Collective modes of identical Kuramoto rotators in a ring-like topology

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Systems of locally coupled identical Kuramoto rotators in a 1D ring-like topology are considered. We investigate the emergent stationary collective modes in such systems. These spatio-temporal patterns are interpreted as generalized synchronization modes of the system, or can be viewed as self-closing rotating waves on a circle with a well defined winding number. Our study investigates the predictability of the final stable state when the phases of the rotators are randomly initialized.

First we summarize the known results regarding this type of collective behavior in circular oscillator ensembles using a novel theoretical framework. Interpreting the system as a gradient system we identify all possible stationary states including a new class of unstable asymptotic solutions. The linear stability of the emergent patterns is also determined and it is linked to the winding number, which labels the stationary states, in agreement with the results in [1, 2, 3]. Computer experiments were considered, confirming that the distribution of the probability of appearance for the stable collective modes is well approximated by a Gaussian envelope curve. We also show that variance of the distributions scales linearly with the system size [1].

New results are obtained by numerically studying the dynamics of the system. Using multidimensional geometry we interpret the phase space of the system as various distinct planes confined in a hypercube. We show that the motion of the characteristic point of the system is only possible on the surface of these planes. Unfortunately for the general $d > 3$ case the actual trajectories cannot be directly visualized, so we present different attempts to picture the time-evolution of the system. We show that the complexity of the dynamics is rapidly increasing with the system size. This feature indicates that predictions on the final state of the system made solely from the random initial conditions become more and more difficult as we consider bigger systems. We argue however, that the final stationary mode is always predictable after a certain time-moment of the dynamics. A simple method is proposed for identifying this time-moment, t_s. We study numerically the scaling of the average time for the selection process as a function of the system parameters.

References

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