Coincidence point results via variational inequalities

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22. 11. 2012

Generalized variational inequalities

Let X be a real Banach space, X^* its topological dual, and for $x^* \in X^*$ denote by $\langle x^*, x\rangle$ the scalar $x^*(x)$. Consider the set $K \subseteq X$ and let $A : K \longrightarrow X^*$ and $a : K \longrightarrow X$ be given operators. Recall that Stampacchia variational inequality, $VI_S(A, K)$, is:

Find an element $x \in K$, such that

$$\langle A(x), y - x\rangle \geq 0 \text{ for all } y \in K,$$

where the set K is convex and closed.

The general variational inequality of Stampacchia type, $VI_S(A, a, K)$, is:

Find an element $x \in K$, such that

$$\langle A(x), a(y) - a(x)\rangle \geq 0, \quad \text{for all } y \in K. \tag{1}$$

Obviously, when $a \equiv id_K$, then (1) reduces to Stampacchia variational inequality $VI_S(A, K)$.

Let $T : K \rightrightarrows X^*$ and $f : K \longrightarrow X$ be given operators. Consider the following problem.

Find an element $x \in K$, such that

$$(\exists) u \in T(x) : (\forall) y \in K \; \langle u, f(y) - f(x) \rangle \geq 0. \tag{2}$$

It is easy to observe, that if T is single valued, then (2) reduces to the general variational inequality of Stampacchia type, $VI_S(T, f, K)$. Let us denote by $S(T, f, K)$ the set of solutions of (2).
Let A and B be two arbitrary sets let $F : A \rightrightarrows B$, respectively $f, g : A \longrightarrow B$ be given mappings. Recall that a point $a \in A$ is a coincidence point of F and g, respectively f and g if

$$g(a) \in F(a), \text{ respectively } f(a) = g(a).$$

If $A = B$ and $g \equiv \mathrm{id}_A$ then the above definition reduces to the definition of a fixed point of F, respectively f.

Let H be a Hilbert space, $K \subseteq H$ and $f, g : K \longrightarrow H$ two given mappings, such that $f(K) \subseteq g(K)$. Assume that (1) admits a solution for the operators $A = g - f$ and $a = g$ respectively. Then obviously there exists $x_0 \in K$ such that $\langle (g - f)(x_0), g(x) - g(x_0) \rangle \geq 0$ for all $x \in K$. Since $f(K) \subseteq g(K)$ there exists $x_1 \in K$ such that $g(x_1) = f(x_0)$, hence

$$\langle g(x_0) - f(x_0), f(x_0) - g(x_0) \rangle \geq 0, \text{ consequently } f(x_0) = g(x_0).$$

Let $F : K \rightrightarrows H$ a setvalued mapping such that $R(F) \subseteq f(K)$, where $R(F) = \cup_{x \in K} F(x)$ is the range of the operator F. Assume that (2) admits a solution for the operators $T = f - F$ and f respectively. Then obviously there exists $x_0 \in S(T, f, K)$. Hence there exists $u_0 \in T(x_0)$ such that $\langle u_0, f(x) - f(x_0) \rangle \geq 0$ for all $x \in K$ or equivalently, there exists $v_0 \in F(x_0)$ such that $\langle f(x_0) - v_0, f(x) - f(x_0) \rangle \geq 0$ for all $x \in K$. Since $R(F) \subseteq f(K)$ there exists $x_1 \in K$ such that $f(x_1) = v_0$, hence

$$\langle f(x_0) - v_0, v_0 - f(x_0) \rangle \geq 0, \text{ consequently } f(x_0) \in F(x_0).$$

Operators of type ql

Let X be a real linear space. For $x, y \in X$ let us denote by $[x, y]$ the closed line segment with the endpoints x respectively y, $[x, y]$ being the set $\{z = (1-t)x + ty : t \in [0,1]\}$. The open line segment with the endpoints x respectively y is defined as
$(x, y) := [x, y] \setminus \{x, y\} = \{z = (1-t)x + ty : t \in (0,1)\}$.

Definition (L.)

Let X and Y be two real linear spaces. One says that the operator $A : D \subseteq X \longrightarrow Y$ is of type ql, if for every $x, y \in D$ and every $z \in [x, y] \cap D$ one has $A(z) \in [A(x), A(y)]$.

Proposition (L.)

