BABEŞ-BOLYAI UNIVERSITY
Faculty of Computer Science and Mathematics

ARTIFICIAL INTELLIGENCE

Solving search problems

Informed search strategies

Global and local

Topics

A. Short introduction in Artificial Intelligence (AI)

A. **Solving search problems**
   A. **Definition of search problems**
   B. **Search strategies**
      A. Uninformed search strategies
      B. **Informed search strategies**
      C. **Local search strategies (Hill Climbing, Simulated Annealing, Tabu Search**, Evolutionary algorithms, PSO, ACO)
      D. Adversarial search strategies

C. Intelligent systems
   A. Rule-based systems in certain environments
   B. Rule-based systems in uncertain environments (Bayes, Fuzzy)
   C. Learning systems
      A. Decision Trees
      B. Artificial Neural Networks
      C. Support Vector Machines
      - Evolutionary algorithms
   D. Hybrid systems

Content

- Solving problem by search
  - Informed search strategies (ISS)
    - Global search strategies
      - Best first search
    - Local search strategies
      - Hill Climbing
      - Simulated Annealing
      - Tabu search

Useful information

- Chapter II.4 of *S. Russell, P. Norvig, Artificial Intelligence: A Modern Approach, Prentice Hall, 1995*

- Chapters 3 and 4 of *C. Groşan, A. Abraham, Intelligent Systems: A Modern Approach, Springer, 2011*

- Chapter 2.5 of http://www-g.eng.cam.ac.uk/mmg/teaching/artificialintelligence

Solving problems by search

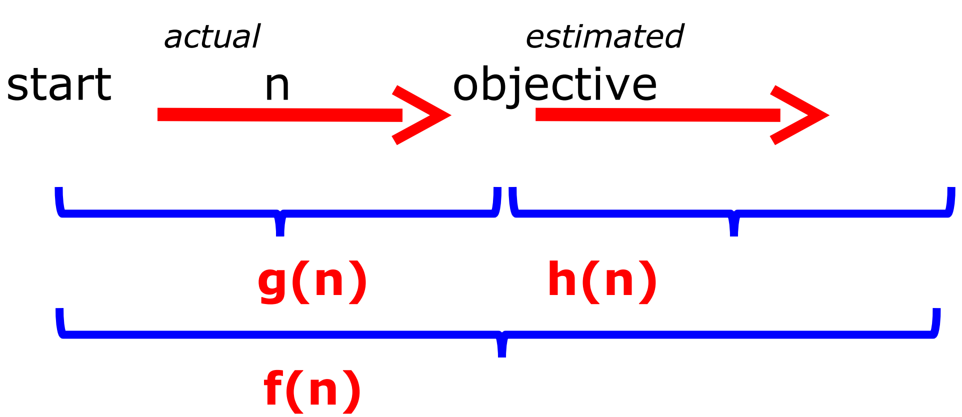
- Search strategies
  - Topology
    - Available information
      - Uninformed search (blind search)
      - Informed search (heuristic search)

Informed search strategies (ISS)

- Characteristics
  - Based on specific information about the problem, trying to bound the search space by intelligent choosing the nodes to be explored
  - An evaluation (heuristic) function sorts the nodes
  - Specific to the problem
- Topology
  - Global search strategies
    - Best-first search
      - Greedy best-first search
      - A* + versions of A*
  - Local search strategies
    - Tabu search
    - Hill climbing
    - Simulated annealing

SS in tree-based structures

- Basic elements
  - *f(n)* – evaluation function for estimating the cost of a solution through node (state) *n*
  - *h(n)* – evaluation function for estimating the cost of a solution path from node (state) *n* to the final node (state)
  - *g(n)* – evaluation function for estimating the cost of a solution path from the initial node (state) to node (state) *n*
  - *f(n) = g(n) + h(n)*



ISS – Best first search

- **Basic elements**
  - Best first search = first, the best element is processed
  - Each state is evaluated by a function *f*
  - The best evaluated state is explored
  - Example of a SS that depends on evaluation function
    - Uniform cost search (from USS)
      - *f* = path cost
    - ISSs use heuristic functions
  - 2 possible BFS strategies
    - Expand the closest node to the objective state
    - Expand the best evaluated (best cost) node

- **Example**
  - See next slides ☺

ISS – Best first search

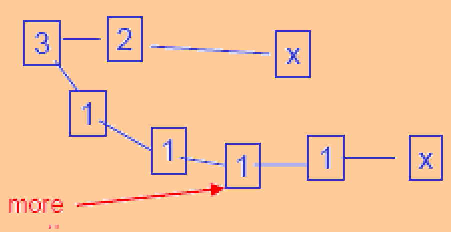
- **Algorithm**

```
bool BestFS(elem, list){
     found = false;
     visited = Φ;
     toVisit = {start}; //FIFO sorted list (priority queue)
     while((toVisit != Φ) && (!found)){
          if (toVisit == Φ)
            return false
          node = pop(toVisit);
          visited = visited U {node};
          if (node == elem)
            found = true;
          else
            aux = Φ;
          for all unvisited children of node do{
            aux = aux U {child};
          }
          toVisit = toVisit U aux; //adding a node into the FIFO list based on its
                                  // evaluation (best one in the front of list)

     }  //while
     return found;
}
```

