model. Further, models may have to cope with the natural intrinsic variability of the system, such as the occurrence of stochastic events.

- Good modeling practice requires that the modeler provides an evaluation of the confidence in the model, possibly assessing the uncertainties associated with the modeling process and with the outcome of the model itself. Uncertainty and Sensitivity Analysis offer valid tools for characterizing the uncertainty associated with a model.
- In models involving many input variables sensitivity analysis is an essential ingredient of model building and quality assurance. National and international agencies involved in impact assessment studies have included section devoted to sensitivity analysis in their guidelines. Examples are the European Commission, the White House Office for Budget and Management, the Intergovernmental Panel on Climate Change and the US Environmental Protection Agency.

Methodology

- There are several possible procedures to perform uncertainty (UA) and sensitivity analysis (**SA**). The most common sensitivity analysis is sampling-based. A sampling-based sensitivity is one in which the model is executed repeatedly for combinations of values sampled from the distribution (assumed known) of the input factors. Sampling based methods can also be used to decompose the variance of the model output (see references).

- In general, UA and **SA** are performed jointly by executing the model repeatedly for combination of factor values sampled with some probability distribution. The following steps can be listed:

  1. Specify the target function and select the input of interest
  2. Assign a distribution function to the selected factors
  3. Generate a matrix of inputs with that distribution(s) through an appropriate design
  4. Evaluate the model and compute the distribution of the target function
  5. Select a method for assessing the influence or relative importance of each input factor on the target function.

Applications

- **S**ensitivity **A**nalysis can be used to determine:

1. The model resemblance with the process under study
2. The quality of model definition
3. Factors that mostly contribute to the output variability
4. The region in the space of input factors for which the model variation is maximum
5. Optimal - or instability - regions within the space of factors for use in a subsequent calibration study
6. Interactions between factors

Sensitivity **A**nalysis is popular in financial applications, risk analysis, signal processing, neural networks and any area where models are developed. **SA** can also be used in model-based policy assessment studies see e.g. [1].

Environmental

- Computer environmental models are increasingly used in a wide variety of studies and applications. For example global climate model are used for both short term weather forecasts and long term climate change.
- Moreover, computer models are increasingly used for environmental decision making at a local scale, for example for assessing the impact of a waste water treatment plant on a river flow, or for assessing the behavior and life length of bio-filters for contaminated waste water.

- In both cases sensitivity analysis may help understanding the contribution of the various sources of uncertainty to the model output uncertainty and system performance in general. In these cases, depending on model complexity, different sampling strategies may be advisable and traditional sensitivity indexes have to be generalized to cover multivariate sensitivity analysis, heteroskedastic effects and correlated inputs.

Business

- In a decision problem, the analyst may want to identify cost drivers as well as other quantities for which we need to acquire better knowledge in order to make an informed decision. On the other hand, some quantities have no influence on the predictions, so that we can save resources at no loss in accuracy by relaxing some of the conditions. See Corporate finance: Quantifying uncertainty.

Sensitivity analysis can help in a variety of other circumstances which can be handled by the settings illustrated below:

- to identify critical assumptions or compare alternative model structures
- guide future data collections
- detect important criteria
- optimize the tolerance of manufactured parts in terms of the uncertainty in the parameters
- optimize resources allocation
- model simplification or model lumping, etc.

However there are also some problems associated with sensitivity analysis in the business context:

- Variables are often interdependent, which makes examining them each individually unrealistic, e.g.: changing one factor such as sales volume, will most likely affect other factors such as the selling price.
- Often the assumptions upon which the analysis is based are made by using past experience/data which may not hold in the future.
- Assigning a maximum and minimum (or optimistic and pessimistic) value is open to subjective interpretation. For instance one persons 'optimistic' forecast may be more conservative than that of another person performing a different part of the analysis. This sort of subjectivity can adversely affect the accuracy and overall objectivity of the analysis.