Let $f : I \subseteq \mathbb{R} \longrightarrow \mathbb{R}$ be a function. Then f is of type ql, if and only if f is monotone (increasing or decreasing).

Proposition (L.)

Let X and Y be two real linear spaces, let $D \subseteq X$ be convex and let $A : D \longrightarrow Y$ be an affine operator. Then A is of type ql.

Proposition (L.)

Let X and Y be two real linear spaces and let $A : D \subseteq X \longrightarrow Y$ be an operator of type ql. Then $\lambda A : D \to Y$ is of type ql for all $\lambda \in \mathbb{R}$.

Proposition (L.)

Let X be a real linear space, let $D \subseteq X$ convex and let $f : D \longrightarrow \mathbb{R}$ be a function. Then f is of type ql if and only if f is quasilinear, i.e. for every $x, y \in D$ and $t \in [0,1]$ we have

$$\min\{f(x), f(y)\} \leq f((1-t)x + ty) \leq \max\{f(x), f(y)\}.$$

Proposition (L.)

Let X, Y, Z be real linear spaces, $D \subseteq X$, $C \subseteq Y$ and let $A : D \longrightarrow Y$, $B : C \longrightarrow Z$, $A(D) \subseteq C$ be two operator of type ql. Then $B \circ A : D \longrightarrow Z$ is of type ql.

At this point is time to give some examples of operators of type ql, that are nontrivial in the sense that are neither linear operators nor quasilinear functions. The first one provides an operator of type ql in finite dimension.

Example

Let us consider the operator $A : [-1,1] \times [-1,1] \longrightarrow \mathbb{R}^3$,

$$A(x,y) = \left(\frac{2x+2y}{(x+y-1)^2+3}, \frac{(x+y)^2+4}{(x+y-1)^2+3}, \frac{(x+y-2)^2}{(x+y-1)^2+3} \right).$$

Then A is a continuous operator of type ql.

Indeed, it can be easily verified that $A = P \circ Q \circ S$, where
$S : [-1,1]^2 \longrightarrow [-1,1]$, $S(x,y) = \dfrac{x+y}{2}$,
$Q : [-1,1] \longrightarrow [-1,1]$, $Q(x) = \dfrac{2x}{x^2+1}$,
$P : [-1,1] \longrightarrow \mathbb{R}^3$, $P(x) = \left(\dfrac{x}{2-x}, \dfrac{2}{2-x}, \dfrac{2-2x}{2-x}\right)$. Since S is linear, according to the previous Proposition, is of type ql. It is obvious that S is continuous as well. It is easy to check that Q is a monotone increasing function, hence is of type ql. Obviously Q is continuous as well. The fact that P is continuous and of type ql can be verified directly. The conclusion follows from the previous Proposition.
The next example provides an operator of type ql in a general infinite dimensional setting.

Example

Let $D = \{f \in C_{[a,b]} | f(a) \geq 0\} \subseteq C_{[a,b]}$ and consider the operator $S : D \longrightarrow \mathbb{R}^{\mathbb{R}}$, $S(f)(x) = (f(a))^2 x$. Then S is a nonlinear operator of type ql.

Theorem (L.)

Let X and Y be two real linear spaces, let $D \subseteq X$ be convex and let $A : D \longrightarrow Y$ be an operator of type ql. Then for every finite number of elements $x_1, x_2, \ldots, x_n \in D$ and for every $x \in \mathrm{co}\{x_1, x_2, \ldots, x_n\}$ we have $A(x) \in \mathrm{co}\{A(x_1), A(x_2), \ldots, A(x_n)\}$.

Definition (A. Amini-Harandi-L.)

Let X and Y be two real linear spaces and let $D \subseteq X$ be a convex set. An operator $B : D \longrightarrow Y$ is said to be of type g-ql if, for every $n \in \mathbb{N}$ and every $x_1, \ldots, x_n \in D$, there exist $y_1, \ldots, y_n \in D$, not necessarily all different, such that for any subset $\{y_{i_1}, \ldots, y_{i_k}\} \subseteq \{y_1, \ldots, y_n\}$, $1 \leq k \leq n$, we have

$$B(\mathrm{co}\{y_{i_1}, \ldots, y_{i_k}\}) \subseteq \mathrm{co}\{B(x_{i_1}), \ldots, B(x_{i_k})\}.$$

Theorem (A. Amini-Harandi-L.)

