Project code: **PN-III-P4-ID-PCE-2016-0190**
Contract: **96/12.07.2017**
Project title: *Equilibrium and optimization problems: theoretical and computational approaches*
Project acronym: EQOPTIM
Project leader: Professor **Gábor KASSAY**, Ph.D.
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SCIENTIFIC RESEARCH REPORT No. 3/2019

I. Research Team

Prof. Gábor KASSAY (project leader), Ph.D.
Prof. Nicolae POPOVICI, Ph.D. habil.
Assoc. Prof. Cornel Sebastian PINTEA, Ph.D.
Assoc. Prof. Szilárd Csaba LÁSZLÓ, Ph.D. habil.
Assoc. Prof. Zsolt DARVAY, Ph.D.
Lect. Prof. Anca GRAD, Ph.D.
Lect. Prof. Livia Mihaela BERCHEŞAN (maiden name MIHOLCA), Ph.D.
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Petra-Renáta RIGÓ (maiden name TAKÁCS) , Ph.D. Student

II.Project summary and objectives

(a) The project is mainly motivated by the growing interest in the literature regarding sensitivity, openness/metric regularity, variational principles, and practical applications concerning equilibrium and optimization problems, interest which clearly shows that these fields are appropriate for applying the modern tools of nonlinear analysis. The aim of the seven objectives of the project is to bring to light new concepts and approaches within the following specific objectives:

Obj. 1 *Parametric equilibrium problems*
Obj. 2 *Variational principles for vector equilibrium problems*
Obj. 3 *Vector optimization problems in connection with differential inclusions*
Obj. 4 *Duality and optimality conditions for set-valued optimization*
Obj. 5 *Numerical methods for equilibrium problems*
Obj. 6 *Interior-point methods for scalar optimization problems*
Obj. 7 *Algorithms for image processing and optimal location*

(b) All objectives planned for 2018 have been achieved as indicated in Section III below.

III. Scientific results

(a) Publications

12 papers have been completed in 2019:

- **5 articles** have been **published/accepted** in **WOS – indexed journals** (A1, A2, A3, A4 and A9 in the next table);
- **3 articles** have been **published** in international **peer-reviewed journals -** (A5, A6 and A12 in the next table);
- **1 article** has been **published** in the proceedings volume of an international conference (A10 in the next table);
- **3 papers** have been **submitted to WOS – indexed journals** (A7, A8 and A11 in the next table);

Ref.	Article	Obiectives
A1	**Zsolt Darvay**, Tibor Illés, Behrouz Kheirfam, **Petra Renáta Rigó**: *A corrector-predictor interior-point method with new search direction for linear optimization*, **Central European Journal of Operations Research**, First Online 15 May 2019, DOI: 10.1007/s10100-019-00622-3, [JCR IF 2018: 1.260]	Obj. 6
A2	Ouayl Chadli, **Gábor Kassay,** Asma Saidi: *On the existence of antiperiodic solutions for hemivariational inequalities: an equilibrium problem approach*, **Optimization Letters**, Published Online 15 October 2019, DOI: 10.1007/s11590-019-01490-1	Obj. 2
A3	Ovidiu Bagdasar, Stuart Berry, **Nicolae Popovici**: *Traffic assignment: On the interplay between optimisation and equilibrium problems*, **Optimization**, Published online 13 January 2020, DOI: 10.1080/02331934.2019.1711082 [JCR IF 2018: 1.206]	Obj. 7
A4	Christian Günther, **Nicolae Popovici**: *Characterizations of explicitly quasiconvex vector functions w.r.t. polyhedral cones*, **Journal of Nonlinear and Convex Analysis**, Special Issue in Memory of Professor Siegfried Schaible, 20 (2019) (12), 2653-2665 [JCR IF 2018: 0.595]	Obj. 3
A5	Christian Günther, Elisabeth Köbis, **Nicolae Popovici**: *Computing minimal elements of finite families of sets w.r.t. preorder relations in set optimization*, **Journal of Applied and Numerical Optimization**, Special Issue Dedicated to Boris Polyak, 1 (2019) (2), 131-144, DOI: 10.23952/jano.1.2019.2.04	Obj. 3
A6	Christian Günther, **Nicolae Popovici**: *The role of nonlinear scalarization functions in characterizing generalized convex vector functions*, Journal of Applied and Numerical Optimization, Special Issue Dedicated to Christiane Tammer**,** 1 (2019) (3), 1 (2019) (3), 325-333, DOI: 10.23952/jano.1.2019.3.09	Obj. 3

