… *Teoria Deciziilor*

- *Metode multicriteriale de analiză a deciziilor*

  …

  ➢ Metoda *ELECTRE*

  …

ELECTRE methods

ELECTRE is a family of multi-criteria decision analysis methods that originated in Europe in the mid-1960s. The acronym ELECTRE stands for: ELimination Et Choix Traduisant la REalité (ELimination and Choice Expressing REality).

The method was first proposed by Bernard Roy and his colleagues at SEMA consultancy company. A team at SEMA was working on the concrete, multiple criteria, real-world problem of how firms could decide on new activities and had encountered problems using a weighted sum technique. Bernard Roy was called in as a consultant and the group devised the ELECTRE method. As it was first applied in 1965, the ELECTRE method was to choose the best action(s) from a given set of actions, but it was soon applied to three main problems: choosing, ranking and sorting. The method became more widely known when a paper by B. Roy appeared in a French operations research journal.[1] It evolved into ELECTRE I (*electre one*) and the evolutions have continued with ELECTRE II, ELECTRE III, ELECTRE IV, ELECTRE IS and ELECTRE TRI (*electre tree*), to mention a few.[2]

Bernard Roy is widely recognized as the father of the ELECTRE method, which was one of the earliest approaches in what is sometimes known as the French School of decision making. It is usually classified as an "*outranking method*" of decision making.

There are two main parts to an ELECTRE application: first, the construction of one or several outranking relations, which aims at comparing in a comprehensive way each pair of actions; second, an exploitation procedure that elaborates on the recommendations obtained in the first phase. The nature of the recommendation depends on the problem being addressed: choosing, ranking or sorting.

Criteria in ELECTRE methods have two distinct sets of parameters: the importance coefficients and the veto thresholds.

References

1. Roy, Bernard (1968). "Classement et choix en présence de points de vue multiples (la méthode ELECTRE)". *la Revue d'Informatique et de Recherche Opérationelle (RIRO)* (8): 57–75.
2. Figueira, José; Salvatore Greco, Matthias Ehrgott (2005). *Multiple Criteria Decision Analysis: State of the Art Surveys*. New York,: Springer Science + Business Media, Inc.. ISBN ISBN 0-387-23081-5.
3. Multicriteria Decision Aid: the Outranking Approach (ppt)

Introduction to Decision Making Methods, János Fülöp
Laboratory of Operations Research and Decision Systems,
Computer and Automation Institute, Hungarian Academy of Sciences
http://academic.evergreen.edu/projects/bdei/documents/decisionmakingmethods.pdf

Outranking methods

The principal outranking methods assume data availability broadly similar to that required for the MAUT methods. That is, they require alternatives and criteria to be specified, and use the same data of the decision table, namely the *aij*.s and *wi*.s.

Vincke (1992) provides an introduction to the best known outranking methods; see also Figueira et al. (2004) for state-of-art surveys. Here, the two most popular families of the outranking methods, the **ELECTRE** and the **PROMETHEE** methods will be briefly outlined.

http://academic.evergreen.edu/projects/bdei/documents/decisionmakingmethods.pdf

The ELECTRE methods

The simplest method of the ELECTRE family is ELECTRE I.

The ELECTRE methodology is based on the *concordance* and *discordance* indices. We start from the data of the decision matrix, and assume here that the sum of the weights of all criteria equals to 1. For an ordered pair of alternatives (**A***j*,**A***k*), the concordance index c_{jk} is the sum of all the weights for those criteria where the performance score of **A***j* is least as high as that of **A***k*, i.e.

$$c_{jk} = \sum_{i:\, a_{ij} \geq a_{ik}} w_i, \quad j,k = 1,..,n, \ j \neq k.$$

Clearly, the concordance index lies between 0 and 1.

The computation of the discordance index d_{jk} is a bit more complicated:

$d_{jk}=0$ if $a_{ij}>a_{ik}$, $i=1,...,m$, i.e. the discordance index is zero if **A***j* performs better than **A***k* on all criteria. Otherwise,

$$d_{jk} = \max_{i=1,..,m} \frac{a_{ik} - a_{ij}}{\max_{j=1,..,n} a_{ij} - \min_{j=1,..,n} a_{ij}}, \quad j,k = 1,..,n, \; j \neq k,$$

i.e. for each criterion where **A***k* outperforms **A***j*, the ratio is calculated between the difference in performance level between **A***k* and **A***j* and the maximum difference in score on the criterion concerned between any pair of alternatives. The maximum of these ratios (which must lie between 0 and 1) is the discordance index.

A concordance threshold c^* and discordance threshold d^* are then defined such that $0<d^*<c^*<1$.

Then, **A***j* *outranks* **A***k* if the $c_{jk}>c^*$ and $d_{jk}<d^*$, (the concordance index is above and the discordance index is below its threshold, respectively).

This outranking defines a partial ranking on the set of alternatives. Consider the set of all alternatives that outrank at least one other alternative and are themselves not outranked. This set contains the promising alternatives for this decision problem. Interactively changing the level thresholds, we also can change the size of this set.

The ELECTRE I method is used to construct a partial ranking and choose a set of promising alternatives. ELECTRE II is used for ranking the alternatives. In ELECTRE III an outranking degree is established, representing an outranking creditability between two alternatives which makes this method more sophisticated (and, of course, more complicated and difficult to interpret).

