

# On Vector Equilibrium Problems Given by a Sum of Two Functions

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## Abstract

The equilibrium problem attracted much attention in recent years, since apart from its theoretical interest includes, as particular cases, scalar and vector optimization problems, saddlepoint (minimax) problems, variational inequalities, Nash equilibria problems, complementarity problems, fixed point problems, etc.

However, a seemingly interesting special case, where the function involved is represented as a sum of two bifunctions captured less attention, although it was investigated already by Blum and Oettli in 1994; when one of the bifunctions vanishes, their result becomes a variant of Ky Fan's theorem (1971), whereas when the second does, one obtains a variant of the Browder-Minty theorem for variational inequalities.

Following the idea and the steps of the proofs given by Blum and Oettli, Kazmi (2000) obtained an existence result for this type of problem (i. e., sum of two bifunctions) in the vector case, but his assumptions on the vector-valued functions turn to be too strong in order to recover the result of Blum and Oettli.

The aim of this paper is to weaken the assumptions of Kazmi in a way to be able to recover Blum-Oettli's results on one hand, and by assuming alternative conditions on the vector functions, to deduce new existence theorems, on the other hand. The special case of reflexive Banach spaces endowed with the weak topology is separately treated; in that case mild sufficient conditions for guaranteeing coercivity are presented.