ISS – best first search

- **Complexity analyse**
  - Time complexity
    - *b* – ramification factor
    - *d* – maximal length (depth) of solution
    - $T(n) = 1 + b + b^2 + \ldots + b^d \Rightarrow O(b^d)$
  - Space complexity
    - $S(n) = T(n)$
  - Completeness
    - No – infinite paths if the heuristic evaluates each node of the path as being the best selection
  - Optimality
    - Possible - depends on heuristic
- **Advantages**
  - Specific information helps the search
  - Good speed to find the final state
- **Disadvantages**
  - State evaluation → effort (computational, physic, etc)
  - Some paths could seem to be good
- **Applications**
  - *Web crawler (automatic indexer)*
  - Games



ISS – heuristic functions

- Etymology: *heuriskein (gr)*
  - *To find, to discover*
  - *Study of methods and rules of discovering and invention*

- Utility
  - Evaluation of the state potential (in the search space)
  - Estimation of path's cost from the current state to the final state

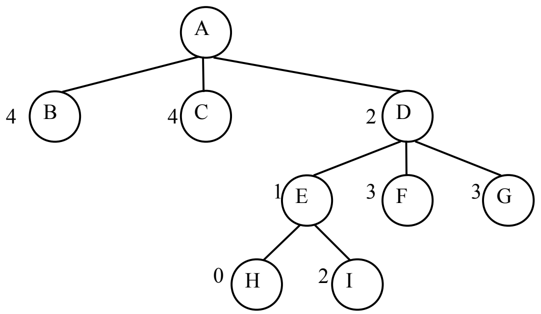
- Characteristics
  - Depends on the problem to be solved
  - New functions for new problems
  - A specific state is evaluated (instead of operators that map a state into another one)
  - Positive functions for each node *n*
    - $h(n) \geq 0$ for all states n
    - $h(n) = 0$ for final state
    - $h(n) = \infty$ for a state that starts a dead end

ISS – heuristic functions

- Examples
  - Missionary and cannibal problem
    - *h(n)* – no of persons from initial river side
  - 8-puzzle
    - *h(n)* – no of pieces that are in wrong places
    - *h(n)* – sum of Manhattan distance (of each piece relative to the final position)
  - Travelling salesman problem
    - *h(n)* – nearest neighbour ! ! !
  - Pay a sum by using a minimal number of coins
    - *h(n)* – choose the coin of best (large) value smaller than the sum to be paid

ISS - Greedy

- **Basic elements**
  - Evaluation function ***f(n) = h(n)***
    - Cost path estimation from the current state to the final one – *h(n)*
    - cost minimization for the path that must be followed
- **Example**
  - A,D,E,H



Vizitate deja	De vizitat
Φ	A
A	D, B, C
A, D	E, F, G, B, C
A, D, E	H, I, F, G, B, C
A, D, E, H	Φ

- **Algorithm**

```
bool BestFS(elem, list){
        found = false;
        visited = Φ;
        toVisit = {start};        //FIFO sorted list (priority queue
        while((toVisit != Φ) && (!found)){
                if (toVisit == Φ)
                    return false
                node = pop(toVisit);
                visited = visited U {node};
                if (node == elem)
                    found = true;
                else
                    aux = Φ;
                for all unvisited children of node do{
                    aux = aux U {child};
                }
                toVisit = toVisit U aux; //adding a node onto the FIFO list based on its evaluation h(n)
                                   //(best one in the front of list)

        }  //while
        return found;
}
```

ISS - Greedy

- **Complexity analyse**
  - Time complexity → DFS
    - *b* – ramification factor
    - d^{max} – maximal length (depth) of an explored tree
    - $T(n) = 1 + b + b^2 + \ldots + b^{dmax} \Rightarrow O(b^{dmax})$
  - Space complexity → BFS
    - *d* - length (depth) of solution
    - $S(n) = 1 + b + b^2 + \ldots + b^d \Rightarrow O(b^d)$
  - Completeness
    - nu
  - Optimality
    - possible

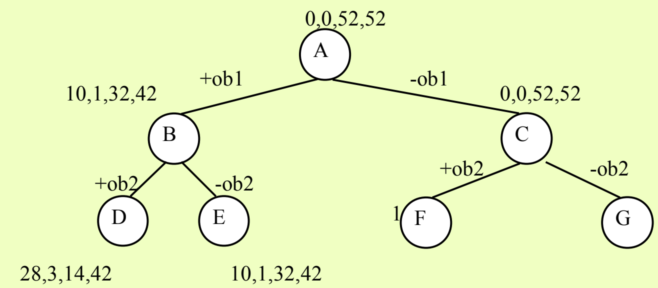
- **Advantages**
  - Quickly finds a solution (possible not-optimal), especially for small problems

- **Disadvantages**
  - Sum of optimal local decisions ≠ global optimal decision
    - Ex. TSP

- **Applications**
  - Planning problems
  - Problem of partial sums
    - Coins
    - knapsack
  - Puzzles
  - Optimal paths in graphs

ISS – A*

- **Basic elements**
  - Combination of positive aspects from
    - Uniform cost search
      - Optimality and completeness
      - Sorted queues
    - Greedy search
      - Speed
      - Sorted based on evaluation
  - Evaluation function ***f(n)***
    - Cost estimation of the path that passes though node *n* ***f(n) = g(n) + h(n)***
    - ***g(n)*** – cost function from the initial state to the current state n
    - ***h(n)*** – cost heuristic function from the current state to the final state
  - Minimisation of the total cost for a path