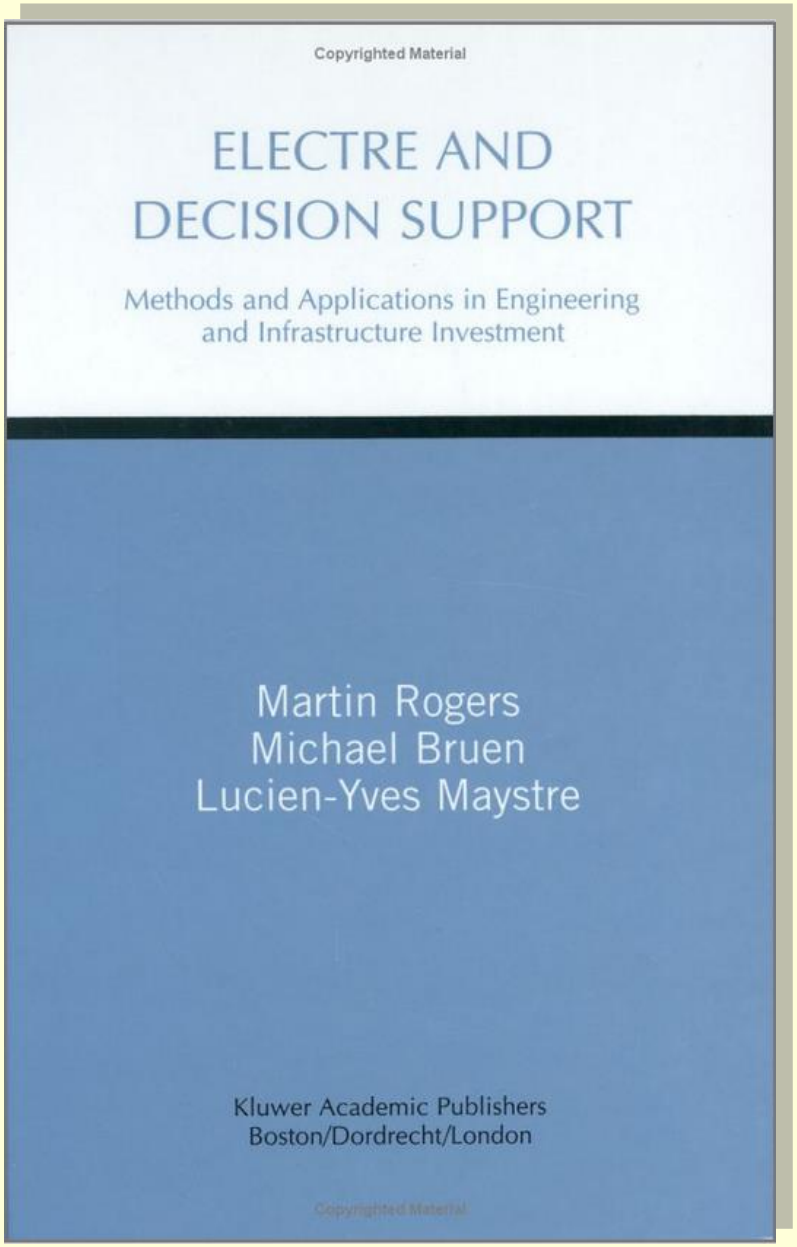
See Figueira et al (2004) for more details and further members of the ELECTRE family.

