Spectral collocation solutions to a class of nonlinear t. p. b. v. p. on the half line.

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In [1] and [3] as well as in some other papers the authors elaborate on the existence, uniqueness and regularity of solutions to the following class of genuinely nonlinear two-point boundary value problems (t. p. b. v. p.)

$$\begin{array}{c} \frac{1}{p(x)}\left(p\left(x\right)u'\left(x\right)\right)' = q\left(x\right)f\left(x, u\left(x\right), p\left(x\right)u'\left(x\right)\right),\ x \in (0,\infty) \\ \alpha u\left(0\right) - \beta u'\left(0\right) = r, \\ \lim_{x\to\infty} u\left(x\right) = 0, \end{array} \tag{1}$$

where $\alpha > 0$, $\beta \geq 0$ and r is a given constant. The r. h. s. map $f : [0,\infty) \times \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ and the coefficients p, $\frac{1}{q} : [0,\infty) \to [0,\infty)$ are assumed continuous.

Our aim is to approximate, as accurate as possible, the solutions of these problems by a high order Laguerre Gauss Radau collocation (LGRC) method (see [2], Ch. 2). We are looking for a solution of the form

$$u_{N-1}\left(x\right) := \sum_{j=1}^{N} \frac{e^{-x/2}}{e^{-x_j/2}} \phi_j\left(x\right) u_j, \tag{2}$$

where $\phi_j\left(x\right)$ are the fundamental Lagrangian interpolating polynomials in barycentric form, $x_j, j = 1, 2, \ldots, N$ are the Laguerre Gauss Radau nodes and $u_j, j = 1, 2, \ldots, N$ are the nodal unknowns.

The problems of type (1) are hard, because they are some nonlinear boundary value ones over an unbounded domain which can be polluted by singularities and/or near-singularities, and no explicit solutions are known. These problems have been usually solved by empirical truncation of the domain coupled with a sort of shooting.

LGRC scheme avoids this technique and offers reliable numerical solutions which can confirm and, in some situations extend and deepen the analytical outputs. Moreover, using the advantage of Laguerre polynomial transform we can transfer the information (nodal values of the numerical solution) from the physical space into the coefficient (phase) space where, in a log-linear plot we can estimate the order of convergence of numerical solution.

In order to exemplify, we solve the so called Kidder's problem,

$$z'' + 2x\left(1 - \alpha z\right)^{\delta} z' = 0,\ z\left(0\right) = 1,\ z\left(\infty\right) = 0,\ \delta = -\frac{1}{2}. \tag{3}$$

Some more examples from the theory of semiconductors and electromagnetic theory of strong interactions are also carried out.

References

[1] Bonanno, G., O'Regan, D.: A Boundary Value problem on the Half-Line via Critical Point Method. Dyn. Syst. Appl. **15**, 395–404 (2006)

[2] Gheorghiu, C.I.: Spectral Collocation Solutions to Problems on Unbounded Domains. Casa Cartii de Stiinta, Cluj-Napoca, 2018

[3] O'Regan, D.: Singular Nonlinear Differential Equations on the Half line. Topol. Method. Nonl.An. **8**, 137–159 (1996)