IN THE MIRROR OF MATHEMATICS (DOAMNA MATEMATICĂ ŞI EU)

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Abstract. Talk in Cluj

1. Introduction

You invited me at a time when it is slightly early for me, to give merely retrospective talks. But you have chosen to honor me as if it were precisely the time for counting the crops ... while I do work to see it is not; yet I must recognize I am deeply moved.

So I will respond to the situation by a lose series of recollections which are neither simple retrospection, nor prospects for the future. They are rather bound by questions like "How does the world look in the mirror of mathematics? How do we see ourselves in that mirror?" and "What do we serve with mathematics" and "what makes out its beauty?" [1].

I should mention from the beginning, that my relationship to *lady Mathematics* could not have started in a more dull way. No choice or passion involved, simple being. It happened that, as a child, I had a gift for numbers: I could for instance perform mental multiplication of two or three digit numbers at the age of four, and there was the reciprocal discovery of the fact that many other people could not do that. This contrast marked somehow my early age: mathematics was supposed by *others* to be part of me, yet I did not know what exactly that could be. I also enjoyed reading, music and travels, more than often more than numbers.

The first moment of attitude towards mathematics that I recall, happened in the early teenage. I crossed at a party of elderly people (around thirty!) on Horia Ene, the specialist of fluid mechanics at that time, and one of the founders of modern Romanian numerical analysis, later. At that party, he was explaining to an audience the platonic attitude to mathematics and opposed to this attitude, his "own" point of view, of mathematics being rather a natural science. In my present understanding, I would guess that he was at that time arguing on the line of thought that was expressed also by

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[1]What you will hear are my opinions and experiences, I take full and temporary responsibility for them, they are not the opinions of my employer or others unnamed. I found them times and times again confirmed in reading. So if you hear some thought or remark that sounds known to you and connect it to some person that I might have failed to quote or mention, chances are you may be right.

major mathematical physicist of the Russian school. For the early teenager that I was, the question was interesting and I heard myself talk out loud beyond all shyness: "Of course it is a natural science, for two reasons: first, the objects it deals with, numbers and figures and all this, are very lively so they must have some kind of nature of their own. And second, it happens to fit the way nature functions, it helps describe nature, so it is a science of nature". Then I fell back in silence, scared by my own audacity. What I had done was somehow *melting* platonic and empiric point of view in some kind of unity.

Without ever getting really entangled in systematic philosophy, and keeping a sane close distance in this discipline, I may say that dancing on the various sides of this presumed unity is something that accompanied me throughout most of my life. Even today, I never end being surprised, how daring and consuming this project can be, when pursuing it in its consequences.

Teenage is a time when we form some of the leading questions that accompany us throughout life. I shall therefore mention one that preoccupied me; useless to say, it was partly influenced by its arising at the end of the destalinization in a country striving for some liberalization. I discovered later, that the scope is quite broader. I was very much in accord with Malraux and his "*le 21-ème siècle sera spirituel ou ne sera pas du tout*" and convinced that technical development would continue, but fail to solve any major problem of life. With respect to science and the more restricted area of mathematics, I was also convinced that science had stepwise taken over the role of religion and in some sense it was natural to think of a *church of science*. Simply because 200 years had passed since natural sciences, far from being questioned by the sharp inquisition, had become the source of accepted understanding of Nature, Life and the Universe, while the Church had been progressively pushed into a defensive position. *This* was excessively true in communist countries, but I found it holds at least in all western type civilizations.

As a natural consequence, like priests had once been the protectors of the *living word*, but pursued this task from their very human limitations, some with grace and creative humility, some with power and despotism, we should likewise be prepared and careful about the fact that the temptations and responsibilities of the carriers of understanding passing over to science, the possibilities of *failure* – e.g. in form abuse of power or influence – necessarily migrated to the field of science too.

These early questions – which have meanwhile become a common place, but were less discussed at that time – had the positive impact of leading me to ponder later about the way the mathematics I was doing *served*, either practice or beauty or both. Briefly, *I was preparing myself for taking responsibility in my way of doing mathematics.*

My evolution took a radical change when I decided to "leave" the country and stayed in Switzerland, two years after Ceauşescu's *small cultural revolution* of 1971. It thus turned out that with 18 I was looking ahead of

building a life in a new country, and in this situation, I was most interested in the way mathematics could be useful in practical life. I needed to know how a pragmatical and industrious country will present the study (at the ETH), so that it serves real economy in a constructive way. The question preoccupied me for decades, until I understood that it has no answer in the structures. There exists one in the precious few people one may find in either field (academic or industry), who are used to look beyond the margin of their glass.

