Stud. Univ. Babeş-Bolyai Math. 60(2015), No. 3, 485–489

Book reviews

Michael Harris, Mathematics without apologies: portrait of a problematic vocation, Princeton University Press, Princeton, NJ, 2015, xxii+438,
ISBN: 978-0-691-15423-7/hbk; 978-1-400-85202-4/ebook.

This is an unusual book written by an eminent professional mathematicians, a well known expert in number theory and algebraic geometry. It reflects author's opinions on mathematics, both pure and applied, and on mathematicians, as seen from the inside of the caste and from outside as well. The author combines his own experience as a mathematician with that obtained from various domains of the human knowledge - philosophy (Plato, Archimedes, Omar Khayam, hindustan philosophy, Witgenstein, Pascal, Russel Kant, Nietsche, etc), literature (a lot of quotations for Goethe's *Faust* and from Shakespeare), history, religion (the case of Pavel Florensky), cinematography and even pop music.

A large part of the book is devoted to the discussion whether mathematics is useful only by the prism of his applications (as considered Fourier) or as a creation of the human spirit, without any reference to practical applications. The conclusion is that pure mathematics, considered by the author a scientific discipline very close to art (an abstract one), deserves to be studied, developed and supported *per se*, without any reference to applications. And besides, some branches of pure mathematics turned to find unexpected applications, in spite of the fact that at the origins they were developed as pure theoretical achievements. As an example, the number theory, a highly theoretical discipline, found applications in cryptography which is basic in bank and e-commerce security. This contradicts Hardy's opinions, who at several occasions toasted "for pure mathematics, and to never find applications", with emphasis on number theory. In fact the title evokes G. H. Hardy's classic *A Mathematician's Apology*, Cambridge Univ. Press, Cambridge, 1940. The author discusses also the allegations blaming mathematicians involved in mathematical economy (another field where results in pure mathematics found deep applications) for the 2008 financial crisis.

The gallery of mathematicians presented in the book is dominated by two giants of the XX century - Alexander Grothendieck and Robert Langlands. Besides the visionary results of Grothendieck in algebraic geometry ("schemas" and "motifs"), the author discusses in several places ideas from *Récoltes te Sémailles*, a collection of reminiscences and reflections about mathematics, philosophy, politics and others, written by Grothendieck. Robert Langlands is best known by his long term "Langlands program", still in progress, the author himself being involved in it. The realization of some intermediary steps led Maxim Kontsevich to a Fields medal in 1998.

The book contains also five sessions entitled "How to Explain Number Theory at a Dinner Party", written as a dialogue between two imagined interlocutors at a dinner party, Performing Artist and Number Theorist, whose challenges and responses elaborate key ideas and themes.

Some practical advices on the career-shaping of a mathematician - the role of "charisma", the quality of publications, and, the last but not the least, the chance - are included as well.

It is difficult to present in a few lines the wealthy of information contained in this marvelous book, our strong advice being to read it (far from being an easy task) and benefit from author's erudition and his charming style of presentation.

P. T. Mocanu

Miroslav Bačák, Convex analysis and optimization in Hadamard spaces, De Gruyter Series in Nonlinear Analysis and Applications, Vol. 22, viii + 185 pp, Walter de Gruyter, Berlin, 2014, ISBN 978-3-11-036103-2/hbk; 978-3-11-036162-9/ebook.

In recent years, concepts and traditional results from convex analysis have been extended to nonlinear settings. Due to their rich geometry, CAT(0) spaces (also known as spaces of non-positive curvature in the sense of Alexandrov) proved to be relevant in this context. The aim of this book is to present a systematic discussion of various topics in convex analysis in the setting of Hadamard spaces (that is, complete CAT(0) spaces). The book contains eight chapters which combine techniques from analysis, geometry, probability and optimization to study different problems in convex analysis.

The first chapter defines Hadamard spaces giving equivalent conditions, examples and construction methods. The second chapter introduces many convexity concepts, properties and results used throughout the book. Special attention is given, among others aspects, to barycenters and resolvents of convex lower semi-continuous functions defined on a Hadamard space. The next chapter deals with a notion of weak convergence defined in terms of asymptotic centers of bounded sequences which recovers the usual weak convergence in Hilbert spaces. Properties of nonexpansive mappings and gradient flows of convex lower semi-continuous functions are the central focus of the following two chapters. Chapter 6 is devoted to convex optimization algorithms used to study convex feasibility problems, to approximate fixed points of a nonexpansive mapping or to find a minimizer of a convex lower semi-continuous function as well as a finite sum thereof when the Hadamard space is in addition locally compact. The next chapter is concerned with random variables with values in Hadamard spaces. In the last chapter, the author gives an example of a Hadamard space, the so-called tree space constructed by L. Billera, S. Holmes and K. Vogtmann, which finds interesting applications in phylogenetics.