A7	Monica Bianchi, **Gábor Kassay**, Rita Pini: *Regularization of Brézis pseudomonotone variational inequalities*, under review	Obj. 2 Obj. 5
A8	Juan Enrique Martínez-Legaz, **Cornel Pintea**: *Closed convex sets with an open or closed Gauss range*, under review	Obj. 3
A9	**Zsolt Darvay**, Behrouz Kheirfam, **Petra Renáta Rigó**: *A new wide neighborhood primal-dual second-order corrector algorithm for linear optimization*, **Optimization Letters**, First Online 24 August 2019, DOI: 10.1007/s11590-019-01468-z, [JCR IF 2018: 1.399]	Obj. 6
A10	**Zsolt Darvay**, **Petra Renáta Rigó**, Eszter Szénási: *Infeasible interior-point algorithm for linear optimization based on a new search direction*. In L. Zadnik Stirn, M. Kljajic Borstnar, J. Zerovnic, S. Drobne, J. Povh (eds.), Proceedings of the 15th International Symposium on Operational Research SOR'19, Bled, Slovenia, September 25-27, pp. 475-480, 2019	Obj. 6
A11	Suliman Al-Homidan, Qamrul Hasan Ansari, **Gábor Kassay**: *Bregman type regularization of variational inequalities with Mosco approximation of the constraint set*, under review	Obj. 2 Obj. 5
A12	Christian Günther, Elisabeth Köbis, **Nicolae Popovici**: *On strictly minimal elements w.r.t. preorder relations in set-valued optimization*, **Applied Set-Valued Analysis and Optimization**, Special Issue Dedicated to Alfred Göpfert, 1 (2019) (3), 205-219, DOI: 10.23952/asvao.1.2019.3.02	Obj. 3

The main results obtained in these eleven papers are described below.

A1. In the paper [*A corrector-predictor interior-point method with new search direction for linear optimization*, **Central European Journal of Operations Research**, First Online 15 May 2019, DOI: 10.1007/s10100-019-00622-3] **Zsolt Darvay** and **Petra Renáta Rigó,** together with Tibor Illés (Budapest University of Technology and Economics, Budapest, Hungary) and Behrouz Kheirfam (Azarbaijan Shahid Madani University,Tabriz, Iran), proposed a feasible corrector–predictor interior-point algorithm for linear optimization, which is based on a new search direction. They derived the iteration bound that matches the best iteration bounds known for these types of methods. Moreover, they proved the practical efficiency of the new algorithm by presenting numerical results.

A2. The hemivariational inequalities are generalizations of variational inequalities. They were introduced and studied first by Panagiotopoulos since the 1980s in order to model nonmonotone phenomena. For instance, hemivariational inequalities arise in the variational formulation of boundary-value problems governed by nonsmooth energy functionals.

Motivated by the studies of Panagiotopoulos, the existence of solutions for hemivariational inequalities has been investigated by many authors in recent years.

Although hemivariational inequalities with antiperiodic condition are of a great importance due to their appearance in the mathematical modeling of many physical processes, the study of this kind of problems captured less attention. Indeed, up to our knowledge, in the two papers by J.

Y. Park and T.G. Ha [*Existence of antiperiodic solutions for hemivariational inequalities*, Nonlinear Anal. 68 (2008) , 747–767] and [*Existence of anti-periodic solutions for quasilinear parabolic hemivariational inequalities*, Nonlinear Anal. 71 (2009), 3203–3217] the authors gave an attempt to study the existence of solutions for this problem by using an approach based on the Browder's surjectivity theorem on pseudomonotone perturbation of maximal monotone operators.

However, their proof seems to be not correct as it is stressed at the end of the paper by Ouayl Chadli, **Gábor Kassay,** Asma Saidi [*On the existence of antiperiodic solutions for hemivariational inequalities: an equilibrium problem approach*, under review]. Due to the previous reasons, it is proposed in this paper to study the existence of antiperiodic solutions for hemivariational inequalities associated to time-dependent pseudomonotone (respectively, quasimonotone) operator in the sense of Brézis by an approach based on the theory of equilibrium problems.