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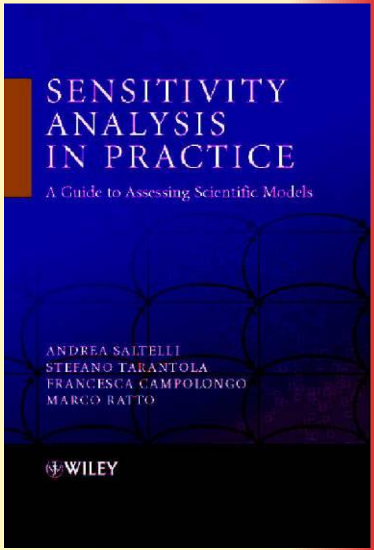
Tutorial on *Sensitivity Analysis*

To start with, we propose the following material :



- Cookbook
- Tutorial 1
- Tutorial 2
- Tutorial 3
- Book on Sensitivity Analysis (Preface)

A REAL TEST CASE

We also suggest some bibliographic material, where the reader will find the grater part of sensitivity analysis studies, together with applications and reviews:

- Sensitivity Analysis for Chemical Models
- Composite Indicators
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An Introduction to Sensitivity Analysis

Prepared for the

MIT System Dynamics in Education Project

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Vensim Examples added October 2001

7.1 LEMONADE STAND MODEL

7.2 EPIDEMICS MODEL

7.3 COFFEEHOUSE MODEL

Lemonade Stand

In the first exploration, let's look at a lemonade stand located on a college campus.

As usual, we are particularly interested in the behavior of the stock, the number of cups of lemonade that are ready to be sold to customers. The stand is open eight hours every day.

Howard, the owner, is the only person working in the stand.

Epidemics

In the second exploration we look at an epidemics model. The model was already used in a previous chapter in Road Maps, so it is possible that you have already built it.

Coffeehouse

We now return to Howard, the owner of the lemonade stand on a college campus.

Howard realized that it could be more profitable for him to sell coffee because students tend to drink more coffee than lemonade, and they drink it at any time of the day and night. Therefore, he closed his lemonade stand and opened a 24-hour Coffeehouse.

Howard bases the Coffeehouse model on the model he used in his lemonade stand to model the number of cups of "Coffee ready." We will run the simulation over a period of two days, or 48 hours.

Conclusions

Specific parameter values can change the appearance of the graphs representing the behavior of the system. But significant changes in behavior do not occur for all parameters. System dynamics models are in general insensitive to many parameter changes. It is the structure of the system, and not the parameter values, that has most influence on the behavior of the system.

Sensitivity analysis is an important tool in the model building process. By showing that the system does not react greatly to a change in a parameter value, it reduces the modeler's uncertainty in the behavior. In addition, it gives an opportunity for a better understanding of the dynamic behavior of the system.

We encourage you to experiment with the three models from this paper (as well as any other models that you have built) on your own. For example, try to change several parameters at the same time, observe the behavior produced, and compare it to the conclusions in this paper. Can you suggest any parameter values that would produce the "optimal," or most desirable behavior? The use of sensitivity analysis in such policy analysis will be explored in a later paper in this series.

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Sensitivity analysis

Sensitivity analysis (SA) is the study of how the variation (uncertainty) in the output of a mathematical model can be apportioned, qualitatively or quantitatively, to different sources of variation in the input of a model [1]. Put another way, it is a technique for systematically changing parameters in a model to determine the effects of such changes.

In more general terms uncertainty and sensitivity analyses investigate the robustness of a study when the study includes some form ofmathematical modelling. Sensitivity analysis can be useful to computer modellers for a range of purposes[2], including:

- support decision making or the development of recommendations for decision makers (e.g. testing the robustness of a result);
- enhancing communication from modellers to decision makers (e.g. by making recommendations more credible, understandable, compelling or persuasive);
- increased understanding or quantification of the system (e.g. understanding relationships between input and output variables); and
- model development (e.g. searching for errors in the model).

While uncertainty analysis studies the overall uncertainty in the conclusions of the study, sensitivity analysis tries to identify what source of uncertainty weights more on the study's conclusions. For example, several guidelines for modelling (see e.g. one from the US EPA) or forimpact assessment (see one from the European Commission) prescribe sensitivity analysis as a tool to ensure the quality of the modelling/assessment.

… Sensitivity analysis

The problem setting in sensitivity analysis has strong similarities with design of experiments. In design of experiments one studies the effect of some process or intervention (the 'treatment') on some objects (the 'experimental units'). In sensitivity analysis one looks at the effect of varying the inputs of a mathematical model on the output of the model itself. In both disciplines one strives to obtain information from the system with a minimum of physical or numerical experiments.

In uncertainty and sensitivity analysis there is a crucial trade off between how scrupulous an analyst is in exploring the input assumptions and how wide the resulting inference may be. The point is well illustrated by the econometrician Edward E. Leamer (1990) [3]:

I have proposed a form of organized sensitivity analysis that I call ‘global sensitivity analysis’ in which a neighborhood of alternative assumptions is selected and the corresponding interval of inferences is identified. Conclusions are judged to be sturdy only if the neighborhood of assumptions is wide enough to be credible and the corresponding interval of inferences is narrow enough to be useful.