- **Example**
  - Knapsack problem – capacity W, n objects $(o_1, o_2, \ldots, o_n)$ each of then having a profit p_i, $i=1,2,\ldots,n$
    - Solution: for W = 5 ➔ o_1 and o_3
  - $g(n) = \Sigma p_i$, for selected objects o_i
  - $h(n) = \Sigma p_j$, for not-selected objects and $\Sigma w_j <= W - \Sigma w_i$
  - Fetch node is a tuple *(p, w, p*, f)*, where:
    - *p* – profit of selected objects (function *g(n)*)
    - *w* – weight of selected objects
    - *p** - maximal profit that can be obtained starting from the current state and tacking into account the available space in the knapsack (function *h(n)*)

	o_1	o_2	o_3	o_4
p_i	10	18	32	14
w_i	1	2	4	3

ISS – A*

- **Algorithm**

```
bool BestFS(elem, list){
     found = false;
     visited = Φ;
     toVisit = {start};     //FIFO sorted list (priority queue
     while((toVisit != Φ) && (!found)){
          if (toVisit == Φ)
            return false
          node = pop(toVisit);
          visited = visited U {node};
          if (node == elem)
            found = true;
          else
            aux = Φ;
          for all unvisited children of node do{
            aux = aux U {child};
          }
          toVisit = toVisit U aux; //adding a node onto the FIFO list
                          // based on its evaluation f(n)= g(n)+ h(n)
                               // (best one in the front of list)

     }  //while
     return found;
}
```

ISS – A*

- **Complexity analyse**
  - Time complexity
    - *b* – ramification factor
    - d^{max} – maximal length (depth) of an explored tree
    - $T(n) = 1 + b + b^2 + \dots + b^{dmax} \Rightarrow O(b^{dmax})$
  - Space complexity
    - *d* - length (depth) of solution
    - $T(n) = 1 + b + b^2 + \dots + b^d \Rightarrow O(b^d)$
  - Completeness
    - Yes
  - Optimality
    - yes

- **Advantages**
  - Expands the fewest nodes of the tree
- **Disadvantages**
  - Large amount of memory

- **Applications**
  - Planning problems
  - Problems of partial sums
    - Knapsack problem
    - Coin's problem
  - Puzzles
  - Optimal paths in graphs

ISS – A*

- **Versions**
  - iterative deepening A* (IDA*)
  - memory-bounded A* (MA*)
  - simplified memory bounded A* (SMA*)
  - recursive best-first search (RBFS)
  - dynamic A* (DA*)
  - real time A*
  - hierarchical A*

- **Bibliography**
  - 02/A_IDA.pdf
  - 02/A_IDA_2.pdf
  - 02/SMA_RTA.pdf
  - 02/Recursive Best-First Search.ppt
  - 02/IDS.pdf
  - 02/IDA_MA.pdf
  - http://en.wikipedia.org/wiki/IDA*
  - http://en.wikipedia.org/wiki/SMA*

Solving problem by search

- Topology of search strategies:
  - Solution **generation**
    - **Constructive** search
      - Solution is identified step by step
      - Ex. TSP
    - **Perturbative** search
      - A possible solution is modified in order to obtain another possible solution
      - Ex. SAT - Propositional Satisfiability Problem
  - Search space **navigation**
    - **Systematic** search
      - The entire search space is visited
        - Solution identification (if it exists) → complete algorithms
    - **Local** search
      - Moving from a point of the search space into a neighbour point → incomplete algorithms
      - A state can be visited more times
  - **Certain** items of the search
    - **Deterministic** search
      - Algorithms that exactly identify the solution
    - **Stochastic** search
      - Algorithms that approximate the solution
  - Search space **exploration**
    - **Sequential** search
    - **Parallel** search

Solving problem by search

Methods

- Step-by-step construction of solution
- Identification of a possible optimal solution
  - Problem’s components
    - Conditions (constraints) that solution must satisfy (partially or totally)
    - Evaluation function of a possible solution → optimal solution identification
  - Search space
    - Set of all possible complete solutions
    - State = a possible complete solution
    - Final state = optimal solution
  - Example
    - Queen’s problem
      - Possible states: boards with 8 queens
      - Operators: change the column of a queen
      - Search objective: the board without attacks
      - Evaluation function: number of attacks
    - Planning problems
    - Design of digital circuits

Solving problem by search

Methods

- Step-by-step construction of solution
- Identification of a possible optimal solution
  - Algorithms
    - Until now → **systematic** exploration of the search space
      - Eg. A* ➔ 10^{100} states ≈ 500 binary variables
    - Real-world problems can have 10 000 – 100 000 variables ➔ require new algorithms that **locally** explore the search space
    - Main idea:
      - Start with a state that does not respect some conditions and
      - Change the state for eliminating these violations
        - The search moves into a neighbourhood of the current solution
      - Such that the search will advance through the optimal state
    - Iterative algorithms
      - Only a state is retained
      - Try to improve this state
    - Intelligent version of brute force algorithm
    - Search past is not retained

```
bool LS(elem, list){
        found = false;
        crtState = initState
        while ((!found) && timeLimitIsNotExceeded){
            toVisit = neighbours(crtState)
            if (best(toVisit) is better than crtState)
         crtState = best(toVisit)
           if (crtState == elem)
                found = true;
        }  //while
        return found;
}
```

Solving problem by search

Methods

- Step-by-step construction of solution
- Identification of a possible optimal solution
  - Advantages
    - Simple implementation
    - Less memory
    - can find good solution in large (continuous) search spaces where other systematic algorithms can not be applied
  - Is useful when
    - Can be generated reasonable complete solutions
    - Can be selected a good starting point
    - Exist operators for solution changing
    - Exists a progress measure (for evaluating how the search advances)
    - Exists an evaluation function for a possible solution

