A SECOND ORDER DYNAMICAL SYSTEM AND A MONOTONE INCLUSION PROBLEM

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DECEMBER 2, 2016

Abstract. In this paper we consider a second order dynamical system of the from $\ddot{x}(t) + \gamma(t)\dot{x}(t) + x(t) - J_{\lambda(t)A}(x(t) - \lambda(t)D(x(t)) - \lambda(t)\beta(t)B(x(t))) = 0$, $x(0) = u_0, \dot{x}(0) = v_0$, where $A : \mathcal{H} \rightrightarrows \mathcal{H}$ is a maximal monotone operator, $J_{\lambda(t)A} : \mathcal{H} \longrightarrow \mathcal{H}$ is the resolvent operator of $\lambda(t)A$, $D, B : \mathcal{H} \to \mathcal{H}$ are cocoercive operators defined on a real Hilbert space $\mathcal{H}$, $\lambda, \beta : [0, +\infty) \to [0, +\infty)$ are relaxation functions and $\gamma : [0, +\infty) \to [0, +\infty)$ a damping function, all depending on time. We show the existence and uniqueness of strong global solutions in the framework of the Cauchy-Lipschitz-Picard Theorem and prove ergodic asymptotic convergence for the generated trajectories to a zero of the operator $A + D + N_C$, where $C = \mathrm{zer}(B)$ and N_C is the normal cone operator, by using Lyapunov analysis combined with the celebrated Opial Lemma in its ergodic continuous version. Furthermore, we show the strong convergence of trajectories to the unique zero of $A + D + N_C$ in case A is a strongly monotone operator. The framework allows to address as particular case the minimization of the sum of a nonsmooth convex function with a smooth convex one and allows us to recover and improve several results from the literature.

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Date: December 3, 2016.