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Power Pompeiu's type inequalities for absolutely continuous functions with applications to Ostrowski's inequality

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Abstract. In this paper, some power generalizations of Pompeiu's inequality for complex-valued absolutely continuous functions are provided. They are applied to obtain some new Ostrowski type results.

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1. Introduction

In 1946, Pompeiu [6] derived a variant of Lagrange's mean value theorem, now known as *Pompeiu's mean value theorem* (see also [8, p. 83]).

Theorem 1.1. (Pompeiu, 1946 [6]) *For every real valued function f differentiable on an interval $[a,b]$ not containing 0 and for all pairs $x_1 \neq x_2$ in $[a,b]$, there exists a point ξ between x_1 and x_2 such that*

$$\frac{x_1 f(x_2) - x_2 f(x_1)}{x_1 - x_2} = f(\xi) - \xi f'(\xi). \tag{1.1}$$

In 1938, A. Ostrowski [4] proved the following result in the estimating the integral mean:

Theorem 1.2. (Ostrowski, 1938 [4]) *Let $f : [a,b] \to \mathbb{R}$ be continuous on $[a,b]$ and differentiable on (a,b) with $|f'(t)| \leq M < \infty$ for all $t \in (a,b)$. Then for any $x \in [a,b]$, we have the inequality*

$$\left| f(x) - \frac{1}{b-a}\int_a^b f(t)\,dt \right| \leq \left[\frac{1}{4} + \left(\frac{x - \frac{a+b}{2}}{b-a} \right)^2 \right] M(b-a). \tag{1.2}$$

The constant $\frac{1}{4}$ *is best possible in the sense that it cannot be replaced by a smaller quantity.*

In order to provide another approximation of the integral mean, by making use of the Pompeiu's mean value theorem, the author proved the following result:

Theorem 1.3. (Dragomir, 2005 [3]) *Let* $f : [a, b] \to \mathbb{R}$ *be continuous on* $[a, b]$ *and differentiable on* (a, b) *with* $[a, b]$ *not containing* 0. *Then for any* $x \in [a, b]$, *we have the inequality*

$$\left| \frac{a+b}{2} \cdot \frac{f(x)}{x} - \frac{1}{b-a} \int_a^b f(t)\, dt \right| \leq \frac{b-a}{|x|} \left[\frac{1}{4} + \left(\frac{x - \frac{a+b}{2}}{b-a} \right)^2 \right] \|f - \ell f'\|_\infty , \tag{1.3}$$

where $\ell(t) = t$, $t \in [a, b]$.

The constant $\frac{1}{4}$ *is sharp in the sense that it cannot be replaced by a smaller constant.*

In [7], E. C. Popa using a mean value theorem obtained a generalization of (1.3) as follows:

Theorem 1.4. (Popa, 2007 [7]) *Let* $f : [a, b] \to \mathbb{R}$ *be continuous on* $[a, b]$ *and differentiable on* (a, b). *Assume that* $\alpha \notin [a, b]$. *Then for any* $x \in [a, b]$, *we have the inequality*

$$\left| \left(\frac{a+b}{2} - \alpha \right) f(x) + \frac{\alpha - x}{b-a} \int_a^b f(t)\, dt \right| \leq \left[\frac{1}{4} + \left(\frac{x - \frac{a+b}{2}}{b-a} \right)^2 \right] (b-a) \|f - \ell_\alpha f'\|_\infty , \tag{1.4}$$

where $\ell_\alpha(t) = t - \alpha$, $t \in [a, b]$.

In [5], J. Pečarić and S. Ungar have proved a general estimate with the p-norm, $1 \leq p \leq \infty$ which for $p = \infty$ give Dragomir's result.

