# SCIENTIFIC RESEARCH REPORT No. 3
## covering the period of 16.12.2012 – 15.12.2013

## I. Research Team

Prof. Dr. **Gabor KASSAY** (project leader),
Assoc. Prof. Dr. **Nicolae POPOVICI**,
Assoc. Prof. Dr. **Cornel PINTEA**,
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Dr. **Erika NAGY**,
PhD Student **Mihaela MIHOLCA** (**BERCHESAN**).

## II. Project summary and objectives

**(a)** The project is mainly motivated by the growing literature in scalar and vector optimization problems, variational inequalities, and equilibrium problems, which neatly shows that these fields are appropriate for applying the modern tools of variational analysis. The following objectives have been proposed in the funding application:

**O1** *Studying condition numbers and metric regularity within parametric variational inequalities and parametric equilibrium problems*
**O2** *Identifying classes of generalized monotone operators for which local and global monotonicity are equivalent and deduce injectivity results*
**O3** *Studying the structure of the solution sets for generalized monotone operators*
**O4** *Characterizing the subdifferential for certain classes of generalized monotone operators*
**O5** *Approaching the sum problem for maximal monotone operators*
**O6** *Constructing algorithms for variational inequalities and equilibrium problems*
**O7** *Extending the proximal point algorithm for equilibrium problems to reflexive Banach spaces*
**O8** *Characterizing generalized convex vector functions by scalarization*
**O9** *Studying the structure of the solution sets of vector variational inequalities and equilibrium problems*

**(b)** All objectives planned for the period 16.12.2012 – 15.12.2013 have been achieved as follows:

    **O1**: 2 papers in preparation**
    **O2**: 1 published paper [A1 in Section III.(a)] and 1 paper under review
    **O3**: 3 published papers [A4, A5 and A6  in Section III.(a)]
    **O5**: 1 published paper [A3 in Section III.(a)]
    **O6**: 2 published papers [A2 and A4 in Section III.(a)], 1 PhD Thesis* and 2 papers in preparation**
    **O8**: 2 papers in preparation**
    **O9**: 1 published paper [A7 in Section III.(a)] and 2 papers in preparation**

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*see Section III.(b)
**see Section III.(c)

## III.  Scientific results

### (a)  Published/submitted papers

    **8 papers** have been **completed** during the period of 16.12.2012 – 15.12.2013:
- **6** articles **appeared in ISI journals** (A1-A5 and A7 in the next table);
- **1** article **appeared in a journal indexed in international databases** (A6 in the next table);
- **1** paper has been **submitted to an ISI journal** (A8 in the next table).

| Ref. | Article | Objectives |
|------|---------|------------|
| A1 | **Cornel Pintea**: *Global injectivity conditions for planar maps*, **Monatsh. Math.**, 172 (2013), 399-413, doi:10.1007/s00605-012-0474-x [JCR Science Edition 2012 IF: 0.698, JCR Science Edition 2013 IF: 0.638] | O2 |
| A2 | **Mihaela Miholca** (**Berchesan**): *Vector variational-like inequalities and vector optimization problems,* **Carpathian J. Math.**, 30 (2014) (1), 97-104 [JCR Science Edition 2012 IF: 0.852, JCR Science Edition 2013 IF: 0.642] | O6 |
| A3 | Radu Ioan Bot, Erno Robert Csetnek and **Erika Nagy**: *Solving systems of monotone inclusions via primal-dual splitting techniques*, **Taiwanese J. Math.**, 17 (2013) (6), 1983-2009, doi:10.11650/tjm.17.2013.3087 [JCR Science Edition 2012 IF: 0.67, JCR Science Edition 2013 IF: 0.658] | O5 |
| A4 | **Gabor Kassay** and **Mihaela Miholca** (**Berchesan**): *Existence results for variational inequalities with surjectivity consequences related to generalized monotone operators*, **J. Optim. Theory Appl.**, 159 (2013) (3), 721–740, doi:10.1007/s10957-013-0383-8 [JCR Science Edition 2012 IF: 1.423, JCR Science Edition 2013 IF: 1.406] | O3 and O6 |
| A5 | Dorin Andrica, Dana Mangra and **Cornel Pintea**: *The circular Morse-Smale characteristic of closed surfaces*, **Bull. Math. Soc. Sci. Math. Roumanie**, Tome 57(105) (2014) (3), 235–242 [JCR Science Edition 2012 IF: 0.419, JCR Science Edition 2013 IF: 0.452] | O3 |
| A6 | Dorin Andrica, Dana Mangra and **Cornel Pintea**: *The minimum number of critical points of circular Morse functions*, **Stud. Univ. Babes-Bolyai Math.,** 58 (2013 (4), 485-495 | O3 |