Let X and Y be two real linear spaces and let $D \subseteq X$ be a convex set. Let $B : D \longrightarrow Y$ be an operator, and assume that there exists a convex subset $D_1 \subseteq D$, such that the restriction of B on D_1, $B|_{D_1} : D_1 \longrightarrow Y$ is of type ql, and $B(D_1) = B(D)$. Then B is of type g-ql.

Next we provide an example of operators of type g-ql which are not of type ql.

Example

Let $X = \mathbb{R}$ and consider the functions $f, g : \mathbb{R} \longrightarrow \mathbb{R}$, $f(x) = x^2$ and $g(x) = \begin{cases} 0, & \text{if } x \neq 0, \\ 1, & \text{if } x = 0. \end{cases}$ Since f and g are not monotone f and g are not of the type ql. On the other hand f and g are of type g-ql (let $\widetilde{I} = [0, \infty)$ and apply the previous Theorem.

The existence of the solutions

In what follows let X be a real Banach space, let X^* be its topological dual, let $K \subseteq X$ be a convex set and let $A : K \longrightarrow X^*$ and $a : K \longrightarrow X$ be two operators. Recall that an operator $T : X \longrightarrow X^*$ is called weak to $\|\cdot\|$-sequentially continuous at $x \in X$, if for every sequence x_n that converge weakly to x we have that $T(x_n) \longrightarrow T(x)$ in the topology of the norm of X^*. If the range of T is a subset of X, we say that T is weak to weak-sequentially continuous at $x \in X$, if for every sequence x_n that converge weakly to x we have that $T(x_n)$ converge weakly to $T(x)$. One of the main results of this section is the following theorem.

Theorem (L.)

If A is weak to $\|\cdot\|$-sequentially continuous, a is of type ql and weak to weak-sequentially continuous and K is weakly compact and convex, then $VI_S(A, a, K)$ admits solutions.

A generalization of the previous result is the following.

Theorem (A. Amini-Harandi-L.)

Let $K \subseteq X$ be a weakly compact and convex set and let a be of type g-ql. Assume that one of the following conditions is fulfilled.

a) *A is weak to $\|\cdot\|$-sequentially continuous and a is weak to weak-sequentially continuous.*

b) *A is weak to weak-sequentially continuous and a is weak to $\|\cdot\|$-sequentially continuous.*

Then $VI_S(A, a, K)$ admits solutions.

The next example shows, that the assumption, that the operator a is of type (g-)ql in the hypothesis of the previous Theorems is essential.

Example

Let $K \subseteq l^2$, $K := \left\{ (x_k) \in l^2 : \sum_{k \geq 1} \left| x_k - \frac{1}{k} \right|^2 \leq 1 \right\}$. Let $A : K \longrightarrow l^2$,

$$(x_k) \longrightarrow A((x_k)) \in l^2,\ A((x_k))((y_k)) := \sum_{k \geq 1} x_k y_k .$$

Let $a : K \longrightarrow l^2$, $a((x_k)) := \dfrac{2x_1}{2x_1^2 + 1}(x_k)$. Then K is weakly compact, and A and a are continuous, but $VI_S(A, a, K)$ has no solutions. In other words, all the assumptions of the previous Theorems are satisfied, excepting the one that a is of type (g-)ql, and consequently their conclusions fail.

Let X and Y be two Banach spaces. An operator $T : X \rightrightarrows Y$ is said to be weak to (weak) weak* upper semicontinuous if, for every $x_0 \in X$ and for every open set $N \subseteq Y$, in the (weak) weak* topology of Y, containing $T(x_0)$, there exists a neighborhood M of x_0, in the weak topology of X, such that $T(M) \subseteq N$.

Theorem (L.)

Let $K \subseteq X$ be nonempty, weakly compact and convex, consider the set-valued map $T : K \rightrightarrows X^*$ and let $f : K \longrightarrow X$ be of type ql. Assume that one of the following conditions is fulfilled.

a) *f is weak to weak-sequentially continuous on K, T is weak to norm upper semicontinuous on K and $T(x)$ is compact and convex for every $x \in K$.*

b) *f is weak to norm-sequentially continuous on K, T weak to weak* upper semicontinuous on K and $T(x)$ is weak* compact and convex for every $x \in K$.*

Then, $S(T, f, K) \neq \emptyset$.

The next example shows that the assumption that f is of type ql is essential in the previous result even in finite dimension.

