Analytical geometry exam (Sample subject)

Problem 1. Given the two vectors $\mathbf{a} = (3, -1, 5)$ and $\mathbf{b} = (1, 2, -3)$, find the vector $\mathbf{x}$ perpendicular to the axis Oz and satisfying the conditions

$$\mathbf{x} \cdot \mathbf{a} = 9, \; \mathbf{x} \cdot \mathbf{b} = -4.$$

Problem 2. Find the equations of the sides of a triangle ABC with $A(1,3)$ as a vertex, if

$$x - y + 1 = 0 \text{ and } y - 1 = 0$$

are the equations of two of its medians.

Problem 3. Prove that the lines

$$\frac{x-1}{2} = \frac{y+2}{-3} = \frac{z-5}{4}$$

and

$$\begin{cases} x = 3t + 7, \\ y = 2t + 2, \\ z = -2t + 1 \end{cases}$$

lie in the same plane and find the equation of this plane.

Problem 4. Write the canonical equations of the rectilinear generators of the surface

$$\frac{x^2}{4} + \frac{y^2}{9} - \frac{z^2}{16} = 1,$$

parallel to the plane $6x + 4y + 3z - 17 = 0$