Local search strategies (LSS)

- Typology
  - Simple local search – a single neighbour state is retained
    - Hill Climbing -> chooses the best neighbour
    - Simulated Annealing -> probabilistically chooses the best neighbour
    - Tabu search -> retains the recent visited solutions
  - Beam local search – more states (population) are retained
    - Evolutionary Algorithms
    - Particle swarm optimisation
    - Ant colony optimisation

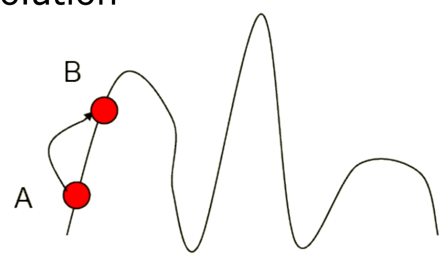
Local search strategies

- Simple local search
  - Special elements:
    - Solution representation
    - Evaluation of a possible solution
    - Neighbourhood of a solution
      - How a neighbour solution is defined/generated
      - How neighbour solutions are identified:
        - Randomly
        - Systematically
    - How a possible solution is accepted
      - First neighbour of the current solution better than the current solution
      - Best neighbour of the current solution better than the current solution
      - Best neighbour of the current solution weaker than the current solution
      - A random neighbour of the current solution

Depends on problem

Local search strategies – Hill climbing (HC)

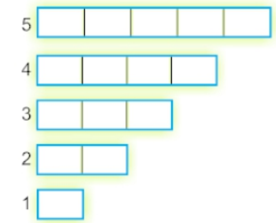
- **Basic elements**
  - Climbing a foggy mountain by an amnesiac hiker :D
  - Continuous moving to better values (larger → mountain climbing)
  - Search advances to improved states until an optimal one is identified
  - How a possible solution is accepted
    - Best neighbour of the current solution better than the current solution
  -
  - Improvement by
    - Maximisation of state's quality → *steepest ascent HC*
    - Minimisation of state's quality → *gradient descent HC*

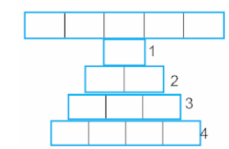


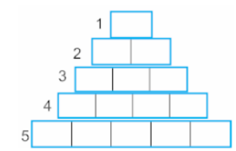
  - HC ≠ *steepest ascent/gradient descent* (SA/GD)
    - HC optimises f(x) with $x \in R^n$ by changing an element of x
    - SA/GD optimises f(x) with $x \in R^n$ by changing all the elements of x

Local search strategies – Hill climbing (HC)

- **Example**
  - Construct tours from different geometrical shapes
    - We have *n* rectangular pieces (of the same width, but different lengths) that are overlapped in a stack. Construct a stable tour of all pieces such that at each move only a piece is moved from the top of the stack (on one of two supplementary stacks).



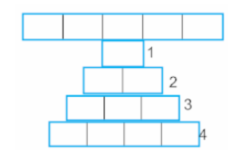




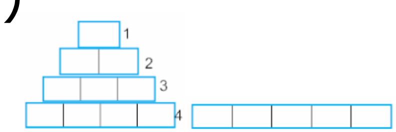
    - Solution representation
      - State x – vector of *n* pairs (*i,j*), where *i is the index of the piece* (*i=1,2,...,n*) and *j* is the index of the stack (*j=1,2,3*)
      - Initial state – vector of the initial tour
      - Final state – vector of the final tour
    - State evaluation
      - $f1$ = # of correctly located pieces → maximisation
        - Conform tot the final tour – $f1 = n$
      - $f2$ = # of wrongly located pieces → minimisation
        - Conform tot the final tour– $f2 = 0$
      - $f = f1 - f2$ → maximization
    - Neighbourhood
      - Possible moves
        - Move a piece *i* from stack *j1* on stack *j2*
    - How a possible solution is accepted
      - Best neighbour of the current solution better than the current solution

Local search strategies – Hill climbing (HC)

- **Example**
  - Iteration 1
    - current state = initial state:
      - x = s_1 = ((5,1), (1,1), (2,1), (3,1), (4,1))
      - Pieces 1, 2 and 3 are correctly located
      - Pieces 4 and 5 are wrongly located
      - $f(s_1) = 3 - 2 = 1$



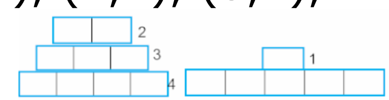
    - $x^* = x$
    - Neighbours of current state x – a single one → piece 5 moves on stack 2 ➔
      - s_2 = ((1,1), (2,1), (3,1), (4,1), (5,2))
      - $f(s_2) = 4-1=3 > f(x)$ ➔ $x = s_2$



Local search strategies – Hill climbing (HC)

- **Example**
  - Iteration 2
    - Current state x = ((1,1), (2,1), (3,1), (4,1), (5,2))
      - f(x) = 3
    - Neighbours of the current state – 2 neighbours:
      - Piece 1 moves on stack 2 ➔ s_3 = ((2,1), (3,1), (4,1), (1,2), (5,2)) ➔ f(s_3) = 3-2=1 < f(x)

      - Piece 1 moves on stack 3 ➔ s_4 = ((2,1), (3,1), (4,1), (5,2), (1,3)) ➔ f(s_4) = 3-2=1 < f(x)