I studied at a renowned university, the ETH, with prestigious names in its past - Minkowski, Herman Weil, Nevanlina, van der Waerden, etc - and several monuments in life - Stiefel, Eckmann, Moser. The curriculum was good - compared with what I lived to see later too, quite much so. And I certainly learned to appreciate more of the formal-abstract and deductive approach to mathematics. I may say without being unfair, that the passion was though missing. And, most of all, it was missing between colleagues that hardly shared their interest for the field, an unpleasant development which seems to have become part of the past. I came to understand that this was more than only my subjective and particular impression. This led at times to the only nostalgia I had: I thought that for my mathematical development, I had missed a lot by not studying in Romania. But of course, I did not spend much time around this thought, the mission being to survive.

In addition, there were sufficient compensations even in my mathematical studies, to provide resources I would not have found in Romania. I interrupted the studies after the second year, in order to see what was asked from mathematics in the industry. My subject became numerical analysis and I followed it in work and for a PhD during half a dozen years. Along with the essential computer skills, I learned two things in this period. On a practical level, the importance of an appealing presentation and visualization of results – computer graphics was just emerging – and writing the first graphical programs that helped visualize the results of involved numerical computations, was a useful and fascinating task. Meanwhile, visual support is something that one became so accustomed to, it cannot be thought out of our life.

In my year of practical work at what then was Brown Boveri AG, I worked in the department of numerical analysis, being exposed to some insights around the simple and beautiful myths about our science, which is supposed to predict precisely the natural processes, etc. Instead of this hardly believable story, I learned a few of the timeless tricks in the art of applied numerical analysis. The first, easier thing is: never trust your tools, try them first out on some *reference data*. More important, even in classical problems, with well understood differential equations as model, the numerical search for solutions remains partly an art. Solutions are given by approximations processes which always converge to *something*, but only in very favorable cases can be proved to converge to the desired solution, even in the lucky cases when a predestinated *desired solution* is known to exist. Often, the

practical cases cannot be known or proved to satisfy the nice theoretical conditions required. So there is always a reminiscence of art, in choosing appropriate starting points, imagining good validation tests to verify that the solution has important chances to correspond to a real life situation, etc.

From a point of view of school mathematics, what engineers do is thus apparently lacking rigor. From a more down to earth perspective, it is a very respectable and responsible art, since they must take full responsibility for the validity and interpretation given to the computed results – and this requires some additional kind of rigor. We speak of course of the few experienced and open minded engineers, who create the standards of some department, the others trusting the trend.

The experience gathered in several practical projects I could bring to an end – namely implementation and field applications – had a lasting effect on my mathematical taste. On the one hand, if I work on some subject of *applied mathematics*, I am most concerned of it being *applicable*, and this quality is estimated with the perspective of an engineer – would the theory or algorithm help make a difference? Would he be incited to use it? One key ingredient for *applicable* approaches, is the *adequate* amount of maths for the informational complexity of the given practical problem. Too much is overkill; insufficient, one should try to do better. There are many others interesting things to learn from such an experience, but let us stop by saying that I will never understand mathematicians who look down to engineers, due to their presumed lack of rigor or mathematical understanding. Sometimes one might wish that a mathematician, confronted with an engineering problem, would know so well to select and apply to the end, the adequate mathematical knowledge: knowing there is a theory which in principle solves the problem is certainly not the right approach. What rigor is to the mathematician, is *confidence* to the practitioner, he must be able to have full confidence in the technical decisions for which he takes responsibility. The two may meet, but are not identical[2]

It happened that numerical analysis did not become my specialty, although I had completed a PhD on the subject. The reason was simply that I discovered computational instability in an algorithm that was the core of my subject, and did not accept to present a Thesis based on such an algorithm. Due to the departure of my PhD advisor, the work was accepted, but should have come to an end, so I preferred to drop it. My attitude was encountered with understanding and I chose to go in the direction of computational number theory. This meant, for a living, also cryptography and IT security, a practical specialty in which I worked for one more dozen years. Although not much mathematics could be used here either, algorithms are always a fascinating subject. The art of programming, the love for which was made immortal by Donald Knuth, became a central attraction in this

[2]There are of course several other arts in the engineering life, which are of less use in mathematics – the art of avoiding responsibility and gathering praise and advantages, etc. Or real importance as it is, this is off-topic here.

time. It was a great experience, to discover how in this crude practical art, aesthetical criteria are in perfect balance with the most desired practical requirements, like efficiency, sustainability, expandability of software etc. Among thousands of IT specialists working in the company, I had the chance to meet two or three real specialists in the art of programming. In discussing this art with them, I was surprised that it was not only my instinct of a semi-professionist, but their full- and long-time experience, that led to the same kind of conclusion. A software system that sounds good, explains clearly and has friendly architecture, will have all the chances to be – after due testing of course – also a reliable and expandable one. And vice-versa, a software that no one can read, but is built with most accurate and advanced flow-chart tools, and semantic testing, has more chances to have hidden problems that will eventually be flushed up – mostly when the developer is not an employee of the company any more, as Murphy[3] wants it.