Note Leamer’s emphasis is on the need for 'credibility' in the selection of assumptions. The easiest way to invalidate a model is to demonstrate that it is fragile with respect to the uncertainty in the assumptions or to show that its assumptions have not been taken 'wide enough'. The same concept is expressed by Jerome R. Ravetz, for whom bad modeling is when *uncertainties in inputs must be suppressed lest outputs become indeterminate.*[4]

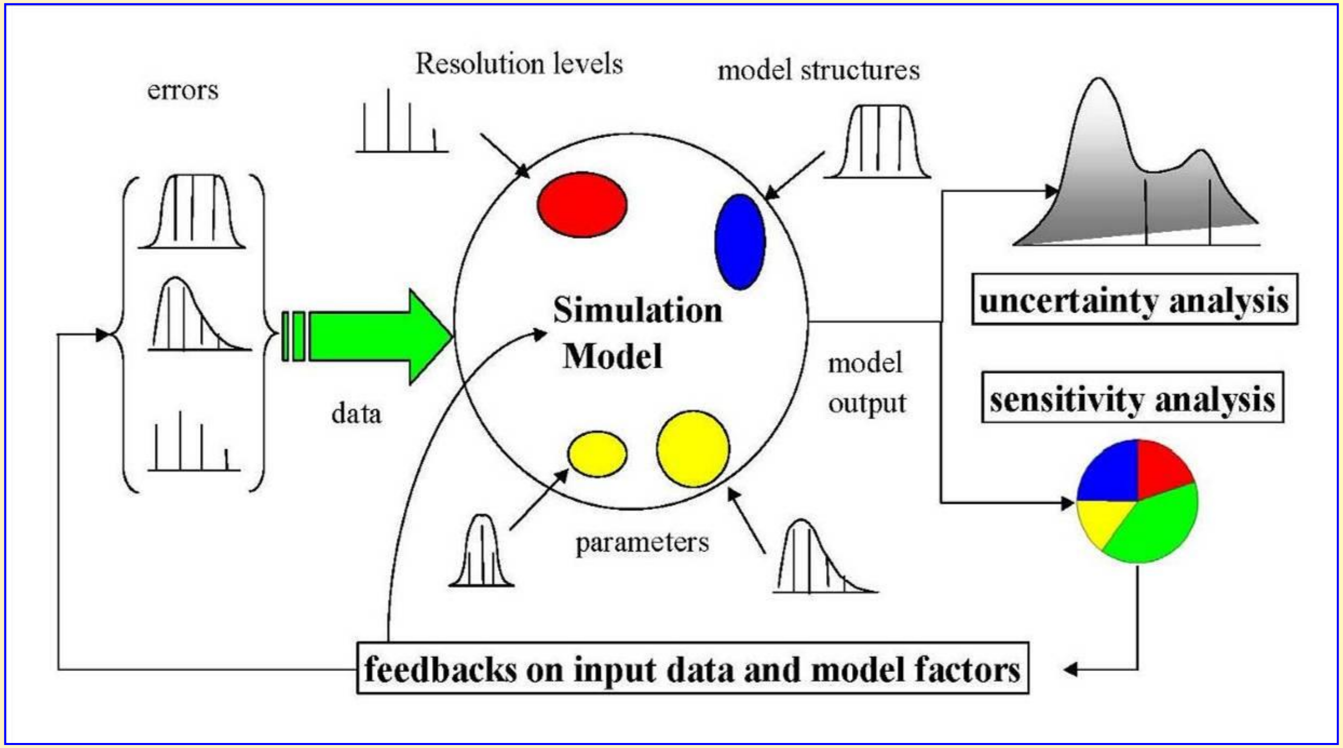
In modern econometrics the use of sensitivity analysis to anticipate criticism is the subject of one of the ten commandments of applied econometrics (from Kennedy, 2007[5]):

Thou shall confess in the presence of sensitivity. Corollary: Thou shall anticipate criticism [⋯] When reporting a sensitivity analysis, researchers should explain fully their specification search so that the readers can judge for themselves how the results may have been affected. This is basically an 'honesty is the best policy' approach, advocated by Leamer, (1978[6]).

The use of mathematical modelling can be the subject of controversies, see Nassim Nicholas Taleb[7] in Economics, and Orrin H. Pilkey and Linda Pilkey Jarvis[8] in Environmental Sciences. As noted by the latter Authors, this increases the relevance of sensitivity analysis in today's modelling practice[1] .