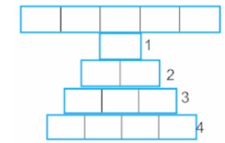
      - There is no neighbour better than x ➔ stop
  - x* = x = ((1,1), (2,1), (3,1), (4,1), (5,2))
  - But x* is a local optimum just (not a global one)

Local search strategies – Hill climbing (HC)

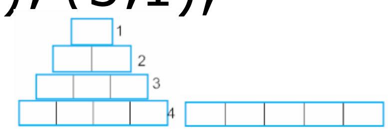
- **Example**
  - Construct tours from different geometrical shapes - other solution
    - State evaluation
      - f1 = sum of stack's height whose all pieces are correctly located (final tour f1 = 10) → maximisation
      - f2 = sum of stack's height whose pieces are wrongly located (final tour f2=0) → minimisation
      - f = f1 – f2 → maximisation
    - Neighbourhood
      - Possible moves
        - Move a piece *i* from stack *j1* on stack *j2*

Local search strategies – Hill climbing (HC)

- **Example**
  - Iteration 1



    - Current state x = initial state s_1 = ((5,1), (1,1), (2,1), (3,1), (4,1))
      - All pieces are wrongly located → f1 = 0, f2 = 3+2 + 1 + 0 + 4 = 10
      - $f(s_1) = 0 - 10 = -10$
    - $x^* = x$
    - Neighbours of current state x– a single one → piece 5 is moved on stack 2 → s_2 = ((1,1), (2,1), (3,1), (4,1), (5,2))

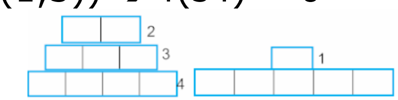


      - $f(s_2) = 0 - (3+2+1+0) = -6 > f(x)$ → $x = s_2$

Local search strategies – Hill climbing (HC)

- **Example**
  - Iteration 2
    - Current state x = ((1,1), (2,1), (3,1), (4,1), (5,2))
      - f(x) = -6
    - Neighbours of the current state – two neighbours:
      - Piece 1 is moved on stack 2 ➔ s3 = ((2,1), (3,1), (4,1), (1,2), (5,2)) ➔ f(s3) = 0 – (0+2+3+0)=-5 > f(x)

      - Piece 1 is moved 3 ➔ s4 = ((2,1), (3,1), (4,1), (5,2), (1,3)) ➔ f(s4) = 0 - (1+2+1) = -4 > f(x)



      - Best neighbour of x is s4 ➔ x = s4
  - Iteration 3
    - ...
  - Local optima are avoided

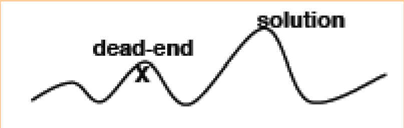
Strategii de căutare locală – Hill climbing (HC)

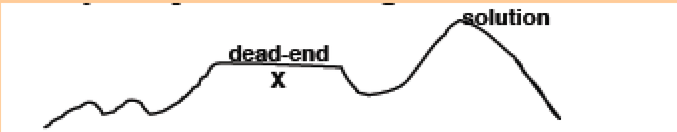
- **Algorithm**

```
Bool HC(S){
    x = s1   //initial state
    x*=x  // best solution (found until now)
    k = 0// # of iterations
    while (not termination criteria){
    k = k + 1
    generate all neighbours of x (N)
    Choose the best solution s from N
    if f(s) is better than f(x) then x = s
    else stop
    }  //while
    x* = x
    return x*;
}
```

Local search strategies – Hill climbing (HC)

- **Search analyse**
  - Convergence to local optima
- **Advantages**
  - Simple implementation -> solution approximation (when the real solution is difficult or impossible to find)
    - Eg. TSP with many towns
  - Does not require memory (does not come back into the previous state)
- **Disadvantages**
  - Evaluation function is difficult to be approximated
  - If a large number of moves are executed, the algorithm is inefficient
  - If a large number of moves are executed, the algorithm can block
    - In a local optimum
    - On a plateau – evaluation is constant
    - On a peak – a skip of more steps can help the search
- **Applications**
  - Cannibal's problem
  - 8-puzzle, 15-puzzle
  - TSP
  - Queen’s problem

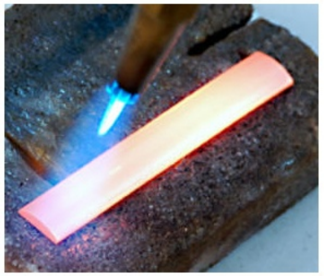




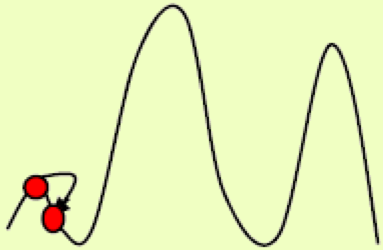
Local search strategies – Hill climbing (HC)

- **Versions**
  - Stochastic HC
    - The next state is randomly selected
  - First-choice HC
    - Randomly generation of successors until a new one is identified
  - Random-restart HC → *beam local search*
    - Restart the search from a randomly initial state when the search does not advance

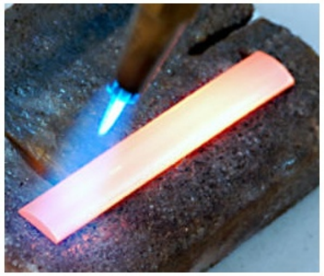
Local search strategies – Simulated Annealing