| A7 | Shaghaf Alzorba, Christian Guenther and **Nicolae Popovici**: *A special class of extended multicriteria location problems*, **Optimization**, doi:10.1080/02331934.2013.869810 [JCR Science Edition 2012 IF: 0.707, JCR Science Edition 2013 IF: 0.771] | O9 |
|---|---|---|
| A8 | **Szilard Laszlo** and Adrian Viorel: *Generalized monotone operators on dense set*, under review | O2 |

The main results obtained in these eight papers are mentioned below.

- In his paper **A1** [*Global injectivity conditions for planar maps*, **Monatsh. Math.**, 172 (2013), 399-413], **C. Pintea** defines the planar maps and produces some quite rich classes of such maps. Afterwards the author proves several global injectivity results for planar local diffeomorphisms based upon the spectra of their Frechet differentials and the spectra of the composed Frechet differential at a certain arbitrary point with the inverse of the Frechet differential at another arbitrary point. In the case of the local diffeomorphisms of the plane defined on its convex open subsets, these results are given in terms of the coefficients of the characteristic polynomials associated to their Frechet differentials of the involved local diffeomorphism.

- **M. Miholca** (**Berchesan**) introduced in paper **A2** [*Vector variational-like inequalities and vector optimization problems*, **Carpathian J. Math.**, 30 (2014) (1), 97-104] several generalized convexity concepts and studied the relationship between them. It is also shown, by means of examples, that the convexity notions introduced here are different from those considered within the literature. In addition, the paper concerns the relationship between the solution sets of certain variational inequalities and a vector optimization problem. These results are obtained by using a generalized subdifferential, namely the Mordukhovich limiting subdifferential, which is known from the literature.

- **E. Nagy** jointly with R. I. Bot and E. R. Csetnek (Technical University of Chemnitz, Germany), considered in paper **A3** [*Solving systems of monotone inclusions via primal-dual splitting techniques*, **Taiwanese J. Math.**, 17 (2013) (6), 1983-2009] a system of monotone inclusions whose coupling operator satisfies a Lipschitz continuity property, along with its dual system of monotone inclusions in an extended sense of the Attouch-Thera duality. It is shown that in order to solve simultaneously the primal and the dual system of monotone inclusions it suffices to solve the problem of finding the zeros of the sum of a maximally monotone operator and a monotone and Lipschitz continuous operator in an appropriate product space. The latter problem is solved by a forward-backward-forward algorithm, fact that allows the authors to provide for the resulting iterative scheme (which proves to have a high parallelizable formulation), both weak and strong convergence assertions. The numerical performances of the proposed splitting algorithm are emphasized through applications in average consensus on colored networks and image classification via support vector machines.

- The concept of monotone operator in the sense of Minty and Browder has been extended in the last decades in several directions. In this way a new topic within the field of nonlinear analysis was born: the theory of generalized monotone operators. These, beside their theoretical interest, shown to be very useful in practice, since a lot of real life problems could better (and more precisely) be modeled. Taking into consideration this fact, the surjectivity results concerning these types of operators become important as they guarantee the existence of solutions for equations defined by means of them. In the paper **A4** [*Existence results for variational inequalities with surjectivity consequences related to generalized monotone operators*, **J. Optim. Theory Appl.**, 159 (2013) (3), 721–740] **the project leader** jointly with **M. Miholca** (**Berchesan**) obtained new

surjectivity results and the existence of zeroes for set-valued monotone mappings in the generalized sense. These results have been proved either using known facts, or establishing some new results concerning equilibrium problems and variational inequalities. In two cases the authors work with generalized monotonicities, but beside, they also obtained a result without any hypothesis of monotonicity. The latter was achieved by strengthening the continuity upon the operator they considered.

- In the paper **A5** [*The circular Morse-Smale characteristic of closed surfaces*, **Bull. Math. Soc. Sci. Math. Roumanie**, Tome 57(105) (2014) (3), 235–242], **C. Pintea** determines, jointly with D. Andrica and D. Mangra (Babes Bolyai University of Cluj-Napoca, Romania), the minimum number of critical points of the circular Morse functions defined on closed surfaces. The authors provide an upper bound for the minimum number of characteristic points of a compact orientable surface relative to the horizontal distribution of the first Heisenberg group, with respect to all embeddings of the considered surface in the first Heisenberg group.