Mathematical problems met in social, economic or natural sciences may entail the use of mathematical models, which generally do not lend themselves to a straightforward understanding of the relationship between input factors (what goes into the model) and output (the model's dependent variables). Such an appreciation, i.e. the understanding of how the model behaves in response to changes in its inputs, is of fundamental importance to ensure a correct use of the models.

- A mathematical model is defined by a series of equations, input factors, parameters, and variables aimed to characterize the process being investigated.



Ideal scheme of a possibly sampling-based sensitivity analysis. Uncertainty arising from different sources — errors in the data, parameter estimation procedure, alternative model structures — are propagated through the model for uncertainty analysis and their relative importance is quantified via sensitivity analysis.

Errors

In sensitivity analysis Type I error is assessing as important a non important factor, and Type II error assessing as non important an important factor. Type III error corresponds to analyzing the wrong problem, e.g. via an incorrect specification of the input uncertainties. Possible pitfalls in sensitivity analysis are:

•Unclear purpose of the analysis. Different statistical tests and measures are applied to the problem and different factors rankings are obtained. The test should instead be tailored to the purpose of the analysis, e.g. one uses Monte Carlo filtering if one is interested in which factors are most responsible for generating high/low values of the output.

•Too many model outputs are considered. This may be acceptable for quality assurance of sub-models but should be avoided when presenting the results of the overall analysis.

•Piecewise sensitivity. This is when one performs sensitivity analysis on one sub-model at a time. This approach is non conservative as it might overlook interactions among factors in different sub-models (Type II error).

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What Does *Sensitivity Analysis* Mean?

A technique used to determine how different values of an independent variable will impact a particular dependent variable under a given set of assumptions. This technique is used within specific boundaries that will depend on one or more input variables, such as the effect that changes in interest rates will have on a bond's price.

Sensitivity analysis is a way to predict the outcome of a decision if a situation turns out to be different compared to the key prediction(s).

Investopedia explains *Sensitivity Analysis*

Sensitivity analysis is very useful when attempting to determine the impact the actual outcome of a particular variable will have if it differs from what was previously assumed. By creating a given set of scenarios, the analyst can determine how changes in one variable(s) will impact the target variable.

For example, an analyst might create a financial model that will value a company's equity (the dependent variable) given the amount of earnings per share (an independent variable) the company reports at the end of the year and the company's price-to-earnings multiple (another independent variable) at that time. The analyst can create a table of predicted price-to-earnings multiples and a corresponding value of the company's equity based on different values for each of the independent variables.

Spreadsheet Sensitivity Analysis

Spreadsheets and the Case Projects

The Dynamic Strategic Planning workbook is accompanied by a number of spreadsheet-based tools for data analysis. We have supplied these tools so that the users of this workbook can concentrate upon the use and implementation of decision analysis and strategic planning, rather than focusing upon the mechanics of the mathematics underlying their use.

The current form of the spreadsheets is a consequence of a combination of factors: academic research, pedagogical design, and in-class experiences. Based upon new developments, they are being routinely improved.

However, no amount of care in tool design can substitute for expertise on the part of the user.

The case projects have been designed assuming that these tools will be used effectively. The purpose of this document is to assure that you, the user of these tools, are prepared to exploit them to their fullest - specifically, that you are able to make use of spreadsheet sensitivity analysis tools.

http://msl1.mit.edu/rdn/d_table.pdf

Sensitivity Analysis Using Excel

The main goal of sensitivity analysis is to gain insight into which assumptions are critical, i.e., which assumptions affect choice. The process involves various ways of changing input values of the model to see the effect on the output value. In some decision situations you can use a single model to investigate several alternatives. In other cases, you may use a separate spreadsheet model for each alternative.

MANUAL WHAT-IF ANALYSIS

Using this approach, you enter values into cells C4:C6 and see what the effect is on net cash flow.

For example, with the predetermined price of $29, you may think that Units Sold will be in the range between 500 and 900 units. Keeping other input assumptions at base case, the corresponding Net Cash Flows are $–1,500 and $6,900. When we vary a single input assumption, keeping all other input assumptions at their base case values, we say we are doing "one at a time" or "singlefactor“ sensitivity analysis.