- **Basic elements**
  - Inspired by physical process modelling
    - Metropolis et al. 1953, Kirkpatrick et al. 1982;
  - Successors of the current state are randomly selected
    - If a successor is better than the current state
      - It becomes the new current state
      - Otherwise, it is retained by a given probability
  - Weak moves are allowed with a given probability *p*
    - Escape from local optima
  - Probability p = $e^{\Delta E/T}$
    - Depends on difference (energy) ΔE
    - Is modelled by a temperature parameter *T*
  - The frequency of weak moves and their size gradually decrease when T is decreasing
    - T = 0 ➔ hill climbing
    - T → ∞ ➔ weak moves are frequently performed
  - Optimal solution is identified only if the temperature slowly decreases
  - How a possible solution is accepted
    - A random neighbour of the current solution better than the current solution or
    - Probabilistic, a random neighbour of the current solution weaker than the current solution

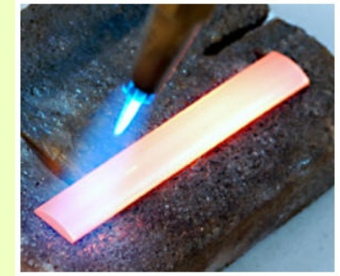


Local search strategies – Simulated Annealing



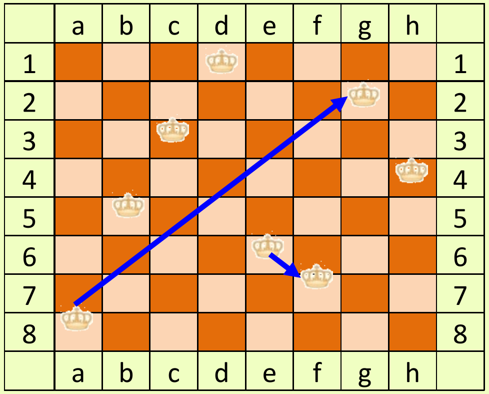
- **Example – 8-queen's problem**
  - Statement
    - Put 8 queens on a chessboard such there are no attacks between queens
  - Solution representation
    - State *x* – permutation of *n* elements x = $(x_1,x_2,..,x_n)$, where x_i – line where the queen of column j is located
      - There are no attacks on lines or on columns
      - It is possible to find diagonal attacks
    - Initial state – a random permutation
    - Final state – a permutation without attacks
  - Evaluation function for a state
    - F – sum of attacked queens by each queen → minimisation
  - Neighbourhood
    - Possible moves
      - Move a queen from a line to a new line (swap 2 elements from permutation)
  - How a possible solution is accepted
    - A random neighbour of the current solution
      - better than the current solution or
      - Weaker than the current solution – by a probability $P(\Delta E) = e^{-\frac{\Delta E}{T}}$,where
        - ΔE – energy (evaluation) difference of two states
        - T – temperature, T(k) = 100/k, where k is the iteration number

Local search strategies – Simulated Annealing



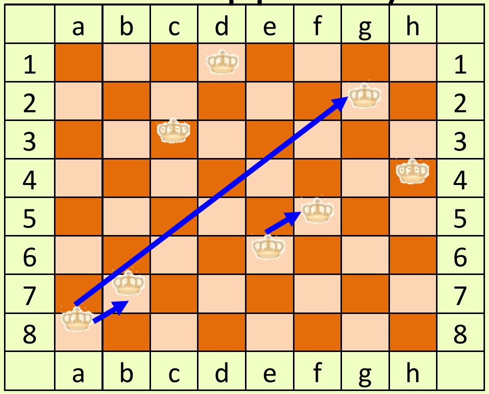
- **Example – 8-queen's problem**
  - Iteration 1 (k = 1)
    - Current state x = initial state

      $s_1 = (8,5,3,1,6,7,2,4)$
      - $f(s_1) = 1+1 = 2$

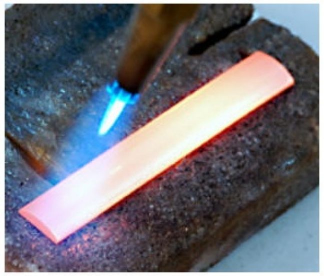


    - $x^* = x$
    - $T = 100/1 = 100$
    - A neighbour of current state x → queen of line 5 is swapped by queen of line 7

      ➔ $s_2 = (8,7,3,1,6,5,2,4)$
      - $f(s_2) = 1+1+1=3 > f(x)$
      - $\Delta E = f(s_2) - f(s_1) = 1$
      - $P(\Delta E)=e^{-1/100}$
      - r = random(0,1)
      - if $r < P(\Delta E)$ ➔ $x = s_2$



Local search strategies – Simulated Annealing

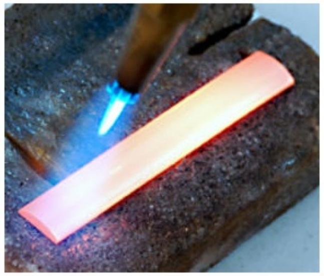


- **Algorithm**

```
bool SA(S){
      x = s1    //initial state
      x*=x // best solution found until a given moment
      k = 0     // iteration number
      while (not termination criteria){
      k = k + 1
      generate a neighbour s of x
      if f(s) is better than f(x) then x = s
      else
           pick a random number r (in (0,1) range)
           if r < P(ΔE) then x = s
      }  //while
      x* = x
      return x*;
}
```