Beyond the immediate use of applications of mathematics, in engineering, computer science or the most various modelings used by sciences and their surrogates, there is a virtue of mathematics which one starts suspecting when working in various environments. One that is becoming more known in the large and slowly reveals a new and important *attractivity for mathematics*.

This has less to do with any explicit mathematical area of knowledge. But it is a consequence of mathematical training and work: the mathematician is used to distinguish premises and axioms on which statements rely. She is also trained to quickly verify if a certain conclusion is *the only one* which can be drawn from a given set of premises. This is uncommon for daily life logics, where people tend to identify "solution" with "the only solution" or "A is not true" with "non-A must be true", etc. With disastrous consequences in communications. It is exactly these kind of faculties which make that one may find oneself as a mathematician in the midst of a hot social debate - or, say, controversy at work - as the only one who sees that the tension relays on some false premises and inconsistent logical deductions on more sides. An other typical example: A and B contemplate two equal vectors in the $x-y$ plane. A sits on the y-coordinate, B on the x-coordinate, both consider the vector closer to them to be the largest. Stated like this, the error is obvious but in real life the coordinate systems are not as evident, and the common frame of reference may simply not be given a priori. But there is this implicit assumption, that if we can talk to someone, then speech itself already gives us a common frame of reference: dangerous!

To my understanding, the subject received a well deserved academic interest following successful experiments and books of Daniel Kahneman - the only psychologist having received a Nobel Prize, by today - and coauthors. Their examples are indeed very interesting and the general attitude is to

[3]or, according to some spread suspicion, also the financial interest of the developer himself

reveal that their is something like an inborn predisposition to *simplification*, that is inherent to the physiology of our brain and escapes rational thinking and good or ill intention. Roughly, the talk is about the System I and System II, the first being responsible for quick identifications of objects, and relating them in some main frame "story" which makes up for the reality perception of the world in which we live. The quick identification is done basically on association and the efficient, yet superficial adaptation of System I is of fundamental importance to our navigation through the challenges of daily life. When all fails, System II is asked for. This is an analytic System and it verifies if hypotheses meet facts, a process that requires effort and change. The dynamics between the two systems are the individual art of surviving of individuals. I can only recommend to all of you the books of Kahneman, like "Quick thinking, slow thinking", and several more. For our context it is enough to underline two facts:

(i) It is relieving to consider many misunderstandings of daily and social life as consequences of such almost physiological causes, rather than overloading them with judgmental emotions.
(ii) As mathematicians we have trained - with fascination and, at times, some actual fight against our nature - the best resources that are given to us, for escaping to the circle of illusion, and enhancing the System II. As a consequence, it acts slightly faster, has a good archive of typical situations and is more willing to do its complete job for ascertaining or improving the perceptions of System I.

I should add that mathematical training does indeed help in this context, but it is never a guarantee. There is namely a work in itself, for bringing the thinking experience back to matters of real life - many mathematicians are seen not to perform very well in this respect. This may have to do with a feeling of loss, when leaving the domain of well posed, clear and challenging mathematical questions and trade it against the real world in which the bigger effort consists in bringing the issues of debate into a sustainable logical form. On the other hand, my experience in the practical world allowed me to encounter precious examples of mathematicians, which could master both the necessary social skills and succeeded to bring in short time the simple core issues of endless discussions.

To make a message short, experience shows that mathematicians can, under some additional favorable conditions, help spread some tools of logics, analysis and deduction, which are well received by people of different skills, which end discovering that they may make their lives and communication aptitudes better too. Useless to say, this does not happen by explaining derivative functors or even some classical fact like the fundamental theorem of algebra, but on working on examples that life offers and deducing their logical tissue.

I did not know about Kahneman when I was making my own experiences in this direction, but I felt much relief and happiness when I discovered his

theories - and also, the more incisive speculations of Nissim Taleb, the author of *The Black Swan*. After all, I believe that it is for a mathematician an on going, background preoccupation to question and verify how the generally assumed facts and theories about the functioning of the most various aspects of life agree or not with facts and our experience. *This preoccupation becomes almost a source of tensions that are transformed into mathematical creativity*. But it can build the set of qualities for which mathematician may become more and more required in the most various aspects of "real life", not only in the classical engineering branches.

Transforming the impulses from everyday life into valuable mathematical work is a matter of disposing freely of one's own time. No surprise that it happened in the year 2000 that I chose, on the long way, the return to Academia. There has been a certain day, when, in Berlin, I had the possibility of a new start in both *worlds*. Choosing between a mysteriously named BATIIa position in Paderborn and one in the research lab of a major US Cryptography enterprise, in favor of the first, I came back to Academia.