- In the paper **A6** [*The minimum number of critical points of circular Morse functions*, **Stud. Univ. Babes-Bolyai Math.,** 58 (2013 (4), 485-495] **C. Pintea** provides, jointly with D. Andrica and D. Mangra (Babes-Bolyai University of Cluj-Napoca, Romania), an elementary proof for the minimum number of critical points of the circular Morse functions defined on closed surfaces. The authors also provide the details on the lower and the upper bounds for the minimum number of characteristics points of a compact orientable surface relative to the horizontal distribution of the first Heisenberg group, with respect to all embeddings of the considered surface in the first Heisenberg group.

- **N. Popovici** together with S. Alzorba and Chr. Guenther (Martin Luther University of Halle-Wittenberg, Germany) considered in their paper **A7** [*A special class of extended multicriteria location problems*, **Optimization**, doi:10.1080/02331934.2013.869810] a special class of multicriteria optimization problems, obtained from a planar location problem by adding a new objective function. The location problem is defined by means of the uniform norm, hence it is Pareto reducibie. More precisely, each weakly efficient solution is an efficient solution of a bicriteria subproblem. By using this property, the authors develop a new algorithm for solving the extended location problem, which is implemented in MATLAB.

- In the paper **A8** [*Generalized monotone operators on dense sets*, under review], **Sz. Laszlo** with Adrian Viorel (Technical University of Cluj-Napoca, Romania) show that the lower semicontinuous multivalued operators which are locally strict quasimonotone, locally pseudomonotone, respectively locally strict pseudomonotone on a self segment-dense subset of their domain, possesses the appropriate generalized monotonicity globally. As applications the authors obtain results that provide a global generalized convexity for functions which are locally generalized convex on a self segment-dense subset of their domain


**(b)  Ph.D. Thesis** related to the project objective **O6**:

**Erika Nagy**: *Numerical methods for approximating zeros of operators and for solving variational inequalities with applications* (Ph.D. Supervisor: Professor Gabor Kassay), Faculty of Mathematics and Computer Science, Babes-Bolyai University, Cluj-Napoca, Romania, Ph.D. Dissertation defended on September 26, 2013, Summary available at the internet address http://www.cs.ubbcluj.ro/~grupanopt/PN-II-ID-PCE-2011-3-0024/thesis-summary-erika-nagy.pdf

**(c)  Work in progress**

Preliminary results have been obtained in the period of 16.12.2012 – 15.12.2013, namely:

- **N. Popovici** with M. Rocca (University of Insubria, Varese, Italy) studied the scalarization and reducibility of vector equilibrium problems (within objective **O9**)

- **G. Kassay** with M. Castellani and M. Giuli (University of L'Aquila, Italy) studied the stability of the solutions of parametric equilibrium problems (within objective **O6**);

- **G. Kassay** with M. Bianchi (University "Cattolica del Sacro Cuore" of Milan, Italy) and R. Pini (University Bicocca of Milan, Italy)have continued their investigations on the sensitivity of the sum of set-valued maps (within objective **O1**);

- **G. Kassay** with B. S. Mordukhovich (Wayne State University, Detroit, USA, who visited the project team in Cluj-Napoca, during the period of June 8-13, 2013) studied some problems related to metric regularity (within objective **O1**);

- **N. Popovici** with M. Rocca (University of Insubria, Varese, Italy) and D. Kuroiwa (Shimane University, Matsue, Japan) obtained a new characterization of convex vector-valued functions by means of linear perturbations (within objective **O8**);

- **Sz. Laszlo** obtained sufficient conditions for the global injectivity of certain operators under generalized monotonicity assumptions (within objectives **O6** and **O9**);

- **N. Popovici** with O. Bagdasar (University of Derby, UK) studied extremal properties of explicitly quasiconvex functions(within objective **O8**).


## IV.  Dissemination of research results

The scientific results mentioned within Section III of this report have been presented by the authors (members of the project research team) at **26** conferences, workshops and research seminars in Romania or abroad, namely:

- **10 international conferences and workshops;**

- **5 research seminars abroad;**

- **11 research seminars in Romania.**

The list of talks is available on the project webpage at

**http://www.cs.ubbcluj.ro/~grupanopt/PN-II-ID-PCE-2011-3-0024/index_eng.htm**


Proiect leader,
Prof. Dr. Gabor Kassay