Image of circles through remarkable analytical functions

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Abstract

- Geometric Function Theory
  = Complex Analysis + Geometry;
- Applied Computer Science in Mathematics;
- Remarkable Analytical Functions Analysis;
- Special Classes of Univalent Functions;
- Special properties seen in function's graphical representation.

SPECIAL CLASSES OF UNIVALENT FUNCTIONS

- The class of univalent functions in the unit disc and normalized ;
- The class of starlike functions;
- The class of convex functions;
- The class of α-convex functions;
- The class of spiralike functions.

Image of circles through remarkable analytical functions

- Observing special properties in function's graphical representation;
- Using Maple for representing these images.

Univalent functions

Definition

An holomorphic function and injective on the domain D from $\mathbb{C}$ is univalent on D.

Remark

We denote by $\mathcal{H}_u(D)$ the set of univalent functions on the domain D

Univalent functions

Application

Given $f_a(z) = z + az^2, a \in \{0.3, 0.5, 0.6\}, r \in \{0.5, 0.7, 0.9, 1\}$, *find the image of circles* $\mathcal{C}_r = \mathcal{C}(O; r)$, *where* r *is the radius and* O *the center of the circle and the origin of the cartesian system.*

Univalent functions

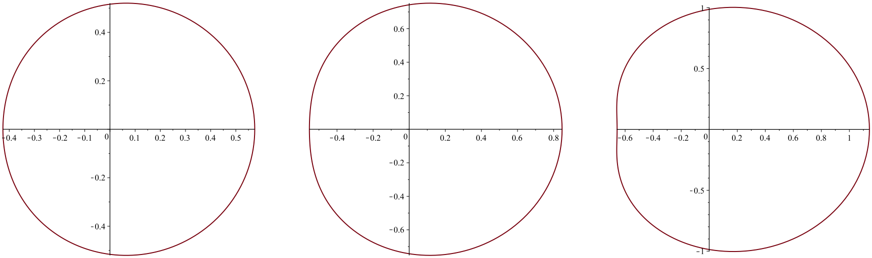

Figure: a=0.3,r=0.5

Figure: a=0.3,r=0.7

Figure: a=0.3,r=0.9

Univalent functions

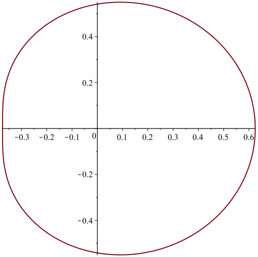

Figure: a=0.5,r=0.5

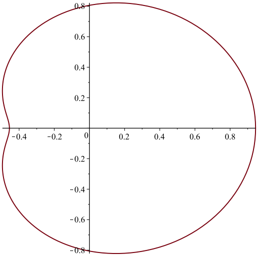

Figure: a=0.5,r=0.7

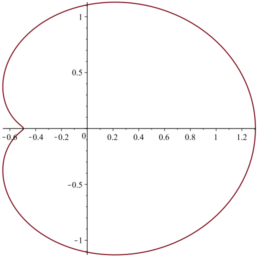

Figure: a=0.5,r=0.9

UNIVALENT FUNCTIONS

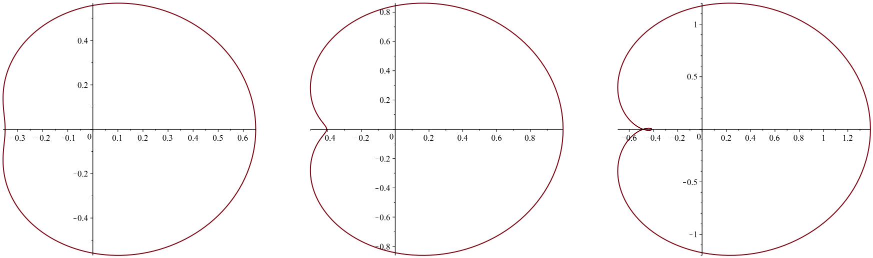

Figure: a=0.6,r=0.5 Figure: a=0.6,r=0.7 Figure: a=0.6,r=0.9

S CLASS
THE CLASS OF UNIVALENT FUNCTIONS IN THE UNIT DISC AND NORMALIZED

Definition

$$S = \{f \in \mathcal{H}_u(U) \mid f(0) = 0,\ f'(0) = 1\}$$

Example

Function $f : \mathbb{C} \to \mathbb{C}, f(z) = az^2 + z$ *with* $a \in (-\frac{1}{2}, \frac{1}{2})$ *is univalent.*

S^* class
The starlike functions

Definition

$$S^* = \left\{ f \in \mathcal{H}(U) \mid f(z) = z + a_2 z^2 + \ldots,\ Re\left(\frac{zf'(z)}{f(z)}\right) > 0,\ z \in U \right\}.$$

S^* CLASS
THE STARLIKE FUNCTIONS

Application

Given the Koebe function $k(z) = \dfrac{z}{(1-z)^2}$,
$r \in \{0.5,\ 0.6,\ 0.7,\ 0.8,\ 0.9\}$ *find the image of circles* $C_r = \mathcal{C}(O; r)$.