References

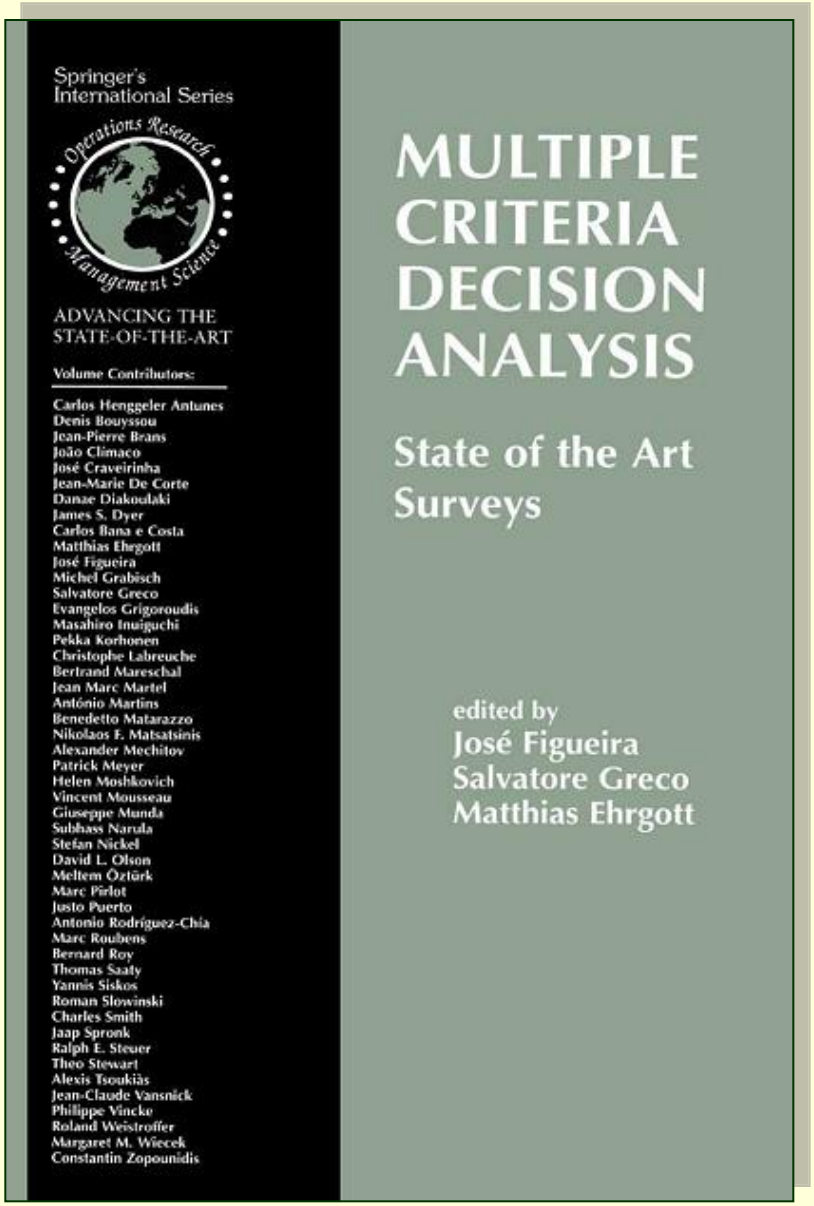
- Aczél, J. and Saaty, T.L. (1983) .Procedures for synthesizing ratio judgements., *Journal of Mathematical Psychology*, 27, 93-102.
- Baker, D., Bridges, D., Hunter, R., Johnson, G., Krupa, J., Murphy, J. and Sorenson, K. (2002) *Guidebook to Decision-*
- *Making Methods*, WSRC-IM-2002-00002, Department of Energy, USA. http://emi-web.inel.gov/Nissmg/Guidebook_2002.pdf
- Barron, F.H. and Barrett, B.E. (1996) .The efficacy of SMARTER . Simple Multi-Attribute Rating Technique Extended to Ranking., *Acta Psychologica*, 93, 23-36.
- Bose, U., Davey, A.M. and Olson, D.L. (1997) .Multi-attribute utility methods in group decision making: Past applications and potential for inclusion in GDSS., Omega, 25, 691-706.
- Brans, J.P. and Vincke, Ph. (1985) "A preference ranking organization method", *Management Science*, 31, 647-656.
- Brans, J.P., Vincke, Ph. and Marechal, B. (1986) "How to select and how to rank projects: The PROMETHEEmethod", *European Journal of Operational Research*, 24, 228- 238.
- Brans, J.-P. and Mareschal, B. (1994) .The PROMCALC & GAIA decision support system for multicriteria decision aid., *Decision Support Systems*, 12, 297-310.
- Csáki, P., Rapcsák, T., Turchányi, P. and Vermes, M. (1995) .Research and development for group decision aid in Hungary by WINGDSS, a Microsoft Windows based group decision support system., *Decision Support Systems* 14, 205-21.
- Dyer, R.F. and Forman, E.H. (1992) .Group decision support with the Analytic Hierarchy Process., *Decision Support Systems*, 8, 99-124.
- Edwards, W. (1977) .How to use multiattribute utility measurement for social decisionmaking., *IEEE Transactions on Systems, Man, and Cybernetics*, SMC-7, 326-340.
- Edwards, W. and Barron, F.H. (1994) .SMARTS and SMARTER: Improved simple methods for multiattribute utility measurements., *Organizational Behavior and Human Decision Processes*, 60, 306-325.
- Ekárt, A. and Németh, S.Z. (2005) .Stability analysis of tree structured decision functions., *European Journal of Operational Research*, 160, 676-695.
- Figueira, J., Greco, S. and Ehrgott, M. (Eds.) (2004) *Multiple Criteria Decision Analysis: State of the Art Surveys*, Springer, New York.
- Forman , E. and Selly, M.A. (2001) *Decision by Objectives*, World Scientific.
- Gass, S. I. and Rapcsák, T. (1998) .A note on synthesizing group decisions., *Decision Support Systems*, 22, 59-63.
- Gass, S. I. and Rapcsák, T. (2004) .Singular value decomposition in AHP., *European Journal of Operational Research*, 154, 573-584.
- Harris, R. (1998) *Introduction to Decision Making*, VirtualSalt. http://www.virtualsalt.com/crebook5.htm
- Keeney, R.L. and Raiffa, H. (1976) *Decisions with Multiple Objectives: Performances and Value Trade-Offs*, Wiley, New York.
- Keeney, R.L. (1976) .A group preference axiomatization with cardinal utility., *Management Science*, 23, 140 .145.