A3. In the paper [*Traffic assignment: On the interplay between optimisation and equilibrium problems*, **Optimization**, Published online 13 Jan 2020, DOI: 10.1080/02331934.2019.1711082] **Nicolae Popovici** together with Ovidiu Bagdasar and Stuart Berry (University of Derby, UK) investigate discrete and continuous optimisation and equilibrium-type problems, for a simplified traffic assignment problem on a simple network with parallel links and _fixed demand. We explore the interplay between solutions of certain optimisation and equilibrium problems which can be solved by dynamic programming. The results are supported by numerical simulations in Matlab, in which the price of anarchy is calculated to highlight the demand levels where there is a change in road choice and usage.

A4. The aim of the paper [*Characterizations of explicitly quasiconvex vector functions w.r.t. polyhedral cones*, Journal of Nonlinear and Convex Analysis, Special Issue in Memory of Professor Siegfried Schaible, 20 (2019) (12), 2653-2665] written by **Nicolae Popovici** jointly with Christian Günther (Martin Luther University, Halle-Wittenberg, Germany) is to present new characterizations of explicitly cone-quasiconvex vector functions with respect to a polyhedral cone of a finite-dimensional Euclidean space. These characterizations are given in terms of classical explicit quasiconvexity of certain real-valued functions, defined by composing the vector-valued function with appropriate scalarization functions, namely the extreme directions of the polar cone or some nonlinear scalarization functions, currently used in vector optimization.

A5. .**Nicolae Popovici** together with Christian Günther and Elisabeth Köbis (Martin Luther University, Halle-Wittenberg, Germany) propose in their paper [*Computing minimal elements of finite families of sets w.r.t. preorder relations in set optimization*, **Journal of Applied and Numerical Optimization**, Special Issue Dedicated to Boris Polyak, 1 (2019) (2), 131-144] new algorithms for computing all minimal elements of a nonempty finite family of sets in a real linear space, with respect to a preorder relation defined on the power set of that space. These algorithms are based on a set-valued counterpart of the well-known Graef-Younes reduction procedure, originally conceived for vector optimization. One of our algorithms consists of two subsequent (forward-backward) reduction procedures, similarly to the classical Jahn-Graef-Younes method. Another algorithm involves a pre-sorting procedure with respect to a strongly increasing real-valued function, followed by a single (forward) reduction procedure. Numerical experiments

in MATLAB allow us to compare our algorithms for special test families of line segments with respect to *l*-type, *u*-type and *s*-type preorder relations, currently used in set optimization.

A6. In the paper . [*The role of nonlinear scalarization functions in characterizing generalized convex vector functions*, Journal of Applied and Numerical Optimization, Special Issue Dedicated to Christiane Tammer**,** 1 (2019) (3), 325-333] **Nicolae Popovici** and . Christian Günther (Martin Luther University, Halle-Wittenberg, Germany) present new characterizations of cone-convex and explicitly cone-quasiconvex vector functions with respect to a proper closed solid convex cone of a real linear topological space. These characterizations are given in terms of classical convexity and explicit quasiconvexity of certain real-valued functions, defined by means of the nonlinear scalarization function introduced by Gerstewitz (Tammer) in the pioneering work [*Nichtkonvexe Dualitaet in der Vektoroptimierung*, Wiss. Zeitschrift Tech. Hochsch. Leuna-Merseburg 25 (1983), 357-364].

A7. Several problems in applied mathematics can be formulated via set-valued variational inequalities and many authors have contributed to the study of these problems. Most of the existence results require some monotonicity of the operator involved, together with some kinds of continuity properties. One is the notion of pseudomonotonicity introduced by Karamardian, arising from and extending the classical notion of monotonicity for operators.

A different kind of pseudomonotonicity for operators, known as Brézis pseudomonotonicity was introduced by Brézis when dealing with integral equations and partial differential equations. This notion is not directly related to any classical monotonicity property, but the operators are required to satisfy some topological conditions.

