Random growth with reset: complex systems in a new perspective

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A simple model [1, 2] based on a master equation that contains a growth term and a reset term to a fundamental state is discussed. In the continuum limit the evolution equation writes:

$$\frac{\partial}{\partial t}\mathcal{P}(x,t) \;=\; -\frac{\partial}{\partial x}\,(\mu(x)\,\mathcal{P}(x,t)) \;-\; \gamma(x)\,\mathcal{P}(x,t). \tag{1}$$

For various $\mu(x)$ growth and $\gamma(x)$ reset rates such processes lead to distributions that are characteristic for complex systems [3, 4, 5]:

$\gamma(x)$	$\mu(x)$	$Q(x)$
γ	μ	Exponential: $\sim \mathrm{e}^{-(\gamma/\mu)x}$
γ	$\sigma(x+b)$	Tsallis–Pareto: $\sim (1+x/b)^{-1-\gamma/\sigma}$
γ	σx^{α}, $\alpha<1$	Weibull: $\sim x^{-\alpha}\mathrm{e}^{-bx^{1-\alpha}}$
γ	$\sigma(x+a)(x+b)$	Pearson: $\sim (x+a)^{-1-v}(x+b)^{-1+v}$
γ	$\sigma\mathrm{e}^{x}$	Gompertz: $\sim \exp\left(\frac{\gamma}{\sigma}\mathrm{e}^{-x}-x\right)$
$\ln(x/a)$	σx	Log-Normal: $\mathcal{Q}(x)\,dx \;\sim\; \mathrm{e}^{-\gamma^2/2\sigma}\,d\gamma$
x	σ^2	Gauss: $\sim \mathrm{e}^{-x^2/2\sigma^2}$
$\sigma(ax-c)$	σx	Gamma: $\sim x^{c-1}\,\mathrm{e}^{-ax}$

Table 1: Common stationary density functions, $Q(x)$, obtained with the $\gamma(x)$ reset and $\mu(x)$ growth rate.

We present several interdisciplinary applications for this simple process: emergence of degree distribution in real-world networks, scientific citations-, Facebook popularity-, income- and wealth distribution, biodiversity indicators and settlement-sizes distribution.

References

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