- Lai, V.S., Bo K.W. and Cheung, W. (2002) .Group decision making in a multiple criteria environment: A case using the AHP in software selection., *European Journal of Operational Research*, 137, 134-144.
- Leyva-López, J-C. and Fernández-González, E. (2003) .A new method for group decision support based on ELECTRE III methodology., *European Journal of Operational Research*, 148, 14-27.
- Linkov, I., Varghese, A., Jamil, S., Seager, T.P., Kiker, G. and Bridges, T. (2004) .Multi-criteria decision analysis: A framework for structuring remedial decisions at the contaminated sites., In: Linkov, I. and Ramadan, A.B. (Eds.)
- *Comparative Risk Assessment and Environmental Decision Making*, Springer, New York, pp. 15-54.
- Macharis, C., Brans, J.P. and Mareschal, B. (1998). .The GDSS PROMETHEE Procedure., *Journal of Decision Systems*, 7, 283-307.
- Macharis, C., Springael, J., De Brucker, K. and Verbeke, A. (2004) .PROMETHEE and AHP: The design of operational synergies in multicriteria analysis.: Strengthening PROMETHEE with ideas of AHP., *European Journal of Operational Research*, 153, 307-317.
- Mareschal, B. (1988) .Weight stability intervals in multicriteria decision aid., *European Journal of Operational Research*, 33, 54-64.
- Mészáros, Cs. and Rapcsák, T. (1996) .On sensitivity analysis for a class of decision systems., *Decision Support Systems* 16, 231-240.
- Munda, G. (1996) .Cost-benefit analysis in integrated environmental assessment: some methodological issues., *Ecological Economics*, 19, 157-168.
- Nemhauser, G.L., Rinnoy Kan, A.H.G. and Todd, M.J. (1989) *Handbooks in Operations Research and Management Science: Volume 1 Optimization*, North-Holland, Amsterdam.
- Rapcsák, T. (2004) *Multiattribute Decision Making*, Lecture notes, Department of Decisions in Economics, Corvinus University, Budapest. (in Hungarian)
- Roy, B. (1968) "Classement et choix en présence de points de vue multiple (la méthode electre)., *RAIRO*, 2, 57-75.
- Saaty, T.L. (1980) *The Analytic Hierarchy Process*, McGraw Hill.
- Saaty, T.L. and Vargas, L.G. (1984) .Comparison of eigenvalue, logarithmic least squares and least squares methods in estimating ratios., *Mathematical Modelling*, 5, 309-324.
- Steuer, R. E. (1986) *Multiple Criteria Optimization: Theory, Computation and Application*, Wiley, New York.
- Triantaphyllou, E. and Sanchez, A. (1997) "A sensitivity analysis approach for some deterministic multi-criteria decision making methods", *Decision Sciences*, 28, 151-194.
- Triantaphyllou, E. (2000) *Multi-Criteria Decision Making Methods: A Comparative Study*, Kluwer Academic Publishers, Dordrecht.
- UK DTLR (2001) *Multi Criteria Analysis: A Manual*, Department for Transport, Local Government and the Regions, UK. http://www.odpm.gov.uk/stellent/groups/odpm_about/documents/page/odpm_about_608524.hcsp
- US EPA (2000) *Guidelines for Preparing Economic Analysis*, United States Environmental Protection Agency, EPA 240-R-00-003. http://yosemite.epa.gov/ee/epa/eed.nsf/webpages/Guidelines.html
- Vincke, P. (1992) *Multi-criteria Decision-Aid,* John Wiley, Chichester.
- Wolters, W.T.M. and Mareschal, B. (1995) .Novel types of sensitivity analysis for additive MCDM methods., *European Journal of Operational Research*, 81, 281-290.‘

http://www.amazon.com/ELECTRE-Decision-Support-Applications-Infrastructure/dp/0792386477



https://commerce.metapress.com/content/g367r44322876223/resource-secured/?target=fulltext.pdf&sid=fbsyj255uh2rhd2xjgybkp55&sh=www.springerlink.com



Chapter 4

ELECTRE METHODS

José Figueira
Faculdade de Economia and INESC-Coimbra
Universidade de Coimbra
Av. Dias da Silva, 165, 3004-512 Coimbra
Portugal
figueira@fe.uc.pt

Vincent Mousseau, Bernard Roy
LAMSADE
Université Paris-Dauphine
Place du Maréchal De Lattre de Tassigny, 75775 Paris Cedex 16
France
{mousseau,roy}@lamsade.dauphine.fr

Abstract Over the last three decades a large body of research in the field of ELECTRE family methods appeared. This research has been conducted by several researchers mainly in Europe. The purpose of this chapter is to present a survey of the ELECTRE methods since their first appearance in mid-sixties, when ELECTRE I was proposed by Bernard Roy and his colleagues at SEMA consultancy company. The chapter is organized in five sections. The first section presents a brief history of ELECTRE methods. The second section is devoted to the main features of ELECTRE methods. The third section describes the different ELECTRE methods existing in the literature according to the three main problematics: choosing, ranking and sorting. The fourth section presents the recent developments and future issues on ELECTRE methods. Finally, the fifth section is devoted to the software and applications. An extensive and up-to-date bibliography is also provided in the end of this chapter.

Keywords: Multiple criteria decision aiding, Outranking approaches, ELECTRE methods.

http://books.google.com/books?id=YM1X0jHrzEMC&printsec=frontcover&dq=ELECTRE&source=gbs_similarbooks_s&cad=1#v=onepage&q=ELECTRE&f=false

The PROMETHEE methods

St. Cloud State University

& LEO (Literacy Education Online)

LEO provides online handouts about a variety of writing topics. Although LEO is affiliated with the Write Place (the writing center at St. Cloud State University), LEO does not offer online tutoring, answer questions about grammar or punctuation, or give feedback about your writing or papers.

If you are interested in information about tutoring or other Write Place services, please head to the writing center homepage at http://www.stcloudstate.edu/writeplace.

LEO is also the online site for the early issues (1990-1998) of *Kaleidoscope*, SCSU's multicultural literary magazine. For current issues, please visit *Kaleidoscope*'s new home on the Write Place web site:http://web.stcloudstate.edu/kaleidoscope/.

http://leo.stcloudstate.edu/index.html

http://leo.stcloudstate.edu/acadwrite/conclude.html

Strategies for Writing a Conclusion

Conclusions are often the most difficult part of an essay to write, and many writers feel that they have nothing left to say after having written the paper. A writer needs to keep in mind that the conclusion is often what a reader remembers best. Your conclusion should be the best part of your paper.

A conclusion should

- stress the importance of the thesis statement,
- give the essay a sense of completeness, and
- leave a final impression on the reader.

Suggestions

- **Answer the question "So What?"** Show your readers why this paper was important. Show them that your paper was meaningful and useful.
- **Synthesize, don't summarize**
  - Don't simply repeat things that were in your paper. They have read it. Show them how the points you made and the support and examples you used were not random, but fit together.
- **Redirect your readers**
  - Give your reader something to think about, perhaps a way to use your paper in the "real" world. If your introduction went from general to specific, make your conclusion go from specific to general. Think globally.
- **Create a new meaning**
  - You don't have to give new information to create a new meaning. By demonstrating how your ideas work together, you can create a new picture. Often the sum of the paper is worth more than its parts.

