Some Minimax Results on Dense Sets and an extension of James' Theorem

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Abstract

Recall that a minimax theorem deals with sufficient conditions under which the equality $\inf_{x\in X}\sup_{y\in Y} f(x,y) = \sup_{y\in Y}\inf_{x\in X} f(x,y)$ holds, where X and Y are arbitrary sets and $f : X \times Y \longrightarrow \mathbb{R}$ is a given bifunction. The most general minimax results are due to Fan [1] and Sion [2], and both assume the compactness of X. As a matter of fact, minimax results on dense sets, (that is X is dense in a subset of a topological vector space), are absent in the literature. In this paper we give a motivation of this absence, Example 4.1 shows that the general results of Fan and Sion cannot be extended on usual dense sets. Nevertheless, we obtain some new minimax results on a special type of dense set that we call self-segment-dense [3, 4, 5]. An interesting proof of James Theorem, by using minimax results was first obtained by Simons [6, 7, 8]. The minimax theorems obtained in this paper also lead to some results which can be viewed as generalizations/extensions of James Theorem. However, our approach significantly differs from those in [6, 7, 8].

Keywords: self-segment-dense set, minimax theorem, convex function, James Theorem
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[1] Fan, K., Minimax theorems, National Academy of Sciences, Washington, DC, Proceedings USA **39**, 42-47 (1953)

[2] Sion, M., On general minimax theorems, Pac. J. Math. 8 (1), 171176, (1958).

[3] László, S., Vector Equilibrium Problems on Dense Sets, JOTA (submitted), http://arxiv.org/abs/1502.00174

[4] László, S., Viorel, A.: Generalized monotone operators on dense sets. Numerical Functional Analysis and Optimization **36**, 901-929 (2015)

[5] László, S., Viorel, A.: Densely defined equilibrium problems. J. Optim. Theory Appl. **166**, 52-75 (2015)

[6] Simons, S., Maximinimax, minimax, and antiminimax theorems and a result of R. C. James, Pacific Journal of Mathematics, Vol. 40, No. 3, (1972).

[7] Simons S., Minimax theorems and their proofs. In: Du DZ, Pardalos PM (eds) Minimax and Applications. Kluwer, Dordrecht, pp 123 (1995).

[8] Simons, S., Minimax and Monotonicity. Lecture Notes in Mathematics, 1693, Springer-Verlag (1998)