	A	B	C
1	Controllable Input		
2		Unit Price	$29
3	Uncontrollable Inputs		
4		Units Sold	700
5		Unit Variable Cost	$8
6		Fixed Costs	$12,000
7	Performance Measure		
8		Net Cash Flow	$2,700

http://www.treeplan.com/chapters/02_decan_20071029_1042.pdf

References: Chapter 6: Sensitivity Analysis

Chapter 6: Sensitivity Analysis

Suppose that you have just completed a linear programming solution which will have a major impact on your company, such as determining how much to increase the overall production capacity, and are about to present the results to the board of directors. How confident are you in the results? How much will the results change if your basic data (e.g. profit per item produced, or availability of a component) is slightly wrong? Will that have a minor impact on your results? Will it give a completely different outcome, or change the outcome only slightly?

These are the kinds of questions addressed by sensitivity analysis. Formally, the question is this: is my optimum solution (both the values of the variables and the value of the objective function) sensitive to a small change in one of the original problem coefficients (e.g. coefficients of the variables in the objective function or constraints, or the right hand side constants in the constraints)? If Z or the x_i change when an original coefficient is changed, then we say that the LP is *sensitive*. We could ask, for example, if the Acme Bicycle Company solution is sensitive to a reduction in the availability of the metal finishing machine from 4 hours per day to only 3 (i.e. a change in the third constraint from $x_1+x_2 \leq 4$ to $x_1+x_2 \leq 3$).

http://www.sce.carleton.ca/faculty/chinneck/po/Chapter6.pdf

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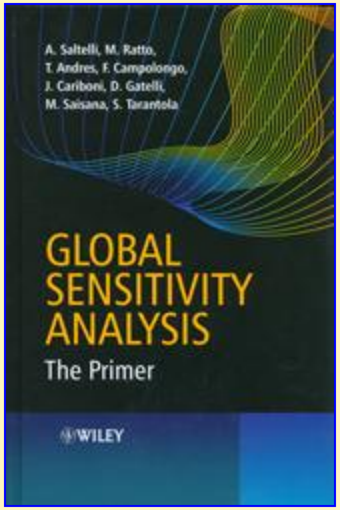
Tutorial on Sensitivity Analysis

SimLab Software for Sensitivity Analysis



What's New

- Sixth **International Conference** on **Sensitivity Analysis of Model Output**, Bocconi University of Milan, **19-22 July 2010**

- Sixth **Summer School** on **Sensitivity Analysis of Model Output**, Villa La Stella, Fiesole - Florence, **14-17 September 2010**

References: Sensitivity Analysis

CA∫DE

Multi-disciplinary Design Optimization

Centre for Aerospace Systems Design & Engineering
Department of Aerospace Engineering
Indian Institute of Technology
Mumbai 400 076

sensitivity.ppt ***(67 slides)***

Sensitivity Analysis – other References

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- *What is sensitivity analysis* ~ http://www.medicine.ox.ac.uk/bandolier/painres/download/whatis/What_is_sens_analy.pdf
- *Sensitivity Analysis* ~ http://web.mit.edu/15.053/www/AMP-Chapter-03.pdf
- *Tutorial_09_Sensitivity_Analysis* ~ http://www.rocscience.com/downloads/slide/webhelp/pdf_files/tutorials/Tutorial_09_Sensitivity_Analysis.pdf
- *Cash Flow Sensitivity Analysis* ~ www.jaxworks.com/Cash%20Flow%20Sensitivity%20Analysis.xls

Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1 X1	0	1.5000	0	-1.5000	at bound	-M	3.0000
2 X2	16,000.0000	2.5000	40,000.0000	0	basic	2.3571	4.5000
3 X3	6,000.0000	3.0000	18,000.0000	0	basic	2.5000	3.7500
4 X4	0	4.5000	0	-0.2000	at bound	-M	4.7000
5 Tass	82,000.0000	0	0	0	basic	-0.2500	1.0000
6 Tpol	50,000.0000	0	0	0	basic	-0.8000	M
7 Tpac	60,000.0000	0	0	0	basic	-0.3000	M
Objective	Function	(Max.) =	58,000.0000				
Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
1 C1	0	=	0	0	0	-18,000.0000	82,000.0000
2 C2	0	=	0	0	0.8000	-10,000.0000	40,000.0000
3 C3	0	=	0	0	0.3000	-26,666.6700	15,000.0000
4 C4	82,000.0000	<=	100,000.0000	18,000.0000	0	82,000.0000	M
5 C5	50,000.0000	<=	50,000.0000	0	0.8000	40,000.0000	90,000.0000
6 C6	60,000.0000	<=	60,000.0000	0	0.3000	33,333.3400	75,000.0000