In the first year I found myself passing most of the free time gained at the cost of a light wallet, with wondering, dreaming and drawing conclusions about the unexpected life of presumed solutions to Diophantine Equations which were expected to have none, although no one had so far succeeded to prove this to be a fact. It is not easy - and probably not worth trying - to speak about the lightness and seriousness of the pursuit of bringing light on mathematical objects *that do not exist*. But I cannot deny the fact that this activity is undoubtedly one of the aspects that make people resent the *beauty* of mathematics. One thing that I can state about it with full conviction and in a way that does not require one to be a mathematician, in order to understand or guess what is meant, is the fact that this process is not much about *discovering* or even less *creating* something new. There may exist domains of mathematics, where such an image is possibly more fit, but number theory is not one of them. The real work is one of consistent thinking and freeing of *expectations and prejudice*: expectation that this or the other method should reveal so little or so much of the presumed result. Prejudices about what "simple" and what "difficult", or even "impossible", might mean. Prejudices about what one knows and what others might or might not know. One accepts that all these side - thoughts are not more than guesses, which may be leading or also misleading guesses. And the journey is led by the sense of life in the mysterious objects, that should not exist. I had written a poem at that time, which described the "dance of ghosts"[4] waiting for the daylight to dissipate them - I have lost the poem. But its major intention which was, of course, to strengthen my conviction that in the end the daylight will dissipate the ghosts, was achieved.

The first ray of light that dissipates the existence of the presumed solutions to an equation that should have not more than one. Towards the

[4]Dansul nălucilor sounds softer, in Romanian

beginning of 2002 I had thus succeeded to gather sufficiently light about possible solutions to a long standing Diophantine Equation $x^u - y^v = 1$, that their existence dissipated in proof. *Briefly what I wished to stress is the fact that in some cases, mathematics is not about creation or invention, it is about elimination of misconceptions, until the necessary clarity gets revealed*. Very much like Michelangelo's work that he was freeing from marble - it is not a nice metaphor, it is the pure truth. This is no news, but I find useful to recall it.

Living through that process to the end, I could pay my ticket to stay on the side of life in which mathematics needs less to explain its right for a living - suffices that it perpetuates and enriches itself in some satisfactorily way. The environment that Latins called *Alma Mater*, and which preserved some maternal protective aspects, although we are very parcemonious about the Soul/Alma.

I will be very brief about this last period of my relation to Lady Mathematics, also because it is more difficult to have the necessary distance to it. There are several important things to say about this time. First, I came in the position to function as an emissary of Her accomplishments and settlements - teach and spread. Unsurprisingly, maybe - but for me with certain enjoyment - I could discover that there are means to transport the excitements and tremors of the game around this Mirror, to young, unknown and a priori so different minds and souls. There is something undeniably solid about it: you use the rigorous form and can transmit the passion for the subject, for the Mirror which gives us a new understanding of life and ourselves. Second, in the hard work of years on new and new questions, I became free of the beginners' presumption about the existence of *those who know*. Mathematics has uncounted facets and fascinating rooms. Along with this maturing, I was also blessed with the developing of dedicated, deep relationships to several mathematicians of various tastes, countries or colors of the eyes: some of them are in this room now. A major gift in this period, along with the experience of teaching, was the possibility to pursue the interest for some fascinating problem, without interruptions due to work in a totally different area. I hope this work leads to new fruits.

During this time and probably to the end of the loneliness, I kept wondering about what and in which way Lady Mathematics succeeds to reflect back about, say, Nature, in lack of more precise terms. It began with the rigid bodies and the movement of the planets. We have spoken about this. About Poincaré's caution that even the simplest classical trajectories should be considered as virtually unpredictable, due to the capricious effects of slight variations in the limit conditions. It took many decades until this observation was understood in its implications – this lead to the development of dynamical systems, a booming discipline of mathematics in these days. We have thus been taught caution in the classical, and, presumedly, most certain citadels of exact applications of mathematics to predictions concerning the physical world. Meanwhile, a request and interest for mathematics grows

in most areas of cognition, from biology and other unformalized disciplines, which can yet to some extent be subject of experimental verification and possible infirmation of hypotheses, all the way through to *social and economical disciplines*, which run the highest risks of falling under dogmatism. In this expanding areas of application, mathematics is in a life and death struggle between a possible misleading use for creation of *accurate illusions and make beliefs* and the strong and timeless use for rigorous understanding and elimination of misconceptions. It becomes more a matter of taste and character of the involved researchers, than of the properties of the mathematics involved, which of the two takes the lead. But there certainly is much new going on – *this can become a subject for a future, really retrospective discussion we may have here.*

References

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