S^* CLASS
THE STARLIKE FUNCTIONS

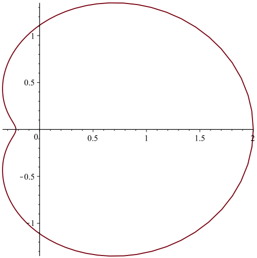

Figure: r=0.5

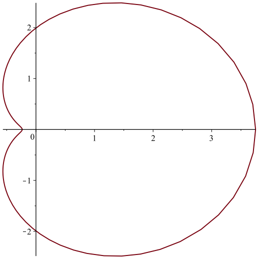

Figure: r=0.6

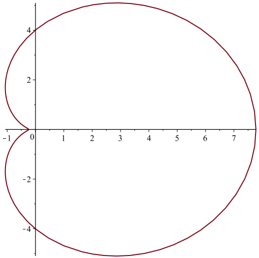

Figure: r=0.7

Clasa S^*
The starlike functions

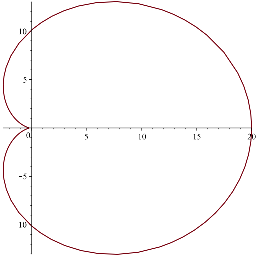

Figure: r=0.8

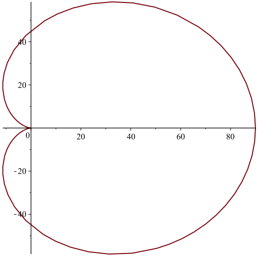

Figure: r=0.9

CLASA S^*
THE STARLIKE FUNCTIONS

Application

Function $f(z) = -2z + z^2 + 6\log\frac{2+z}{z}$ *is a function with a positive real part of the derivative. This function is not a starlike function*

Clasa S^*
The starlike functions

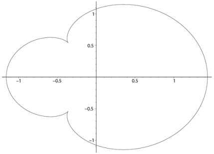

Figure: r=1

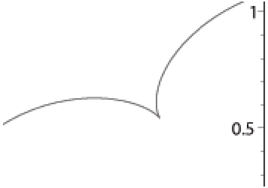

Figure: r=1,zoom

K class
The class of convex functions

Definition

$$\mathcal{K} = \{f \in A : Re\frac{zf''(z)}{f'(z)} + 1 > 0, z \in U\}$$

K class
The class of convex functions

Application

Given the Koebe convex function $s(z) = \dfrac{z}{1-z}$,
$r \in \{0.5, 0.6, 0.7, 0.8, 0.9\}$ *find the image of circles* $\mathcal{C}_r = \mathcal{C}(0; r)$.
The s *function is called the Koebe convex function because*
$zs'(z) = k(z)$.

K CLASS
THE CLASS OF CONVEX FUNCTIONS

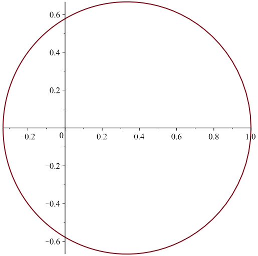

Figure: r=0.5

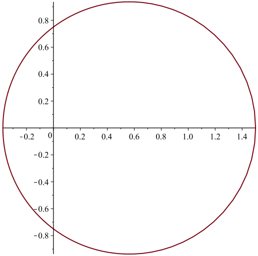

Figure: r=0.6

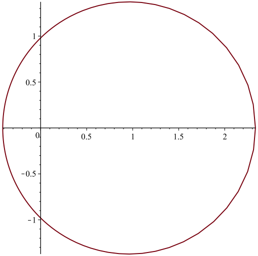

Figure: r=0.7

K CLASS
THE CLASS OF CONVEX FUNCTIONS

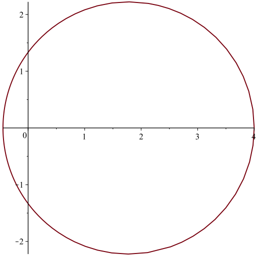

Figure: r=0.8

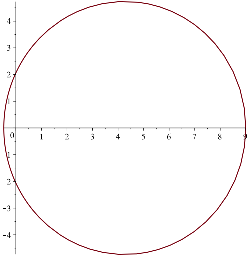

Figure: r=0.9

The class of spiralike functions

Definition

A function f is a γ-spiralike function in U if $f \in \mathcal{H}_u(U)$, with $f(0) = 0$, and U is a γ-spiralike domain.