Example

Let $K = [-1,1] \times [-1,1] \subseteq \mathbb{R}^2$ and consider the operators

$$T : K \rightrightarrows \mathbb{R}^2,\ T(x,y) = [-2, -1 - x^2y^2] \times \{xy\},$$

respectively

$$f : K \longrightarrow \mathbb{R}^2,\ f(x,y) = (-xy, xy).$$

Then all the assumptions of the previous Theorem are satisfied excepting the one that f is of type ql, hence its conclusion fails.

Coincidence points I.

Theorem (A. Amini-Harandi-L.)

Let H be a real Hilbert space identified with its dual. Let $K \subseteq H$ be a convex and weakly compact set, let $f : K \longrightarrow H$ be weak to weak sequentially continuous, $g : K \longrightarrow H$ be weak to norm sequentially continuous and of type g-ql. Assume that $f(K) \subseteq g(K)$. Then there exists $x_0 \in K$ such that $f(x_0) = g(x_0)$.

It is well known that in finite dimensional spaces the weak and strong topologies coincide. As a corollary we obtain the following coincidence point result, a generalization of Brouwer fixed point theorem.

Corollary

Let $K \subseteq \mathbb{R}^n$ be a convex and compact set and let $f, g : K \longrightarrow \mathbb{R}^n$ be continuous. Assume that $f(K) \subseteq g(K)$ and that g is of type g-ql. Then there exists $x_0 \in K$ such that $f(x_0) = g(x_0)$.

Theorem (L.)

Let $K \subseteq H$ be nonempty, weakly compact and convex, consider the weak to weak upper semicontinuous set-valued map $F : K \rightrightarrows H$ with weakly compact and convex values and let $f : K \longrightarrow H$ be weak to norm continuous and of type ql. Assume that $R(F) \subseteq f(K)$. Then, there exists $x \in K$ such that $f(x) \in F(x)$.

As an immediate consequence, in finite dimension we obtain a result that can be viewed as an extension of Kakutani's fixed point theorem.

Corollary

Let $K \subseteq \mathbb{R}^n$ be nonempty, compact and convex, consider the upper semicontinuous set-valued map $F : K \rightrightarrows \mathbb{R}^n$ with compact and convex values and let $f : K \longrightarrow \mathbb{R}^n$ be continuous and of type ql. Assume that $R(F) \subseteq f(K)$. Then, there exists $x \in K$ such that $f(x) \in F(x)$.

Theorem (L.)

Let $K \subseteq H$ be nonempty, weakly compact and convex, consider the set-valued map $F : K \rightrightarrows H$ with weakly compact and convex values and let $f : K \longrightarrow H$ be weak to weak-sequentially continuous and of type ql. Assume that $R(F) \subseteq f(K)$ and that the map $f - F$ is weak to norm upper semicontinuous. Then, there exists $x \in K$ such that $f(x) \in F(x)$.

In virtute of weak to weak-sequential continuity of the map $\mathrm{id}_K : K \longrightarrow K$, $\mathrm{id}_K(x) = x$, as a corollary we have the following fixed point result.

Corollary

Let $K \subseteq H$ be nonempty, weakly compact and convex, consider the set-valued map $F : K \rightrightarrows K$ with weakly compact and convex values. Assume that the map $\mathrm{id}_K - F$ is weak to norm upper semicontinuous. Then F has a fixed point, that is there exists $x \in K$ such that $x \in F(x)$.

Relations between $VI_S(A, a, K)$ and $VI_S(A \circ b, a(K))$

The following result ensures the equivalence between the existence of the solution of $VI_S(A, a, K)$ and the existence of the solution of a particular Stampacchia variational inequality.

Lemma (A. Amini-Harandi-L.)

Let $b : a(K) \longrightarrow K$ be a single valued selection of a^{-1}. Then $u \in a(K)$ is a solution of $VI_S(A \circ b, a(K))$ if and only if, $b(u) \in K$ is a solution of $VI_S(A, a, K)$.

Proposition (L.)

If A is weak to $\|\cdot\|$-sequentially continuous and K is weakly compact and convex, then Stampacchia variational inequality, $VI_S(A, K)$, admits solutions.

Theorem (A. Amini-Harandi-L.)

