Predicting with chaos: dynamical features reveal the global optimum of NP-hard problems

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From the ground-state problem of spin-glasses, through Sudoku puzzles to protein folding, a huge set of real-world optimization problems and various industrial applications can be translated to constraint satisfaction (SAT) mathematical form. Many times these problems do not have a solution, but they still have an optimal state. These problems are the maximum satisfiability (max-SAT) problems, where the task is to find the set of Boolean variables which can satisfy the maximum (hence the name) possible number of the given logical constraints. The max-SAT problem is an NP-hard problem. Exact solvers are extremely inefficient on these types of problems, heuristic solvers on the other hand can never provide information on the correctness of the found optimum.

In our previous work we mapped Boolean satisfiability to transiently chaotic continuous-time dynamical systems with a one-to-one correspondence between SAT solutions and stable attractors. Modifying the dynamics to be applicable to max-SAT we obtain a chaotic dynamical system, which has no stable attractors but performs an efficient search for the global optimum. We show that the scaling of an invariant of the solver's dynamics, the escape rate, as function of the number of unsatisfied clauses can predict the global optimum value, often well before reaching the corresponding state. A rough estimation of the time needed to find more optimal states can also be obtained.

The performance of our solver is demonstrated on hard max-SAT competition problems and we also consider the two-color Ramsey number $R(m, m)$, translate it to SAT and apply our algorithm to the still unknown $R(5, 5)$. We find edge colorings without monochromatic 5-cliques for complete graphs up to 42 vertices, while on 43 vertices we find colorings with only two monochromatic 5-cliques, the best coloring found so far, supporting the conjecture that $R(5, 5) = 43$.