

A monotone inclusion problem solved through an inertial algorithm with two backward steps

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Continuing a recent paper by R.I. Boţ, A. Grad and E.R. Csetnek dealing with inertial algorithms used to solve monotone inclusion problems, we present a new inertial algorithm, with two backward steps. We prove the ergodic convergence of this algorithm to an optimal solution of the problem, and then compare it to the algorithm with just one backward step. The monotone inclusion problem under consideration is

$$0 \in Ax + Dx + N_C(x)$$

where $A, B : \mathcal{H} \rightrightarrows \mathcal{H}$ are two maximally monotone operators, $D : \mathcal{H} \rightarrow \mathcal{H}$ is an η - cocoercive operator with $\eta > 0$ and $C := \text{zer}B \neq \emptyset$.

A cornerstone in the proof of the convergence is a regularity condition stated with the help of the Fitzpatrick function associated to the operator B .

$$(H_{\text{fitz}}) \quad \begin{cases} (i) A + N_M \text{ is a maximally monotone operator and } \text{zer}(A + D + N_M) \neq \emptyset \\ (ii) \forall p \in \text{range}N_M, \sum_{n=1}^{\infty} \lambda_n \beta_n \left[\sup_{\tilde{u} \in C} \varphi_B \left(\tilde{u}, \frac{p}{\beta_n} \right) - \sigma_C \left(\frac{p}{\beta_n} \right) \right] < \infty \\ (iii) (\lambda_n)_{n \geq 1} \in l^2 \setminus l^1. \end{cases} \quad (1)$$

The algorithm is implemented in Matlab. Moreover, applications to convex optimization problems are also presented.

References

- [1] R. I. Boţ, E. R. Csetnek, Penalty schemes with inertial effects for monotone inclusion problems, *Optimization* 66(6) (2017), 965-982
- [2] R. I. Boţ, E. R. Csetnek, An inertial Tseng's type proximal algorithm for nonsmooth and nonconvex optimization problems, *Journal of Optimization Theory and Applications* 171(2) (2016), 600-616
- [3] S. Banert, R. I. Boţ, Backward penalty schemes for monotone inclusion problems, *Journal of Optimization Theory and Applications* 166(3) (2015), 930-948