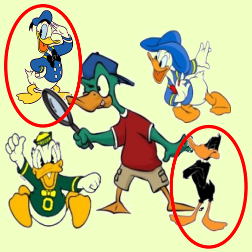
- Stop conditions
  - The solution is found
  - A number of iterations is reached
  - The frozen temperature (T=0) is hit
- How a small probability is chosen?
  - p = 0.1
  - P decreases along the iterations
  - P decreases along the iterations and while the “error” |f(s) – f(x)| is increasing
  - $p = \exp(- |f(s) - f(x)|/T)$
  - Where T –temperature (that increases)
    - For a large T almost any neighbour is accepted
    - For a small T, only neighbours better that s are accepted
  - If the error is large, then the probability is small

Local search strategies – Simulated Annealing



- **Search analyse**
  - Convergence (complete, optimal) through global optima is slowly
- **Advantages**
  - Statistic-based algorithm → it is able to identified the optimal solution, but it requires many iterations
  - Easy to implement
  - Generally, if find a good (global) solution
  - Can solve complex problems (with noise and many constraints)
- **Disadvantages**
  - Slowly algorithm – convergence to solution takes a long time
    - Trade-off between the solution's quality and the time required to find it
  - Depends on some parameters (temperature)
  - The provided optimal solution could be local or global
  - The solution's quality depends on the precision of variables involved in the algorithm
- **Applications**
  - Combinatorial optimisation problems → knapsack problem
  - Design problems → digital circuits design
  - Planning problems → production planning, tenis game planning

Local search strategies – Tabu search

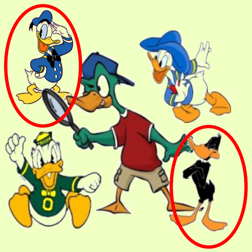


- **Basic elements**
  - "Tabu" → things that cannot be touched because they are sacred
  - Proposed in 1970 by F. Glover

  - Main idea
    - starts with a state that violates some constraints and
    - Performs changes for eliminating them (the search moves into the best neighbour solution of the current solution) in order to identify the optimal solution
    - Retains
      - Current state
      - Visited states and performed moves (limited list of states that must be avoided)
    - How a possible solution is accepted
      - Best neighbour of the current solution better than the current solution and un-visited until that moment

  - 2 important elements
    - Tabu moves (T) – determined by a non-Markov process that uses information obtained during last generations of search process
    - Tabu conditions – linear inequalities or logical links that depend on current solution
      - Influence the selection of tabu moves

Local search strategies – Tabu search



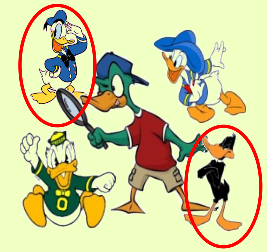
- **Example**
  - Statement
    - Pay a sum S by using n coins of values v_i as many as possible (each coin has b_i copies)

  - Solution representation
    - State x – vector of n integers $x = (x_1, x_2, ..., x_n)$ with $x_i \in \{0, 1, 2, ..., b_i\}$
    - Initial state – randomly

  - State evaluation
    - f_1 = S – total value of selected coins → minimisation
      - If the total value of coins > S → penalisation (eg. 500 units)
    - f_2 = number of selected coins → maximisation
    - $f = f_1 - f_2$ → minimisation

  - neighbourhood
    - Possible moves
      - Including in the sum of j copies of coin i (plus$_{,}$)
      - Eliminating from the sum of j copies of coin i (minus$_{,}$)
    - Tabu list retains performed moves of an iteration
      - \- move = the added/eliminated coin

Local search strategies – Tabu search

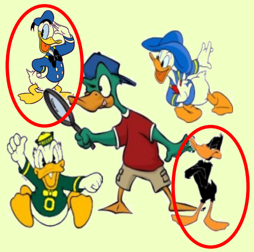


- **Example**
  - S = 500, penalisation= 500, n = 7

S=500	m_1	m_2	m_3	m_4	m_5	m_6	m_7
v_i	10	50	15	20	100	35	5
b_i	5	2	6	5	5	3	10

Stare curentă	Val. f	Listă tabu	Stări vecine	Mutări	Val. f
2 0 1 0 0 2 1	384	∅	2 0 1 3 0 2 1	$plus_{4,3}$	321
			2 0 1 0 0 3 1	$plus_{6,1}$	348
			0 0 1 0 0 2 1	$minus_{1,2}$	406
2 0 1 3 0 2 1	321	$plus_{4,3}$	2 0 1 3 5 2 1	$plus_{5,5}$	316
			2 0 1 1 0 2 1	$minus_{4,2}$	363
			2 1 1 3 0 2 1	$plus_{2,1}$	270
2 1 1 3 0 2 1	270	$plus_{4,3}$ $plus_{2,1}$	…		

Local search strategies – Tabu search



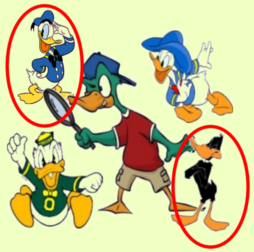
- **Example**
  - S = 500, penalisation = 500, n = 7

