Second order differentiability of the intermediate-point function in Cauchy's mean-value theorem

Beatrix-Mihaela Pop

If the functions $f, g : I \to \mathbb{R}$ are differentiable on the interval $I \subseteq \mathbb{R}$, $a \in I$, then there exists a function $\overline{c} : I \to I$ such that

$$[f(x) - f(a)]\, g^{(1)}(\overline{c}(x)) = [g(x) - g(a)]\, f^{(1)}(\overline{c}(x)), \text{ for } x \in I.$$

In this paper we study the differentiability of the function $\overline{c}$, when

$$f^{(k)}(a)\, g^{(1)}(a) = f^{(1)}(a)\, g^{(k)}(a), \text{ for all } k \in \{1, ..., n-1\}$$

and

$$f^{(n)}(a)\, g^{(1)}(a) \neq f^{(1)}(a)\, g^{(n)}(a)$$