The purpose of the paper [*Regularization of Brézis pseudomonotone variational inequalities*, under review], written by **Gábor Kassay** together with Monica Bianchi (Universita Cattolica del Sacro Cuore di Milano, Italy) and Rita Pini (Universita degli Studi di Milano Bicocca, Italy), is to prove the solvability of set-valued variational inequalities by approximating the operator and the feasible set with a sequence of more regular operators and feasible sets. The authors assume that the sequence of these more regular operators converges to the initial operator by means of the Hausdorff distance between the values of these operators, while the sequence of regular feasible sets converges in Mosco sense to the initial feasible set. It is hown that under some suitable conditions, the sequence of solutions of the regularized problems converges weakly to a solution of the initial problem, while under some additional hypotheses, the convergence is strong.

A8. The Gauss map of a closed convex subset C of R^n, as defined by Laetsch, generalizes the Gauss map of an orientable regular hypersurface of R^n. While the shape of such a regular hypersurface is well encoded by the Gauss map, the range of this map, equally called the *spherical image* of the hypersurface, is used to study various aspects of the smooth convex hypersurfaces both in the finite dimensional setting and in the infinite dimensional setting as well. In this paper, we rely on the potential of the Gauss map of an unbounded closed convex set to characterize the unbounded exposed faces through the boundary points of its Gauss range that lie in the range. For unbounded closed convex sets containing no lines and with nonempty interior one can therefore evaluate how much is infinity accessible through proper exposed faces using the boundary points

of its Gauss range that lie in the range. At one extreme we have those sets for which infinity is accessible through many unbounded proper faces. An important class of such unbounded closed convex sets are the Motzkin decomposable ones, and their Gauss ranges happens to be closed, i.e. all boundary points of the Gauss range are contained in the range. The other extreme is represented by the unbounded closed convex sets with compact proper faces, as no boundary point of the Gauss range lie in the range for such a set. For them the infinity is obviously not accessible through any of their proper faces. The Gauss ranges of such unbounded closed convex sets with nonempty interior happens to be open and the sets themselves are Minkowski sets.

The main purpose of the paper [*Closed convex sets with an open or closed Gauss range*, under review], written by **Cornel Pintea** jointly with Juan Enrique Martínez-Legaz (Universitat Autònoma de Barcelona, Spain) is to characterize the closed convex subsets of R^n that have an open or a closed Gauss range. We will emphasize the case of epigraphs of lower semicontinuous proper convex functions. While the characterizations of general closed convex subsets of R^n with open or closed Gauss ranges have some geometric flavor, in the particular case of epigraphs they become more abstract.

A9. In the paper [*A new wide neighborhood primal-dual second-order corrector algorithm for linear optimization*, **Optimization Letters**, First Online 24 August 2019, DOI: 10.1007/s11590-019-01468-z] **Zsolt Darvay** and **Petra Renáta Rigó,** together with Behrouz Kheirfam (Azarbaijan Shahid Madani University,Tabriz, Iran), introduced a new large-step primal-dual second-order corrector interior-point method for linear optimization. At each iteration, the method uses the new wide neighborhood introduced by Darvay and Takács [*Large-step interior-point algorithm for linear optimization based on a new wide neighbourhood*, Central European Journal of Operations Research, 26 (2018) (3), 551–563]. In this paper the authors improved the directions proposed by Zsolt Darvay and Petra Renáta Takács (Rigó) by adding a second-order corrector direction. To the best knowledge of the authors, this is the first primal-dual second-order corrector interior-point algorithm based on Darvay–Takács's new wide neighborhood, which has the same complexity as the best short-step algorithms for linear optimization. Furthermore, the authors provided numerical results in order to show the efficiency of the algorithm.

A10. In the paper [*Infeasible interior-point algorithm for linear optimization based on a new search direction*. In L. Zadnik Stirn, M. Kljajic Borstnar, J. Zerovnic, S. Drobne, J. Povh (eds.), Proceedings of the 15th International Symposium on Operational Research SOR'19, Bled, Slovenia, September 25-27, pp. 475-480, 2019] **Zsolt Darvay** and **Petra Renáta Rigó**, jointly with Eszter Szénási (Budapest University of Technology and Economics, Budapest, Hungary) study the technique of algebraic equivalent transformation of the central path in order to obtain new search directions of interior-point algorithms for solving linear optimization problems. They consider for the first time a new function in order to use it in the process of the algebraic equivalent transformation. They deal with the infeasible starting points as well. The new method consists of a feasibility and a centering step. Both are obtained using the new direction.