Theorem 1.5. (Pečarić & Ungar, 2006 [5]) *Let* $f : [a, b] \to \mathbb{R}$ *be continuous on* $[a, b]$ *and differentiable on* (a, b) *with* $0 < a < b$. *Then for* $1 \leq p, q \leq \infty$ *with* $\frac{1}{p} + \frac{1}{q} = 1$ *we have the inequality*

$$\left| \frac{a+b}{2} \cdot \frac{f(x)}{x} - \frac{1}{b-a} \int_a^b f(t)\, dt \right| \leq PU(x, p) \|f - \ell f'\|_p , \tag{1.5}$$

for $x \in [a,b]$, *where*

$$PU(x,p) \quad : \quad = (b-a)^{\frac{1}{p}-1}\left[\left(\frac{a^{2-q}-x^{2-q}}{(1-2q)(2-q)}+\frac{x^{2-q}-a^{1+q}x^{1-2q}}{(1-2q)(1+q)}\right)^{1/q} + \left(\frac{b^{2-q}-x^{2-q}}{(1-2q)(2-q)}+\frac{x^{2-q}-b^{1+q}x^{1-2q}}{(1-2q)(1+q)}\right)^{1/q}\right].$$

In the cases $(p,q)=(1,\infty),(\infty,1)$ *and* $(2,2)$ *the quantity* $PU(x,p)$ *has to be taken as the limit as* $p\to 1,\infty$ *and* 2, *respectively.*

For other inequalities in terms of the p-norm of the quantity $f-\ell_\alpha f'$, where $\ell_\alpha(t)=t-\alpha$, $t\in[a,b]$ and $\alpha\notin[a,b]$ see [1] and [2].

In this paper, some power Pompeiu's type inequalities for complex valued absolutely continuous functions are provided. They are applied to obtain some new Ostrowski type inequalities.

2. Power Pompeiu's type inequalities

The following inequality is useful to derive some Ostrowski type inequalities.

Corollary 2.1. (Pompeiu's Inequality) *With the assumptions of Theorem* 1.1 *and if* $\|f-\ell f'\|_\infty=\sup_{t\in(a,b)}|f(t)-tf'(t)|<\infty$ *where* $\ell(t)=t$, $t\in[a,b]$, *then*

$$|tf(x)-xf(t)|\le \|f-\ell f'\|_\infty |x-t| \tag{2.1}$$

for any $t,x\in[a,b]$.

The inequality (2.1) was stated by the author in [3].

We can generalize the above inequality for the power function as follows.

Lemma 2.2. *Let* $f:[a,b]\to\mathbb{C}$ *be an absolutely continuous function on the interval* $[a,b]$ *with* $b>a>0$. *If* $r\in\mathbb{R}$, $r\neq 0$, *then for any* $t,x\in[a,b]$ *we have*

$$|t^r f(x)-x^r f(t)| \tag{2.2}$$
$$\le\begin{cases}\frac{1}{|r|}\|f'\ell-rf\|_\infty |t^r-x^r|, \text{ if } f'\ell-rf\in L_\infty[a,b],\\[2ex] \|f'\ell-rf\|_p \\ \times\begin{cases}\frac{1}{|1-q(r+1)|}\left|\frac{t^r}{x^{1-q(r+1)-r}}-\frac{x^r}{t^{1-q(r+1)-r}}\right|, \text{ for } r\neq-\frac{1}{p}\\ t^r x^r|\ln x-\ln t|, \text{ for } r=-\frac{1}{p}\end{cases}\\ \text{if } f'\ell-rf\in L_p[a,b],\\[2ex] \|f'\ell-rf\|_1\frac{t^r x^r}{\min\{x^{r+1},t^{r+1}\}}\end{cases}$$

or, equivalently

$$\left|\frac{f(x)}{x^r}-\frac{f(t)}{t^r}\right| \tag{2.3}$$
$$\leq \begin{cases} \frac{1}{|r|}\|f'\ell-rf\|_\infty \left|\frac{1}{x^r}-\frac{1}{t^r}\right|, \ \textit{if } f'\ell-rf\in L_\infty[a,b], \\ \\ \|f'\ell-rf\|_p \\ \times \begin{cases} \frac{1}{|1-q(r+1)|}\left|\frac{1}{x^{1-q(r+1)}}-\frac{1}{t^{1-q(r+1)}}\right|, \ \textit{for } r\neq -\frac{1}{p} \\ \\ |\ln x-\ln t|, \ \textit{for } r=-\frac{1}{p} \end{cases} \\ \textit{if } f'\ell-rf\in L_p[a,b], \\ \\ \|f'\ell-rf\|_1 \frac{1}{\min\{x^{r+1},t^{r+1}\}}, \end{cases}$$