Strategies

- **Echoing the introduction:** Echoing your introduction can be a good strategy if it is meant to bring the reader full-circle. If you begin by describing a scenario, you can end with the same scenario as proof that your essay was helpful in creating a new understanding.

Example

Introduction

From the parking lot, I could see the towers of the castle of the Magic Kingdom standing stately against the blue sky. To the right, the tall peak of The Matterhorn rose even higher. From the left, I could hear the jungle sounds of Adventureland. As I entered the gate, Main Street stretched before me with its quaint shops evoking an old-fashioned small town so charming it could never have existed. I was entranced. Disneyland may have been built for children, but it brings out the child in adults.

Conclusion

I thought I would spend a few hours at Disneyland, but here I was at 1:00 A.M., closing time, leaving the front gates with the now dark towers of the Magic Kingdom behind me. I could see tired children, toddling along and struggling to keep their eyes open as best they could. Others slept in their parents' arms as we waited for the parking lot tram that would take us to our cars. My forty-year-old feet ached, and I felt a bit sad to think that in a couple of days I would be leaving California, my vacation over, to go back to my desk. But then I smiled to think that for at least a day I felt ten years old again.

- **Challenging the reader:** By issuing a challenge to your readers, you are helping them to redirect the information in the paper, and they may apply it to their own lives.

  Example

  Though serving on a jury is not only a civic responsibility but also an interesting experience, many people still view jury duty as a chore that interrupts their jobs and the routine of their daily lives. However, juries are part of America's attempt to be a free and just society. Thus, jury duty challenges us to be interested and responsible citizens.

- **Looking to the future:** Looking to the future can emphasize the importance of your paper or redirect the readers' thought process. It may help them apply the new information to their lives or see things more globally.

  Example

  Without well-qualified teachers, schools are little more than buildings and equipment. If higher-paying careers continue to attract the best and the brightest students, there will not only be a shortage of teachers, but the teachers available may not have the best qualifications. Our youth will suffer. And when youth suffers, the future suffers.

- **Posing questions:** Posing questions, either to your readers or in general, may help your readers gain a new perspective on the topic, which they may not have held before reading your conclusion. It may also bring your main ideas together to create a new meaning.

  Example

  Campaign advertisements should help us understand the candidate's qualifications and positions on the issues. Instead, most tell us what a boob or knave the opposing candidate is, or they present general images of the candidate as a family person or God-fearing American. Do such advertisements contribute to creating an informed electorate or a people who choose political leaders the same way they choose soft drinks and soap?

Metoda ELECTRE

Metoda ELECTRE (*ELimination Et Choix Traduisant la REalité*), dezvoltată în Franţa şi aplicată în special în ţările europene [Roy, 1968; Roy şi Vanderpooten, 1996], se bazează pe **conceptul de *surclasare*** pentru eliminarea alternativelor care sunt într-un anumit sens "*dominate*". Noţiunea de "*dominanţă*" în cadrul oferit de surclasare este o generalizare a dominanţei clasice şi foloseşte ponderi pentru a ierarhiza criteriile (unele dintre ele au influenţă mai mare decât altele asupra deciziei).

La fel ca şi celelalte metode de analiză multicriterială, metoda ELECTRE are aceleaşi **etape**:

- specificarea alternativelor şi criteriilor,
- evaluarea performanţelor în funcţie de criterii şi
- stabilirea ponderilor asociate criteriilor, care stabilesc importanţa relativă a acestora.

Conceptul de *surclasare* a fost definit de Roy astfel: alternativa A_i ***surclasează*** alternativa A_k dacă, fiind date preferinţele decidentului, calitatea evaluării alternativelor şi contextul problemei, există suficiente argumente să se decidă că A_i **este cel puţin la fel de bună ca** A_k şi nu există nici un motiv evident care să contrazică afirmaţia.

Pe baza acestei idei, au fost dezvoltate o serie de proceduri pentru concretizarea conceptului de ***surclasare*** ca modalitate de asistare a deciziilor multicriteriale.

In general este nevoie de **două etape**:

a) specificarea unei modalităţi precise de ***stabilire a existenţei surclasării*** între două alternative şi
b) combinarea evaluărilor de surclasare a alternativelor pentru ***a le ierarhiza***.

Esenţa metodei ELECTRE constă în ***identificarea relaţiilor de dominanţă***.

Ea determină o submulţime de alternative $E \subseteq A$ astfel încât orice alternativă $A_k \notin E$ este surclasată de cel puţin o alternativă $A_i \in E$.

Obiectivul este ca submulţimea E să aibă cât mai puţine elemente, care vor reprezenta alternative candidat pentru decizia finală.

Pasul 1. *Determinarea indicilor de concordanţă şi discordanţă*

Pentru determinarea *gradului de concordanţă şi discordanţă* ale alternativelor se folosesc ***indicii de concordanţă şi discordanţă***. *Indicele de concordanţă* c_{ik} se calculează pentru fiecare pereche de alternative (A_i, A_k) ca sumă a ponderilor acelor criterii de decizie C_j pentru care performanţa alternativei A_i este mai mare decât performanţa alternativei A_k. *Indicele de discordanţă* d_{ik} se determină ca elementul maxim al mulţimii rapoartelor dintre diferenţa performanţelor alternativelor A_k şi A_i şi diferenţa maximă de performanţă în raport cu fiecare dintre criteriile j pentru care performanţa alternativei A_k este cel puţin egală performanţei alternativei A_i:

$$c_{ik} = \sum_{\substack{j=1 \\ r_{ij} > r_{kj}}}^{n} p_j \qquad d_{ik} = \max_{\substack{1 \le j \le n \\ r_{ij} \le r_{kj}}} \left\{ \frac{r_{kj} - r_{ij}}{\max\limits_{1 \le l \le m} r_{lj} - \min\limits_{1 \le l \le m} r_{lj}} \right\} \qquad (1 \le i, k \le m).$$

În acest fel, ***indicele de discordanţă*** este 0 dacă performanţa alternativei A_i este mai bună decât performanţa alternativei A_k, respectiv un număr subunitar în caz contrar.