A11. *Bregman functions* have been used to define the so called *Bregman distance* (not a distance in the usual sense of the term: in general, it is not symmetric and does not satisfy the triangular inequality), which turned to be very useful in different proximal and projection methods for solving minimization problems, variational inequalities, equilibrium problems, etc.

Several problems in applied mathematics can be formulated via set-valued variational inequalities and many authors have contributed to the study of this problem.

In the paper [*Bregman Type Regularization of Variational Inequalities with Mosco Approximation of the Constraint Set*, under review] the **project leader** together with Suliman Al-Homidan (King Fahd University of Petroleum and Minerals, Dhahran, Saudi Arabia) and Qamrul Hasan Ansari (Aligarh Muslim University, Aligarh, India) considered a set-valued variational inequality governed by a *premonotone operator*, a notion introduced previously by Iusem, Kassay and Sosa (2009) which is more general than the classical monotonicity of Minty and Browder. The idea is to approximate the initial problem by a sequence of regularized variational inequalities defined by means of Bregman functions and to prove the existence of their solutions by using equilibrium problem techniques.

The authors of this paper established sufficient conditions which guarantee the existence of strong solutions of the regularized problems, they showed that the sequence of strong solutions of these problems admits weak cluster points, and each weak cluster point of this sequence is a strong solution of the initial variational inequality. Under some additional assumptions, they proved that the sequence of strong solutions of the regularized problems converges strongly to a minimal norm solution of the initial variational inequality.

A12. By continuing the research initiated in the article [A5: *Computing minimal elements of finite families of sets w.r.t. preorder relations in set optimization*, Journal of Applied and Numerical Optimization, Special Issue Dedicated to Boris Polyak, 1 (2019) (2), 131-144], **Nicolae Popovici** jointly with Christian Günther and Elisabeth Köbis (Universitatea Martin Luther, Halle-Wittenberg, Germania) have developed in the new paper [*On strictly minimal elements w.r.t. preorder relations in set-valued optimization*, Applied Set-Valued Analysis and Optimization, Special Issue dedicated to the 85th birthday of Alfred Göpfert, 1 (2019) (3), 205-219] two algorithms for computing all strictly minimal elements of a nonempty finite family of subets of a real linear space with respect to a preorder relation. In particular, by considering certain families of rectangles in the Euclidean plane and the *l*-type and *u*-type preorder relations in the sense of Kuroiwa, the authors have implemented the proposed algorithms in MATLAB.

(b) Habilitation Thesis:

Cornel Pintea: *The size of critical and tangency sets*, Habilitation Thesis, Babeş-Bolyai University, Faculty of Mathematics and Computer Science, Cluj-Napoca, Romania, 2019 [172 pages].

IV. Dissemination of research results

The scientific results mentioned within Section III of this report have been presented by the authors (members of the project research team) at **32** conferences, workshops and research seminars in Romania or abroad, namely:

- **14 conferences and workshops abroad [**CS2 - CS5, CS15, CS16, CS18, CS19, CS23- CS27 and CS29 in the next table, among which **3 plenary/invited talks];**
- **5 research seminars abroad [**CS6 - CS10, in the next table**];**

- **3 conferences in Romania [**CS14, CS21 and CS22 in the next table**];**
- **10 research seminars in Romania [**CS1, CS11-CS13, CS17, CS20, CS28 and CS30-CS32, in the next table**].**