Assume that $a(K)$ is weakly compact and convex. Assume further, that for every sequence $\{x_n\} \subseteq K$ the following condition is satisfied: if the sequence $\{a(x_n)\} \subseteq a(K)$ converges weakly to $a(x) \in a(K)$ then the sequence $\{A(x_n)\} \subseteq X^*$ is norm convergent to $A(x) \in X^*$. Then $VI_S(A, a, K)$ admits solutions.

The following classical result concerning on the existence of the solution of $VI(A, K)$, is due to Hartmann and Stampacchia.

Proposition

Let X be a reflexive Banach space, let K be a weakly compact convex nonempty subset of X. If $A : K \longrightarrow X^*$ is a monotone operator, continuous on finite dimensional subspaces then $VI_S(A, K)$ admits solutions.

Theorem (A. Amini-Harandi-L.)

Let the Banach space X be reflexive. Assume that A is monotone relative to a and $a(K)$ is weakly compact and convex. Assume further, that for every finite dimensional subset $L \subseteq a(K)$ and for every sequence $\{x_n\} \subseteq K$, such that $a(x_n) \in L$ for every $n \in \mathbb{N}$, the following condition holds: if the sequence $\{a(x_n)\} \subseteq L$ converges to $a(x) \in a(K)$ then the sequence $\{A(x_n)\} \subseteq X^*$ is weakly convergent to $A(x) \in X^*$. Then $VI_S(A, a, K)$ admits solutions.

In what follows, we introduce the concept of an α-a inverse strongly monotone operator in a Banach space context.

Definition

Let X be a Banach space and let $A : K \longrightarrow X^*$ and $a : K \longrightarrow X$ be two given operators. We say that A is α-a inverse strongly monotone if there exists $\alpha > 0$ such that

$$\langle A(x) - A(y), a(x) - a(y)\rangle \geq \alpha \|A(x) - A(y)\|^2, \text{ for all } x, y \in K.$$

The next result is due to Takahashi and Toyoda.

Proposition

Let K be a bounded, closed and convex subset of a real Hilbert space H and let $A : K \longrightarrow H$ be an α inverse strongly monotone operator. Then $VI_S(A, K)$ admits solutions.

Theorem (A. Amini-Harandi-L.)

Let K be a subset of a real Hilbert space H and let $a : K \longrightarrow H$ and $A : K \longrightarrow H$ be given operators. Assume that $a(K)$ is a bounded closed convex of H and that A is α-a inverse strongly monotone. Then $VI_S(A, a, K)$ admits solutions.

Coincidence points II.

The following result ensures the equivalence between the existence of a coincidence point of two mappings and the existence of the solution of a particular general variational inequality of Stampacchia type.

Lemma (A. Amini-Harandi-L.)

Let $K \subseteq H$ and let $f, g : K \longrightarrow H$ be two given mappings. Assume that $f(K) \subseteq g(K)$. Then $x \in K$ is a solution of $VI_S(g - f, g, K)$ if, and only if, $f(x) = g(x)$.

Theorem (A. Amini-Harandi-L.)

Let $K \subseteq H$ and let $f, g : K \longrightarrow H$ be two given mappings. Assume that $f(K) \subseteq g(K)$ and $g(K)$ is weakly compact and convex. Assume further, that for every sequence $\{x_n\} \subseteq K$ the following condition holds: if the sequence $\{g(x_n)\} \subseteq g(K)$ converges weakly to $g(x) \in g(K)$ then the sequence $\{g(x_n) - f(x_n)\} \subseteq H$ converges to $g(x) - f(x) \in H$ in the topology of the norm of H. Then f and g have a coincidence point.

The next corollary provides sufficient conditions for the existence of a fixed point of a given mapping.

Corollary

Let $K \subseteq H$ be a weakly compact and convex set, let $f : K \longrightarrow H$ be a given mapping such that $f(K) \subseteq K$. Assume that for every sequence $\{x_n\} \subseteq K$ the following condition holds: if the sequence $\{x_n\} \subseteq K$ converges weakly to $x \in K$ then the sequence $\{x_n - f(x_n)\} \subseteq H$ is norm convergent to $x - f(x) \in H$. Then f has a fixed point.

Theorem (A. Amini-Harandi-L.)