where $p>1, \frac{1}{p}+\frac{1}{q}=1.$

Proof. If f is absolutely continuous, then $f/(\cdot)^r$ is absolutely continuous on the interval $[a,b]$ and

$$\int_t^x \left(\frac{f(s)}{s^r}\right)' ds = \frac{f(x)}{x^r}-\frac{f(t)}{t^r}$$

for any $t,x\in[a,b]$ with $x\neq t$.

Since

$$\int_t^x \left(\frac{f(s)}{s^r}\right)' ds = \int_t^x \frac{f'(s)s^r-rs^{r-1}f(s)}{s^{2r}}ds = \int_t^x \frac{f'(s)s-rf(s)}{s^{r+1}}ds,$$

then we get the following identity

$$t^r f(x)-x^r f(t) = x^r t^r \int_t^x \frac{f'(s)s-rf(s)}{s^{r+1}}ds \tag{2.4}$$

for any $t,x\in[a,b]$.

Taking the modulus in (2.4) we have

$$\begin{aligned} |t^r f(x)-x^r f(t)| &= x^r t^r \left|\int_t^x \frac{f'(s)s-rf(s)}{s^{r+1}}ds\right| \\ &\leq x^r t^r \left|\int_t^x \frac{|f'(s)s-rf(s)|}{s^{r+1}}ds\right| := I \end{aligned} \tag{2.5}$$

and utilizing Hölder's integral inequality we deduce

$$I \leq x^r t^r \begin{cases} \sup_{s\in[t,x]([x,t])} \left|f'(s)\, s - rf(s)\right| \left|\int_t^x \frac{1}{s^{r+1}} ds\right|, \\ \left|\int_t^x \left|f'(s)\, s - rf(s)\right|^p ds\right|^{1/p} \left|\int_t^x \frac{1}{s^{q(r+1)}} ds\right|^{1/q}, \\ \left|\int_t^x \left|f'(s)\, s - rf(s)\right| ds\right| \sup_{s\in[t,x]([x,t])} \left\{\frac{1}{s^{r+1}}\right\}, \end{cases} \tag{2.6}$$

$$\leq x^r t^r \begin{cases} \frac{1}{|r|} \left\|f'\ell - rf\right\|_\infty \left|\frac{1}{x^r} - \frac{1}{t^r}\right|, \\ \left\|f'\ell - rf\right\|_p \\ \quad \times \begin{cases} \frac{1}{|1-q(r+1)|} \left|\frac{1}{x^{1-q(r+1)}} - \frac{1}{t^{1-q(r+1)}}\right|, r \neq -\frac{1}{p} \\ \left|\ln x - \ln t\right|, r = -\frac{1}{p}, \end{cases} \\ \left\|f'\ell - rf\right\|_1 \frac{1}{\min\{x^{r+1}, t^{r+1}\}}, \end{cases}$$

where $p > 1$, $\frac{1}{p} + \frac{1}{q} = 1$, and the inequality (2.2) is proved. □

3. Some Ostrowski type results

The following new result also holds.

Theorem 3.1. *Let $f : [a, b] \to \mathbb{C}$ be an absolutely continuous function on the interval $[a, b]$ with $b > a > 0$. If $r \in \mathbb{R}$, $r \neq 0$, and $f'\ell - rf \in L_\infty[a, b]$, then for any $x \in [a, b]$ we have*

$$\left|\frac{b^{r+1} - a^{r+1}}{r+1} f(x) - x^r \int_a^b f(t)\, dt\right| \tag{3.1}$$

$$\leq \frac{1}{|r|} \left\|f'\ell - rf\right\|_\infty \times \begin{cases} \frac{2rx^{r+1} - x^r(a+b)(r+1) + b^{r+1} + a^{r+1}}{r+1}, & \text{if } r > 0 \\ \frac{x^r(a+b)(r+1) - 2rx^{r+1} - b^{r+1} - a^{r+1}}{r+1}, & \text{if } r \in (-\infty, 0) \setminus \{-1\}. \end{cases}$$

Also, for $r = -1$, we have

$$\left|f(x) \ln\frac{b}{a} - \frac{1}{x}\int_a^b f(t)\, dt\right| \leq 2\left\|f'\ell + f\right\|_\infty \left(\ln\frac{x}{\sqrt{ab}} + \frac{\frac{a+b}{2} - x}{x}\right) \tag{3.2}$$

for any $x \in [a, b]$, provided $f'\ell + f \in L_\infty[a, b]$.