Ref.	Conference/workshop /research seminar
CS1	**Gábor Kassay**: *Takahashi's minimization theorem and some related results in quasi-metric spaces*, Seminar of the Research Group on Analysis and Optimization, Babeş-Bolyai University, Cluj-Napoca, Romania, December 6, 2018, www.cs.ubbcluj.ro/~grupanopt/
CS2	**Zsolt Darvay**, Tibor Illés, Behrouz Kheirfam, **Petra Renáta Rigó**: *A new predictor-corrector interior-point algorithm for linear optimization problem*, VOCAL Optimization Conference: Advanced Algorithms, Esztergom, Hungary, December 10-12, 2018, http://vocal.p-graph.org/
CS3	**Petra Renáta Rigó**: *New trends in interior-point algorithms*, VOCAL Optimization Conference: Advanced Algorithms, Esztergom, Hungary, December 10-12, 2018, http://vocal.p-graph.org/
CS4	**Zsolt Darvay**: *A new adaptive predictor-corrector interior-point algorithm for sufficient linear complementarity problems*, VOCAL Optimization Conference: Advanced Algorithms, Esztergom, Hungary, December 10-12, 2018, http://vocal.p-graph.org/
CS5	**Szilárd Csaba László**: *A second order dynamical approach with variable coefficients to nonconvex smooth minimization*, Modern Maximal Monotone Operator Theory: From Nonsmooth Optimization to Differential Inclusions – Workshop on Nonsmooth and Variational Analysis, January 28-February 1, 2019, Vienna, Austria, https://www.esi.ac.at/events/e29/
CS6	**Nicolae Popovici**: *A systematization of convexity and quasiconvexity concepts for set-valued maps, defined by l-type and u-type preorder relations*, Bocconi University, Milan, Italy, Department of Decision Sciences, Research Seminar, February 8, 2019
CS7	**Nicolae Popovici**: *A general local-global extremality principle in vector optimization,* University of Insubria, Varese, Italy, Department of Economics, Research Seminar, February 13, 2019, www.eco.uninsubria.it/site/dipartimento/ricerca/seminari/
CS8	**Nicolae Popovici**: *New algorithms for solving discrete vector optimization problems,* University of Insubria, Varese, Italy, Department of Economics, Research Seminar, February 14, 2019, www.eco.uninsubria.it/site/dipartimento/ricerca/seminari/
CS9	**Nicolae Popovici**: *New algorithm for solving planar multiobjective location problems involving the Manhattan norm,* University of Insubria, Varese, Italy, Department of Economics, Research Seminar, February 15, 2019, www.eco.uninsubria.it/site/dipartimento/ricerca/seminari/
CS10	**Zsolt Darvay**: *A long-step feasible predictor-corrector interior-point algorithm for symmetric cone optimization*, Research Seminar, Department of Differential Equations, Budapest University of Technology and Economics, Budapest, Hungary, February 28, 2019
CS11	**Szilárd Csaba László**: *On a class of second order dynamical systems with variable*

	coefficients associated to a nonconvex minimization, Seminar of the Research Group on Analysis and Optimization, Babeş-Bolyai University, Cluj-Napoca, Romania, March 21, 2019, www.cs.ubbcluj.ro/~grupanopt/
CS12	**Cornel Pintea**: *Convex decompositions of some nonconvex open sets*, Seminar of the Research Group on Analysis and Optimization, Babeş-Bolyai University, Cluj-Napoca, Romania, March 28, 2019, www.cs.ubbcluj.ro/~grupanopt/
CS13	**Gábor Kassay**: *On sensitivity of vector equilibria by means of the diagonal subdifferential operator*, Seminar of the Research Group on Analysis and Optimization, Babeş-Bolyai University, Cluj-Napoca, Romania, April 4, 2019, www.cs.ubbcluj.ro/~grupanopt/
CS14	**Szilárd Csaba László**: *A primal-dual dynamical approach to a nonsmooth convex minimization*, Games, Dynamics and Optimization 2019 COST Action CA16228 "European Network for Game Theory", April 9 -11, 2019, Cluj-Napoca, Romania, http://gdo2019.com/
CS15	**Gábor Kassay** (**plenary speaker**): *Vectorial form of Ekeland's variational principle with applications to vector equilibrium problems*, I Conference on Minimax Inequalities and Equilibrium Poblems, University of Granada, Spain, 6-7 May 2019, https://congresos.ugr.es/minequmax/program
CS16	**Cornel Pintea**: *A nonlinear elliptic eigenvalue-transmission problem with Neumann boundary condition*, I Conference on Minimax Inequalities and Equilibrium Poblems, University of Granada, Spain, 6-7 May 2019, https://congresos.ugr.es/minequmax/program
CS17	**Szilárd Csaba László**: *On quasilinear operators*, Seminar of the Research Group on Analysis and Optimization, Babeş-Bolyai University, Cluj-Napoca, Romania, May 16, 2019, www.cs.ubbcluj.ro/~grupanopt/
CS18	Eszter Szénási, **Zsolt Darvay**, **Petra Renáta Rigó**: *Interior-point algorithm for linear optimization based on a new search direction*, XXXIII. Hungarian Operations Research Conference, Szeged, Hungary, May 20-22, 2019, www.mot.org.hu/mok2019
CS19	Tibor Illés, **Zsolt Darvay**, **Petra Renáta Rigó**, Janez Povh: *Interior-point algorithms for linear complementarity problems with transformed central path*, XXXIII. Hungarian Operations Research Conference, Szeged, Hungary, May 20-22, 2019, www.mot.org.hu/mok2019
CS20	**Gábor Kassay**: *Vectorial form of Ekeland's Variational Principle with applications to vector equilibrium problems*, Seminar of the Research Group on Analysis and Optimization, Babeş-Bolyai University, Cluj-Napoca, Romania, May 23, 2019, www.cs.ubbcluj.ro/~grupanopt/
CS21	**Anca Grad**: *Inertial algorithms for monotone inclusion problems*, 16th International Conference on Applied Mathematics and Computer Science, Technical University of Cluj-Napoca, Romania, July 3-6, 2019, http://tucn.ro/angheluta2019/
CS22	**Livia Mihaela Bercheşan**: *Cyclically antimonotone vector equilibrium problems*, 16th International Conference on Applied Mathematics and Computer Science, Technical University of Cluj-Napoca, Romania, July 3-6, 2019, http://tucn.ro/angheluta2019/