Indicii de concordanţă c_{ik} şi ***discordanţă*** d_{ik} definesc ***matricile de concordanţă MC*** şi ***discordanţă MD***, matrici pătratice de ordinul m = numărul de alternative decizionale.

Pasul 2. *Combinarea indicilor de concordanţă şi discordanţă*

Pentru combinarea indicilor de concordanţă şi discordanţă se introduce un *prag* (*relativ mare*) *de concordanţă* c^* şi un *prag* (*relativ mic*) *de discordanţă* d^*.

Cu ajutorul acestora se poate defini concret conceptul de ***surclasare***:

alternativa A_i ***surclasează*** alternativa A_k dacă $c_{ik} > c^*$ şi $d_{ik} < d^*$.

Mulţimea E a ***alternativelor promiţătoare*** este formată din acele alternative decizionale care surclasează cel puţin o altă alternativă şi nu sunt la rândul lor surclasate.

Dacă mulţimea E conţine **prea multe** elemente, ea se poate restrânge prin **majorarea pragului de concordanţă** şi **micşorarea pragului de discordanţă**.

Dacă E este **prea mică** (de exemplu mulţimea vidă), atunci **se micşorează pragul de concordanţă** şi **se majorează pragul de discordanţă**.

Exemplu: [Dodgson, 2000]

Alternative decizionale: $A = \{A_1, A_2, A_3, A_4, A_5, A_6, A_7, A_8, A_9, A_{10}\}$ (m=10),
Criterii de decizie: $C = \{C_1, C_2, C_3, C_4, C_5, C_6\}$ (n = 6),
Ponderile asociate criteriilor: $P = \{0.25,\ 0.10,\ 0.15,\ 0.25,\ 0.05,\ 0.20\}$.
Matricea performanţelor:

Alternative decizionale	*Criterii de decizie*					
	$p_1 = 0.25$	$p_2 = 0.1$	$p_3 = 0.15$	$p_4 = 0.25$	$p_5 = 0.05$	$p_6 = 0.2$
	C_1	C_2	C_3	C_4	C_5	C_6
A_1	6	300	27	18	570	12
A_2	2	450	21	19	400	23
A_3	16	350	27	12	420	18
A_4	10	500	20	12	450	20
A_5	11	380	23	20	400	16
A_6	5	250	31	10	430	18
A_7	16	390	24	18	510	21
A_8	17	400	22	26	380	23
A_9	10	410	16	23	410	20
A_{10}	5	250	18	21	400	22
diferenţe max – min	17-2= 15	500-250= 250	31-16= 15	26-10= 16	570-380= 190	23-12= 11

Pasul 1. *Determinarea indicilor de concordanţă şi discordanţă*

În această fază se calculează matricile indicilor de concordanţă şi de discordanţă. Pentru exemplificare, se consideră alternativele A_3 şi A_4. Criteriile de decizie pentru care se calculează indicele de concordanţă sunt $\{C_1, C_3\}$ (acele criterii pentru care performanţa alternativei A_3 este mai bună decât performaţa alternativei A_4: $r_{31} > r_{41}$, respectiv $r_{33} > r_{43}$), iar cele pentru care se calculează indicele de discordanţă sunt $\{C_2, C_4, C_5, C_6\}$ (acele criterii pentru care performanţa alternativei A_3 *nu* este mai bună decât performaţa alternativei A_4: $r_{32} < r_{42}$, $r_{34} = r_{44}$, $r_{35} < r_{45}$, respectiv $r_{36} < r_{46}$). Conform definiţiei, indicele de concordanţă $c_{34} = p_1 + p_3 = 0.25 + 0.15 = 0.4$. Pentru determinarea indicelui de discordanţă d_{34} se determină elementul maxim al mulţimii:

$$\left\{\frac{r_{42}-r_{32}}{250}, \frac{r_{44}-r_{34}}{16}, \frac{r_{45}-r_{35}}{190}, \frac{r_{46}-r_{36}}{11}\right\} = \left\{\frac{500-350}{250}, \frac{12-12}{16}, \frac{450-420}{190}, \frac{20-18}{11}\right\} =$$

$$= \left\{\frac{150}{250}, \frac{0}{16}, \frac{30}{190}, \frac{2}{11}\right\} = \left\{\frac{3}{5}, 0, \frac{3}{19}, \frac{2}{11}\right\},$$

prin urmare $d_{34} = 0.6$. Elementele mulţimii din care s-a calculat d_{34} sunt rapoartele dintre diferenţa performanţelor alternativelor A_3 şi A_4 şi diferenţele maxime de performanţă, determinate pentru criteriile $\{C_2, C_4, C_5, C_6\}$ în raport cu care A_3 nu are performanţă mai bună decât A_4.