This book is written in a very lucid way and can be used both by students and researchers interested in analysis in Hadamard spaces. Each chapter contains a set of exercises and ends with detailed bibliographical remarks, where the author carefully refers to the sources of the presented results. Some comments and challenging questions are also included.

Adriana Nicolae

Francesco Altomare, Mirella Cappelletti Montano, Vita Leonessa, Ioan Raşa; Markov operators, positive semigroups and approximation processes, De Gruyter Studies in Mathematics, vol. 61, Walter de Gruyter, Berlin, 2014, xi+ 313 pp.
ISBN 978-3-11-037274-8/hbk; 978-3-11-036697-6/ebook.

Let $C(X), C(Y)$ be the Banach spaces (with respect to the uniform norm $\|\cdot\|_\infty$) of real- or complex-valued continuous functions on compact Hausdorff spaces X, Y, respectively. A positive linear operator $T : C(X) \to C(Y)$ is called a Markov operator if $T1_X = 1_Y$, where 1_Z denotes the function identically equal to 1 on Z. It follows $\|T\| = \|T1_X\|_\infty = 1$. As a special class of positive linear operators, the Markov operators inherit their properties. For reader's convenience, the authors present in the first chapter, *Positive linear operators and approximation problems*, the main notions, tools and results from the theory of linear operators – positive Radon measures, Choquet boundaries, Bauer simplices, Korovkin-type approximation, etc. Good sources for results of this kind are the book by F. Altomare and M. Campiti, *Korovkin-type approximation theory and its applications*, de Gruyter Studies in Mathematics, vol. 17, W. de Gruyter, Berlin, 1994, and the survey paper by F. Altomare, *Korovkin-type theorems and approximation by positive linear operators*, Surv. Approx. Theory vol. 5 (2010), 92-164.

The main theme of the book is the theory of Markov semigroups and the approximation processes which can be generated by a Markov operator acting on $C(K)$, where K is a compact convex subset of a (possibly infinite dimensional) Hausdorff locally convex space.

After a first contact with semigroups of Markov operators in Section 1.4 of the first chapter, their real study starts in Chapter 2, *C_0-semigroups of operators and linear evolution equations*, including a presentation of general properties of semigroups of operators on Banach spaces and their relations with Markov processes and multidimensional second-order differential operators.

Of particular interest in the theory of semigroups of Markov operators are the Bernstein-Schnabl operators associated with Markov operators, whose study begins in the third chapter. After giving several interpretations of these operators – probabilistic, via tensor products – several key examples are discussed: Bernstein-Schnabl operators on [0,1] (which turn to classical Bernstein polynomials), on Bauer simplices, associated with strictly elliptic differential operators, with tensor and convolution products, or with convex combinations of Markov operators. Then one discusses the approximation properties of this class of operators and the rate of convergence. A special attention is paid to their preservation properties – of Hölder and Lipschitz continuity, of convexity and of monotonicity.

Chapters 4, *Differential operators and Markov semigroups associated with Markov operators*, and 5, *Perturbed differential operators and modified Bernstein-Schnabl operators*, treat other main theme of the book. It is shown that "under suitable assumptions on T, the associated (abstract) differential operator is closable and its closure generates a Markov semigroup $(T(t))_{t\geq 0}$ on $C(K)$ which, in turn, is the transition semigroup of a suitable right-continuous Markov process with state space K" (from Introduction, page 2).

Two appendices, A1, *A classification of Markov operators on two dimensional compact convex subsets*, and A2, *Rate of convergence for the limit semigroup of Bernstein operators*, complete the main text.

The bibliography contains 210 items, including many papers by the authors of the book.

Based mainly on the original results of the authors, the book is very well written and contains a lot of interesting material. It can be viewed as an extension of the book by Altomare and Campity mentioned above, and can serve as a reference for researchers in various domain of approximation theory, functional analysis and probability theory. Taking into account the detailed presentation of the results, it can be also used by novices as an accessible introduction to this fertile area of investigation.