THE CLASS OF SPIRALIKE FUNCTIONS

Application

Given $f(z) = \dfrac{z}{(1-z)^{2e^{-i\gamma}\cos\gamma}}$, *find the image of circles* $\mathcal{C}_r = \mathcal{C}(0; r)$ *for different values of* γ *and* r.

THE CLASS OF SPIRALIKE FUNCTIONS

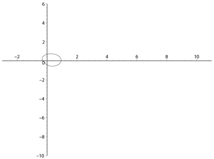

Figure: $\gamma = \frac{\pi}{6}$, r=0.4

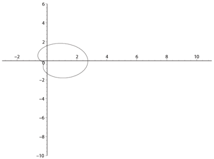

Figure: $\gamma = \frac{\pi}{6}$,r=0.6

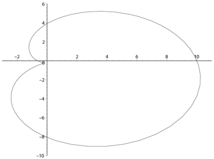

Figure: $\gamma = \frac{\pi}{6}$,r=0.8

THE CLASS OF SPIRALIKE FUNCTIONS

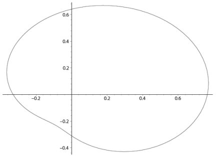

Figure: $\gamma = \dfrac{\pi}{4}$, r=0.4

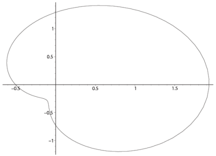

Figure: $\gamma = \dfrac{\pi}{4}$,r=0.6

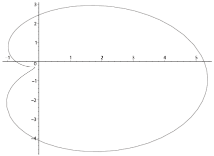

Figure: $\gamma = \dfrac{\pi}{4}$,r=0.8

THE CLASS OF SPIRALIKE FUNCTIONS

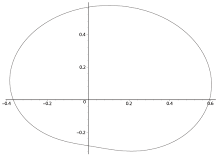

Figure: $\gamma = \dfrac{\pi}{3}$, r=0.4

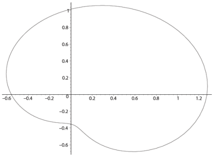

Figure: $\gamma = \dfrac{\pi}{3}$,r=0.6

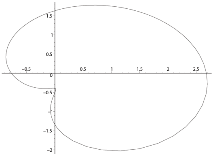

Figure: $\gamma = \dfrac{\pi}{3}$,r=0.8

Extremal functions

Application

Given the functions $f(z) = z + az^2$ *with negative coefficients, find the image of the unit disc for* $a \in \left\{ \frac{-1}{2}, \frac{-1}{3}, \frac{-1}{4} \right\}$

EXTREMAL FUNCTIONS

- $f_2(z) = z - \frac{z^2}{2}$ is an extremal function for the class of starlike functions;
- for $n = 2$ the image of the unit disc through this function is a starlike set;
- $f_4(z) = z - \frac{z^2}{4}$ is an extremal function for the class of convex functions;
- pentru $n = 4$ the image of the unit disc through this function is a convex set;
- $f_3(z) = z - \frac{z^2}{3}$ is not an extremal function.

Extremal functions

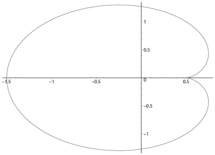

Figure: r=1,a=-1/2

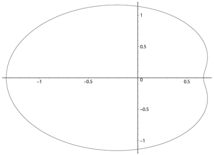

Figure: r=1,a=-1/3

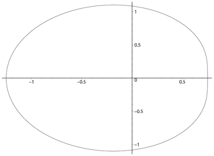

Figure: r=1,a=-1/4

Open-door function

Application

Function $f(z) = \frac{1+z}{1-z} + \frac{2n\alpha z}{1-z^2}$ *is called the open-door function. Given that, find the image of circles through this function.*

The open-door function was named by S. S. Miller si P. T. Mocanu, because through it, the unity disc is transformed into the complex plane cut by a little door, formed by: $Re\omega = 0$ and $|Im\omega| \geq \sqrt{n\alpha(n\alpha+2)}$, so left-half and right-half planes are united through an open door.

Open-door function

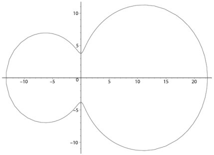

Figure:
r=0.8,$\alpha = 1$,n=3

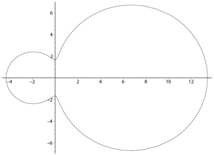

Figure:
r=0.8,$\alpha = 1$,n=1

Open-door function

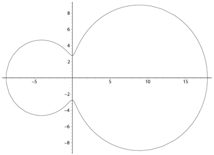

Figure:
r=0.8,$\alpha = 1$,n=2

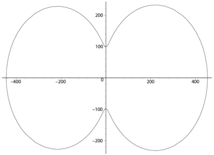

Figure:
r=0.8,$\alpha = 100$,n=1

Thank you!

Q & A