S=50	m_1	m_2	m_3	m_4	m_5	m_6	m_7
v_i	10	50	15	20	100	35	5
b_i	5	2	6	5	4	3	10

Stare curentă	**Val. f**	**Listă tabu**	**Stări vecine**	**Mutări**	**Val. f**
2 0 1 0 0 2 1	384	∅	1 0 1 4 0 2 1	$minus_{1,1}$,$plus_{4,4}$	311
			2 0 4 0 1 2 1	$plus_{3,3}$,$minus_{5,1}$	235
			2 0 1 0 4 2 6	$plus_{5,4}$, $plus_{7,5}$	450
2 0 4 0 1 2 1	235	$plus_{3,3}$, $minus_{5,1}$	2 0 5 0 5 2 1	$plus_{3,1}$, $plus_{5,4}$	315
			5 0 4 0 4 2 1	$plus_{1,3}$, $plus_{5,3}$	399
			2 2 4 0 5 2 1	$plus_{2,2}$, $plus_{5,4}$	739
2 0 4 0 1 2 1	235	$plus_{3,3}$, $minus_{5,1}$	...		

  - Final solution: 4 1 5 4 1 3 10 (f = -28)

Local search strategies – Tabu search

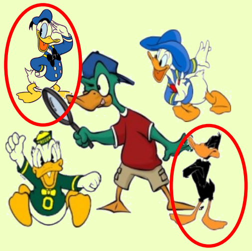


- **Algorithm**

```
bool TS(S){
    Select x∈S //S - search space
    x*=x        //best solution until a moment
    k = 0   //iteration number
    T = ∅   //list of tabu moves
    while (not termination criteria){
    k = k + 1
    generate a subset of solutions in the neighbourhood N-T of x
    choose the best solution s from N-T and set x=s.
    if f(x)<f(x*) then x*=x
    update T with moves of generating x
    }  //while
    return x*;
}
```

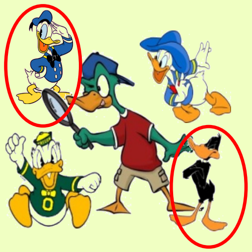
- Stop conditions
  - Fix number of iterations
  - A given number of iterations without improvements
  - Sufficient proximity to the solution (if it is known)
  - Depletion unvisited elements of a neighbourhood

Local search strategies – Tabu search



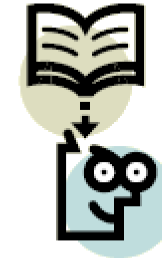
- **Search analyse**
  - Quickly convergence to global optima
- **Advantages**
  - The algorithm is general and can be easy implemented
  - Quickly algorithm (can find in a short time the optimal solution)
- **Disadvantages**
  - Identify the neighbours in continuous search spaces
  - Large number of iterations
  - Global optima identification is not guaranteed

Local search strategies – Tabu search



- **Applications**
  - Determination of three-dimensional structure of proteins in amino acid sequences
  - Traffic optimisation in communication networks
  - Planning in production systems
  - Network design in optical telecommunication
  - Automatic routing of vehicles
  - Graph problems (partitioning)
  - Planning in audit systems

Review



- ISS best first search
  - The best evaluated nodes are firstly expanded

  - Greedy ISS
    - Minimisation of the cost from the current state to the final state – h(n)
    - Search time < USS
    - incomplete

      not optimal

  - A* ISS
    - Minimisation of the cost from the initial state to the current state – g(n) and of the cost from the current state to the final state – h(n)
    - Avoid to re-visit a state
    - Without supra-estimation of h(n)
    - Large time and space complexity → depends on used heuristic
    - Complete
    - Optimal

Review

- Local SS
  - Iterative algorithms
    - Work with a possible solution → optimal solution
    - Can block in local optima

	Nex state selection	Acceptance criteria	Convergence
HC	Best neighbour	Neighbour is better than current state	Local or global optima
SA	Random neighbour	Neighbour is better than current state or neighbour is weaker than current state (prababilistic acceptance)	Global optima (slowly)
TS	Best un-visited neighbour	Neighbour is better than current state	Global optima (quickly)

Next lecture

A. Short introduction in Artificial Intelligence (AI)

A. **Solving search problems**
   A. **Definition of search problems**
   B. **Search strategies**
      A. Uninformed search strategies
      B. Informed search strategies
      C. **Local search strategies (**Hill Climbing, Simulated Annealing, Tabu Search**, Evolutionary algorithms**, PSO, ACO)
      D. Adversarial search strategies

C. Intelligent systems
   A. Rule-based systems in certain environments
   B. Rule-based systems in uncertain environments (Bayes, Fuzzy)
   C. Learning systems
      A. Decision Trees
      B. Artificial Neural Networks
      C. Support Vector Machines
      - Evolutionary algorithms
   D. Hybrid systems

Next lecture – Useful information

- Chapter 14 of *C. Groşan, A. Abraham, Intelligent Systems: A Modern Approach, Springer, 2011*
- *M. Mitchell, An Introduction to Genetic Algorithms, MIT Press, 1998*
- Chapter 7.6 of *A. A. Hopgood, Intelligent Systems for Engineers and Scientists, CRC Press, 2001*
- Chapter 9 of *T. M. Mitchell, Machine Learning, McGraw-Hill Science, 1997*

- Presented information have been inspired from different bibliographic sources, but also from past AI lectures taught by:
  - PhD. Assoc. Prof. Mihai Oltean – www.cs.ubbcluj.ro/~moltean
  - PhD. Assoc. Prof. Crina Groşan - www.cs.ubbcluj.ro/~cgrosan
  - PhD. Prof. Horia F. Pop - www.cs.ubbcluj.ro/~hfpop