S. Cobzaş

Lawrence Craig Evans and Ronald F. Gariepy, Measure theory and fine properties of functions, 2nd revised ed., Textbooks in Mathematics, CRC Press, Boca Raton, FL, 2015, xi+309 pp., ISBN: 978-1-4822-4238-6/hbk.

This is a new revised edition of a very successful book (published by CRC Press in 1992) dealing with measure theory in $\mathbb{R}^n$ and some special properties of functions, usually omitted from books dealing with abstract measure theory, but which a working mathematician analyst must to know. Among these special topics we mention: Vitali's and Besicovitch's covering theorems, Hausdorff measures and capacities (for classifying classes of negligible sets for various fine properties of functions), Rademacher's Theorem on the a.e. differentiability of Lipschitz functions (a topic of great actual interest in connection with its extension to infinite dimensions), the area and coarea formulas (yielding change-of-variable rules for Lipschitz functions between $\mathbb{R}^n$ and $\mathbb{R}^m$), the Lebesgue-Besicovitch differentiation theorem, the precise structure of BV and Sobolev functions, Alexandrov's theorem on a.e. twice differentiability of convex functions, Whithney's extension theorem with applications to approximation of Sobolev and BV functions, etc. The book is clearly written with complete proofs, including all technicalities. One assumes that the reader is familiar with Lebesgue measure and abstract measure theory.

The new edition benefits from LaTeX retyping, yielding better cross-references, as well as of numerous improvements in notation, format and clarity of exposition. The bibliography has been updated and several new sections were added: on π-λ-Theorem (on the relations between σ-algebras and Dynkin systems), weak compactness criteria in L^1, the method of Young measures in the study weak convergence, etc.

Undoubtedly that this welcome updated and revised edition of a very popular book will continue to be of great interest for the community of mathematicians interested in mathematical analysis in $\mathbb{R}^n$.

Valeriu Anisiu

Bernardo Lafuerza Guillén, Panackal Harikrishnan; Probabilistic normed spaces, Imperial College Press, London 2014, World Scientific, London-Singapore-Hong Kong 2014, xi+220 pp, ISBN 978-1-78326-468-1/hbk; 978-1-78326-470-4/ebook.

Probabilistic metric (PM) spaces are spaces on which there is a "distance function" taking as values distribution functions - the "distance" between two points p, q is a distribution function (in the sense of probability theory) $F(p, q)$, whose value $F(p, q)(t)$ at $t \in \mathbb{R}$ can be interpreted as the probability that the distance between p and q be less than t. Probabilistic metric spaces were first considered by K. Menger in 1942, who made important contributions to the subject, followed almost immediately by A. Wald in 1943. A good presentation of results up to 1983 is given in the book by B. Schweizer and A. Sklar, *Probabilistic metric spaces*, North Holland, Amsterdam 1983 (reprinted and updated by Dover Publications, New York 2012).

Probabilistic normed (PN) spaces entered the stage only in 1962, introduced by Šerstnev and developed in a series of papers by him and other Russian mathematicians from the Probability School of the University of Kazan. After that the theory laid into lethargy until 1983 when Alsina, Schweizer and Sklar proposed a new approach to probabilistic normed spaces, which is more general and more adequate for developing a consistent theory. Menger PN spaces are particular cases of those defined by Alsina, Schweizer and Sklar, and Šerstnev PN spaces are particular cases of Menger PN spaces.

The first chapter, 1. *Preliminaries*, includes some background material from probability theory (distribution functions) and on copulas, triangular norms, probabilistic metric spaces.

The rest of the book is devoted to a systematic presentation of various aspects of the theory of PN spaces, which are well reflected by the headings of the chapters: 2. *Probabilistic normed spaces*; 3. *The topology of* PN *spaces*; 4. *Probabilistic norms and convergence*; 5. *Products and quotients of* PN *spaces*; 6. *$\mathcal{D}$-Boundedness and $\mathcal{D}$-compactness*; 7. *Normability*; 8. *Invariant and semi-invariant* PN *spaces*; 9. *Linear operators*.

Applications to functional equations are given in Chapter 10. *Stability of some functional equations in* PN *spaces*, while Chapter 11. *Menger's* 2*-probabilistic normed spaces*, presents a probabilistic version of 2-metric spaces introduced by S. Gähler, Mathematische Nachrichten (1964).

The book is clearly written and can be used as an introductory text to this area of research. Based on recent results, including authors' contributions, it is a good reference in the domain. Most of the chapters ends with a list of open problems, inviting the reader to further investigation and new developments in this active area of research.

S. Cobzaş