The constant 2 in (3.2) is best possible.

Proof. Utilising the first inequality in (2.2) for $r \neq -1$ we have

$$\left|\frac{b^{r+1}-a^{r+1}}{r+1} f(x) - x^r \int_a^b f(t)\,dt\right| \leq \int_a^b |t^r f(x) - x^r f(t)|\,dt \leq \frac{1}{|r|} \|f'\ell - rf\|_\infty \int_a^b |t^r - x^r|\,dt. \tag{3.3}$$

Observe that

$$\int_a^b |t^r - x^r|\,dt = \begin{cases} \int_a^x (x^r - t^r)\,dt + \int_x^b (t^r - x^r)\,dt, & \text{if } r > 0, \\ \int_a^x (t^r - x^r)\,dt + \int_x^b (x^r - t^r)\,dt, & \text{if } r \in (-\infty, 0) \setminus \{-1\}. \end{cases}$$

Then for $r > 0$ we have

$$\begin{aligned} &\int_a^x (x^r - t^r)\,dt + \int_x^b (t^r - x^r)\,dt \\ &= x^r (x-a) - \frac{x^{r+1} - a^{r+1}}{r+1} + \frac{b^{r+1} - x^{r+1}}{r+1} - x^r (b-x) \\ &= 2x^{r+1} - x^r (a+b) + \frac{b^{r+1} + a^{r+1} - 2x^{r+1}}{r+1} \\ &= \frac{2rx^{r+1} + 2x^{r+1} - x^r (a+b)(r+1) + b^{r+1} + a^{r+1} - 2x^{r+1}}{r+1} \\ &= \frac{2rx^{r+1} - x^r (a+b)(r+1) + b^{r+1} + a^{r+1}}{r+1} \end{aligned}$$

and for $r \in (-\infty, 0) \setminus \{-1\}$ we have

$$\begin{aligned} &\int_a^x (t^r - x^r)\,dt + \int_x^b (x^r - t^r)\,dt \\ &= -\frac{2rx^{r+1} - x^r (a+b)(r+1) + b^{r+1} + a^{r+1}}{r+1}. \end{aligned}$$

Making use of (3.3) we get (3.1).

Utilizing the inequality (2.2) for $r = -1$ we have

$$\left|t^{-1} f(x) - x^{-1} f(t)\right| \leq \|f'\ell + f\|_\infty \left|t^{-1} - x^{-1}\right|$$

if $f'\ell + f \in L_\infty [a,b]$.

Integrating this inequality, we have

$$\left|f(x) \ln \frac{b}{a} - x^{-1} \int_a^b f(t)\,dt\right| \leq \int_a^b \left|t^{-1} f(x) - x^{-1} f(t)\right| dt \leq \|f'\ell + f\|_\infty \int_a^b \left|t^{-1} - x^{-1}\right| dt. \tag{3.4}$$

Since

$$\int_a^b \left|\frac{1}{x}-\frac{1}{t}\right| dt = \left[\int_a^x \left(\frac{1}{t}-\frac{1}{x}\right) dt + \int_x^b \left(\frac{1}{x}-\frac{1}{t}\right) dt\right]$$
$$= \left(\ln\frac{x}{a} - \frac{x-a}{x} + \frac{b-x}{x} - \ln\frac{b}{x}\right)$$
$$= \ln\frac{x^2}{ab} + \frac{a+b-2x}{x}$$
$$= 2\left(\ln\frac{x}{\sqrt{ab}} + \frac{\frac{a+b}{2}-x}{x}\right),$$

then by (3.4) we get the desired inequality (3.2).