Let $K \subseteq H$ and let $f, g : K \longrightarrow H$ be two given mappings, such that $\|g(x) - g(y)\|^2 \geq \langle f(x) - f(y), g(x) - g(y)\rangle$ for all $x, y \in K$. Assume that $f(K) \subseteq g(K)$ and $g(K)$ is weakly compact and convex. Assume further, that for every finite dimensional subset $L \subseteq g(K)$ and for every sequence $\{x_n\} \subseteq K$, such that $g(x_n) \in L$ for every $n \in \mathbb{N}$, the following condition holds: if the sequence $\{g(x_n)\} \subseteq L$ converges to $g(x) \in g(K)$ then the sequence $\{f(x_n)\} \subseteq H$ converges to $f(x) \in H$ in the weak topology of H. Then f and g have a coincidence point.

As an immediate consequence we obtain the following fixed point result.

Corollary

Let $K \subseteq H$ be a weakly compact and convex set, let $f : K \longrightarrow H$ be a given mapping such that $f(K) \subseteq K$. Assume that f is continuous on finite dimensional subspaces and $\|x - y\|^2 \geq \langle f(x) - f(y), x - y \rangle$ for all $x, y \in K$. Then f has a fixed point.

Theorem (A. Amini-Harandi-L.)

Let $K \subseteq H$ and let $f, g : K \longrightarrow H$ be two given mappings. Assume that there exists $\alpha > 0$, such that $(2\alpha - 1)\langle f(x) - f(y), g(x) - g(y) \rangle \geq (\alpha - 1)\|g(x) - g(y)\|^2 + \alpha\|f(x) - f(y)\|^2$ for all $x, y \in K$. Assume further, that $f(K) \subseteq g(K)$ and $g(K)$ is bounded closed and convex. Then f and g have a coincidence point.

Let $K \subseteq H$ and let $f, g : K \longrightarrow H$ be two given mappings. Assume that $f(K) \subseteq g(K)$. We say that f is strict $g-$pseudocontractive on K, if there exists a real number k, with $0 \leq k < 1$, such that $\|f(x) - f(y)\|^2 \leq \|g(x) - g(y)\|^2 + k\|(g - f)(x) - (g - f)(y)\|^2$ for all $x, y \in K$. If $g \equiv \mathrm{id}_K$ we obtain the notion of strict pseudocontractive mapping (see [7]).
Note that if $k = 0$ then f is $g-$nonexpansive, i.e. $\|f(x) - f(y)\| \leq \|g(x) - g(y)\|$ for all $x, y \in K$.

Corollary

Let $K \subseteq H$ and let $f, g : K \longrightarrow H$ be two given mappings and assume that f is strict $g-$pseudocontractive on K with some $1 > k \geq 0$. Assume further, that $f(K) \subseteq g(K)$ and $g(K)$ is bounded closed and convex. Then f and g have a coincidence point.

As immediate consequences we obtain the following fixed point results.

Corollary

Let $K \subseteq H$ be a bounded closed and convex set, let $f : K \longrightarrow H$ be a given mapping such that $f(K) \subseteq K$. Assume there exists $\alpha > 0$ such that $(2\alpha - 1)\langle f(x) - f(y), x - y\rangle \geq (\alpha - 1)\|x - y\|^2 + \alpha\|f(x) - f(y)\|^2$ for all $x, y \in K$. Then f has a fixed point.

Corollary

Let $K \subseteq H$ be bounded closed and convex set and let $f : K \longrightarrow H$ be a given mapping. Assume that f is strict pseudocontractive on K with some $1 > k \geq 0$. Assume further, that $f(K) \subseteq K$. Then f has a fixed point.

- Hartmann, G., Stampacchia, G.: On some nonlinear elliptic differential functional equations. Acta Mathematica 115, 271-310 (1966)
- S. László, *Some Existence Results of Solutions for General Variational Inequalities*, Journal of Optimization Theory and Applications, 150 (2011), 425-443
- S. László, *Existence of solutions of inverted variational inequalities*, Carpathian Journal of Mathematics 28 (2) (2012), 271-278
- A. Amini-Harandi, S. László, *A coincidence point result via variational inequalities*, Fixed Point Theory (accepted 2012)
- S. László, *Multivalued variational inequalities and coincidence point results* (submitted)
- A. Amini-Harandi, S. László, *General variational inequalities and coincidence points* (submitted)
- Takahashi, W., Toyoda, M.: Weak convergence theorems for nonexpansive mappings and monotone mappings. J. Optim. Theory Appl. 118, 417-428 (2003)

Mulțumesc pentru atenție!