... Pasul 1. *Determinarea indicilor de **concordanţă** şi discordanţă*

Matricea indicilor de ***concordanţă*** este următoarea:

Matricea indicilor de concordanţă c_{ik}										
	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9	A_{10}
A_1	-	0.45	0.30	0.45	0.20	0.65	0.20	0.20	0.20	0.55
A_2	0.55	-	0.55	0.60	0.30	0.55	0.55	0.15	0.45	0.45
A_3	0.55	0.45	-	0.40	0.65	0.60	0.15	0.20	0.45	0.55
A_4	0.55	0.40	0.35	-	0.35	0.85	0.10	0.15	0.30	0.55
A_5	0.80	0.65	0.35	0.65	-	0.60	0.25	0.20	0.40	0.50
A_6	0.35	0.45	0.02	0.15	0.40	-	0.15	0.20	0.20	0.20
A_7	0.55	0.45	0.60	0.90	0.75	0.85	-	0.20	0.65	0.55
A_8	0.80	0.65	0.80	0.85	0.80	0.80	0.80	-	0.85	0.95
A_9	0.80	0.55	0.55	0.25	0.60	0.80	0.35	0.15	-	0.65
A_{10}	0.45	0.50	0.45	0.45	0.45	0.45	0.45	0.05	0.35	-

... Pasul 1. *Determinarea indicilor de concordanţă şi* ***discordanţă***

Matricea indicilor de ***discordanţă*** este :

Matricea indicilor de discordanţă d_{ik}										
	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9	A_{10}
A_1	-	1.00	0.67	0.80	0.36	0.55	0.82	1.00	0.73	0.91
A_2	0.89	-	0.93	0.53	0.60	0.67	0.93	1.00	0.53	0.20
A_3	0.79	0.45	-	0.60	0.50	0.27	0.47	0.88	0.69	0.56
A_4	0.63	0.44	0.47	-	0.50	0.73	0.40	0.88	0.69	0.56
A_5	0.89	0.64	0.33	0.48	-	0.53	0.58	0.64	0.36	0.55
A_6	0.74	0.80	0.73	1.00	0.63	-	0.95	1.00	0.81	0.69
A_7	0.32	0.24	0.20	0.44	0.13	0.47	-	0.50	0.31	0.19
A_8	1.00	0.20	0.33	0.40	0.10	0.60	0.68	-	0.16	0.11
A_9	0.84	0.33	0.73	0.76	0.47	1.00	0.53	0.47	-	0.18
A_{10}	0.89	0.80	0.73	1.00	0.52	0.87	0.73	0.80	0.64	-

Pasul 2. *Combinarea indicilor de concordanţă şi discordanţă*

Indicii de concordanţă şi discordanţă se combină pentru a defini conceptul (relativ) de surclasare. Pentru aceasta, mai este nevoie de stabilirea pragurilor de concordanţă c^* şi discordanţă d^*. De obicei, valorile de start ale acestora se setează la media elementelor din matricile indicatorilor de concordanţă şi discordanţă:

$$c^* = \frac{1}{m(m-1)} \sum_{\substack{i,k=1 \\ i \neq k}}^{m} c_{ik}, \quad d^* = \frac{1}{m(m-1)} \sum_{\substack{i,k=1 \\ i \neq k}}^{m} d_{ik}.$$

Pentru exemplul curent, valorile acestora sunt $c^* = 0.48$ and $d^* = 0.61$. Prin definiţie, alternativa A_i surclasează alternativa A_k dacă

$$c_{ik} > c^* \quad \textbf{şi} \quad d_{ik} < d^*.$$

... Pasul 2. *Combinarea indicilor de concordanţă şi discordanţă*

Examinând matricile indicatorilor de concordanţă şi discordanţă, rezultă că:

- $A1$ surclasează $A6$;
- $A2$ surclasează $A4$;
- $A3$ surclasează $A5$, $A6$, $A10$;
- $A4$ surclasează $A10$;
- $A5$ surclasează $A4$, $A6$, $A10$;
- $A7$ surclasează $A1$, $A3$, $A4$, $A5$, $A6$, $A9$, $A10$;
- $A8$ surclasează $A2$, $A3$, $A4$, $A5$, $A6$, $A9$, $A10$;
- $A9$ surclasează $A2$, $A5$, $A10$.

Pentru că o alternativă este *dominantă* dacă surclasează cel puţin o altă alternativă şi nu este surclasată de nici o altă alternativă, alternativele dominante sunt doar A_7 şi A_8 (relaţia de surclasare nu este tranzitivă).

Pentru a determina o mulţime mai redusă (de preferat cu un singur element) de alternative dominante, procedeul de mai sus se poate repeta, folosind praguri mai stricte.

Uzual, c^* se măreşte cu 10%, iar d^* se micşorează cu 10%, dar se pot folosi şi alte scheme.

Tabelul alăturat ilustrează modificarea relaţiilor de dominanţă dacă c^* se măreşte, iar d^* se micşorează.

Rezultatele precedente reprezintă ***Iteraţia* 1**).

Modificarea relaţiilor de dominanţă dacă c^* se măreşte, iar d^* se micşorează				
Iteraţia	***c^****	***d^****	***Alternativa***	***surclasează alternativele***
2	**0.53**	**0.54**	A_2	A_4
			A_3	A_5, A_6
			A_5	A_4, A_6
			A_7	$A_1, A_3, A_4, A_5, A_6, A_9, A_{10}$
			A_8	$A_2, A_3, A_4, A_5, A_9, A_{10}$
			A_9	A_2, A_5, A_{10}
3	**0.58**	**0.49**	A_3	A_6
			A_5	A_4
			A_7	A_3, A_4, A_5, A_6, A_9
			A_8	$A_2, A_3, A_4, A_5, A_9, A_{10}$
			A_9	A_5, A_{10}
4	**0.64**	**0.44**	A_7	A_5, A_9
			A_8	$A_2, A_3, A_4, A_5, A_9, A_{10}$
			A_9	A_{10}
5	**0.70**	**0.40**	A_7	A_5
			A_8	A_3, A_5, A_9, A_{10}
6	**0.75**	**0.38**	A_8	A_3, A_5, A_9, A_{10}

În primele cinci iteraţii, mulţimea alternativelor dominante nu are mai puţin de două elemente, A_7 şi A_8. Se observă că doar la ***iteraţia* 6** s-a obţinut alternativa dominantă A_8 .

