Ising-type phase transitions in simple evolutionary coordination games

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In multiplayer evolutionary matrix games player-player interactions are defined by so-called payoff matrices. Recently, the concept of payoff matrix decomposition revealed that all such games are combinations of elementary games of just four distinct classes. Three of these classes—namely self-dependent, cross-dependent, and coordination-type games—span the set of potential games, which become equivalent to classical spin models when strategy updates are governed by the logit rule [1, 2]. In this analogy, coordination-type components correspond to ferromagnetic or antiferromagnetic spin-spin interactions, the symmetric parts of self- and cross-dependent components are analogous to an external magnetic field, while the remaining antisymmetric parts are responsible for the emergence of social dilemmas. This classification scheme allows for a systematic analysis of matrix games that can hopefully provide insights into the characteristics of general evolutionary games. We have recently studied the properties of simple combinations of elementary coordination games to further our understanding of the interplay between elementary game components. In a general n-strategy elementary coordination game, the Ising model is extended by $n-2$ neutral strategies (distinguishable noninteracting spin states) that provide zero payoff regardless of the opponent's choice of strategy. When players are located at the sites of a square lattice and only interact with their nearest neighbors, this game model exhibits an order-disorder phase transition at a strategy number–dependent critical temperature. If the number of neutral strategies is below a threshold value, the transition is continuous and belongs to the Ising universality class, but it is of the first order otherwise [3]. This robustness of the Ising-type critical behavior can also manifest itself in games composed of a small number of elementary coordinations. The four-state clock model, made up of two independent elementary coordinations, is equivalent to a system of two uncoupled identical Ising models and as a result undergoes an Ising-type phase transition [4]. In a similar five-strategy combination of an elementary coordination game and a three-state Potts model, both Ising- and Potts-type critical behaviors can be observed [5].

References

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