CS23	**Gábor Kassay**: *Vectorial form of Ekeland Variational Principle with applications to vector equilibrium problems*, Positivity X, Pretoria, South Africa, July 8-12, 2019, http://positivitymathematics.org/
CS24	**Zsolt Darvay**, Tibor Illés, Janez Povh, **Petra Renáta Rigó**: *Predictor-corrector interior-point algorithm for sufficient linear complementarity problems based on a new search direction*, Sixth International Conference on Continuous Optimization, Berlin, Germany, August 5-8, 2019, https://iccopt2019.berlin/
CS25	**Gábor Kassay** (**invited speaker**): *The Ekeland's Variational Principle in optimization and equilibria*, Veszprém Optimization Workshop dedicated to Béla Vizvári on the occasion of his 70th birthday, Veszprém, Hungary, August 26 – 27, 2019, http://vow2019.mik.uni-pannon.hu/.
CS26	**Zsolt Darvay** (**invited speaker**), Gizella Noémi Márk: *New interior-point algorithm for sufficient linear complementarity problem*, Veszprém Optimization Workshop dedicated to Béla Vizvári on the occasion of his 70th birthday, Veszprém, Hungary, August 26 – 27, 2019, http://vow2019.mik.uni-pannon.hu/
CS27	**Zsolt Darvay**, **Petra Renáta Rigó**: *Infeasible interior-point algorithm for linear optimization based on a new search direction*, The 15th International Symposium on Operational Research SOR'19, Bled, Slovenia, September 25-27, 2019, http://sor19.fov.uni-mb.si/
CS28	**Nicolae Popovici**: *New algorithms for computing the minimal elements of finite families of sets w.r.t. preorder relations*, Seminar of the Research Group on Analysis and Optimization, Babeş-Bolyai University, Cluj-Napoca, Romania, October 17, 2019, http://www.cs.ubbcluj.ro/~grupanopt/
CS29	**Nicolae Popovici**: *Local-global extremality properties in vector optimization*, Colloquium Vector- and Set-Valued Optimization, Wittenberg, Germany, October 25-26, 2019
CS30	**Cornel Pintea**: *The convex decomposition number and the valence of some functions*, Seminar of the Research Group on Analysis and Optimization, Babeş-Bolyai University, Cluj-Napoca, Romania, November 21, 2019, http://www.cs.ubbcluj.ro/~grupanopt/
CS31	**Anca Grad**: *Relaxed inertial algorithms for monotone inclusion problems*, Seminar of the Research Group on Analysis and Optimization, Babeş-Bolyai University, Cluj-Napoca, Romania, November 28, 2019, http://www.cs.ubbcluj.ro/~grupanopt/
CS32	**Szilárd Csaba László**: *Possible convergence rates of order $O(1/n^2)$ for some inertial algorithms obtained via the explicit Euler method applied to a perturbed version of the second order dynamical system that models Nesterov's accelerated convex gradient method*, Seminar of the Research Group on Analysis and Optimization, Babeş-Bolyai University, Cluj-Napoca, Romania, December 5, 2019, http://www.cs.ubbcluj.ro/~grupanopt/

Project leader,
Prof. Gábor Kassay, Ph.D.