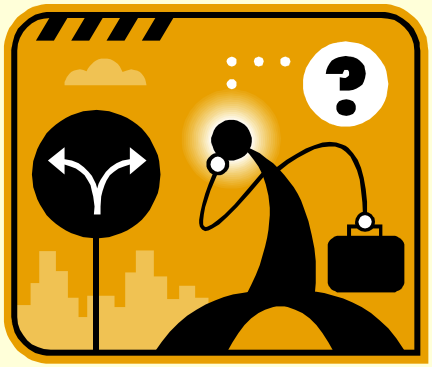
Dacă mulţimea alternativelor dominante **nu se reduce la un singur element şi dacă se cere recomandarea unei singure alternative**, atunci trebuie să se recurgă **la alte tehnici**.

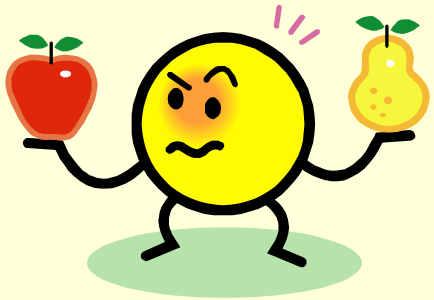
Dacă însă se cere ierarhizarea tuturor alternativelor, se poate proceda în două moduri:

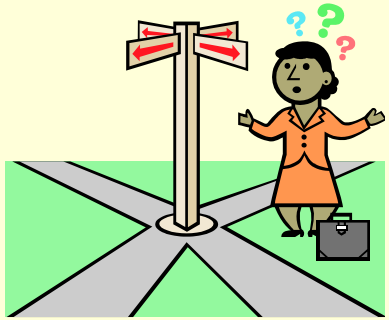
1. **Prima variantă**, iterativă, elimină la fiecare pas din mulţimea alternativelor pe cea considerată "optimă". Pasul iniţial foloseşte mulţimea tuturor alternativelor, iar după m-1 iteraţii se obţine ierarhizarea dorită: alternativa eliminată la pasul k ocupă poziţia k în clasament.
2. **A doua variantă** ia în considerare *meritele relative* ale alternativelor, folosind analiza efectuată anterior. Un **clasament *brut*** al alternativelor este:

   $A_8, A_7, A_9, A_3, A_5, A_2, A_1, A_4, A_6, A_{10}$.

Decision Theory







Unicriterion or multicriteria model ?

- Unicriterion model:

$$\text{Optimise}\left\{ g(a) \middle| a \in A \right\}$$

- Multicriteria model:

$$\text{Optimise}\left\{ g_1(a), g_2(a), ..., g_k(a) \middle| a \in A \right\}$$

Multicriteria table

- Actions:
  - Possible decisions,
  - items to evaluate.
- Criteria:
  - quantitative,
  - qualitative.

	Crit. 1 (/20)	Crit. 2 (rating)	Crit. 3 (qual.)	Crit. 4 (Y/N)	…
Action 1	18	135	G	Yes	…
Action 2	9	147	B	Yes	…
Action 3	15	129	VG	No	…
Action 4	12	146	VB	?	…
Action 5	7	121	G	Yes	…
…	…	…	…	…	…

An example

Purchase of a car

Objectives :

- Economy (price),
- Usage (fuel consumption),
- Performance (power),
- Space,
- Comfort.

Multicriteria table

Car	Price	Power	Consum.	Space	Comfort
Tour. A	360,000	75	8.0	A	A
Sport	390,000	**110**	9.0	VB	B
Tour. B	355,000	85	**7.0**	G	A
Lux. 1	480,000	90	8.5	G	**VG**
Economic	**250,000**	50	7.5	B	VB
Lux. 2	450,000	85	9.0	**VG**	G

- Best buy?
- Best compromise?
- Priorities of buyer?

Modeling

1. Defining the actions: A the set of actions. A can be defined by extension (by enumeration of its elements), by comprehension.

2. Defining the criteria: function g defined on A, taking its values in a totally ordered set, and representing an objective of the decision-maker.

3. Modeling preferences: How to compare two actions a and b to each other?

Problematics

	c_1	c_2	c_3	…
A_1	$g_1(A_1)$	$g_2(A_1)$	$g_3(A_1)$	…
A_2	$g_1(A_2)$	$g_2(A_2)$	$g_3(A_2)$	…
A_3	…			
…	…			

1. choice: determine a subset of actions (the *best ones*).
2. sorting: sort actions in predefined categories.
3. ranking: rank from the best to the worst action.
4. description: describe actions and their consequences.

Dominance and efficience

Objective:

Based on a unanimity principle:

$$a \text{ dominates } b \Leftrightarrow g_h(a) \geq g_h(b) \ \forall \ h$$

Efficiency: *a* is efficient if it is not dominated by any other action.

Problems:

Dominance is poor (few dominances),

Many actions are efficient.

Global value for *a* : $V(a) = w_1\, g_1(a) + w_2\, g_2(a) + \ldots$

a is preferred to *b* if: $V(a) > V(b)$ (if all criteria are to maximise)

Outranking methods

1. Majority principle (vs unanimity for dominance).
2. Pairwise comparison of actions.
3. Closer to the decision problem.
4. ELECTRE methods (1968-).
5. PROMETHEE & GAIA methods (1983-).