Now, assume that (3.2) holds with a constant $C>0$, i.e.

$$\left|f(x)\ln\frac{b}{a} - x^{-1}\int_a^b f(t)\,dt\right| \le C\left\|f'\ell + f\right\|_\infty \left(\ln\frac{x}{\sqrt{ab}} + \frac{\frac{a+b}{2}-x}{x}\right) \tag{3.5}$$

for any $x\in[a,b]$.

If we take in (3.5) $f(t)=1, t\in[a,b]$, then we get

$$\left|\ln\frac{b}{a} - \frac{b-a}{x}\right| \le C\left(\ln\frac{x}{\sqrt{ab}} + \frac{\frac{a+b}{2}-x}{x}\right) \tag{3.6}$$

for any for any $x\in[a,b]$.

Making $x=a$ in (3.5) produces the inequality

$$\left|\ln\frac{b}{a} - \frac{b-a}{a}\right| \le C\left(\frac{b-a}{2a} - \frac{1}{2}\ln\frac{b}{a}\right)$$

which implies that $C\ge 2$.

This proves the sharpness of the constant 2 in (3.2). □

Remark 3.2. Consider the r-Logarithmic mean

$$L_r = L_r(a,b) := \left[\frac{b^{r+1}-a^{r+1}}{(r+1)(b-a)}\right]^{1/r}$$

defined for $r\in\mathbb{R}\setminus\{0,-1\}$ and the Logarithmic mean, defined as

$$L = L(a,b) := \frac{b-a}{\ln b - \ln a}.$$

If $A=A(a,b):=\frac{a+b}{2}$, then from (3.1) we get for $x=A$ the inequality

$$\left|L_r^r(b-a)f(A) - A^r\int_a^b f(t)\,dt\right| \tag{3.7}$$
$$\le \frac{2}{|r|}\left\|f'\ell - rf\right\|_\infty \begin{cases} \frac{A\left(b^{r+1},a^{r+1}\right)-A^{r+1}}{r+1}, & \text{if } r>0,\\[2mm] \frac{A^{r+1}-A\left(b^{r+1},a^{r+1}\right)}{r+1}, & \text{if } r\in(-\infty,0)\setminus\{-1\}, \end{cases}$$

while from (3.2) we get

$$\left|L^{-1}(b-a) f(A)-A^{-1} \int_a^b f(t)\, dt\right| \leq 2\left\|f^{\prime} \ell+f\right\|_{\infty} \ln \frac{A}{G}. \tag{3.8}$$

The following related result holds.

Theorem 3.3. *Let $f:[a, b] \rightarrow \mathbb{C}$ be an absolutely continuous function on the interval $[a, b]$ with $b>a>0$. If $r \in \mathbb{R}$, $r \neq 0$, then for any $x \in[a, b]$ we have*

$$\begin{aligned} & \left|\frac{f(x)}{x^r}(b-a)-\int_a^b \frac{f(t)}{t^r} dt\right| \\ & \leq \frac{1}{|r|}\left\|f^{\prime} \ell-r f\right\|_{\infty} \\ & \times \begin{cases}\frac{2 x^{1-r}-a^{1-r}-b^{1-r}}{1-r}+\frac{1}{x^r}(b+a-2 x), & r \in(0, \infty) \diagdown\{1\} \\ \\ \frac{a^{1-r}+b^{1-r}-2 x^{1-r}}{1-r}+\frac{1}{x^r}(2 x-a-b), & \text{if } r<0.\end{cases}\end{aligned} \tag{3.9}$$

Also, for $r=1$, we have

$$\left|\frac{f(x)}{x}(b-a)-\int_a^b \frac{f(t)}{t} dt\right| \leq 2\left\|f^{\prime} \ell-f\right\|_{\infty}\left(\ln \frac{x}{\sqrt{a b}}+\frac{\frac{a+b}{2}-x}{x}\right) \tag{3.10}$$

for any $x \in[a, b]$, provided $f^{\prime} \ell-f \in L_{\infty}[a, b]$.

The constant 2 is best possible in (3.10).

Proof. From the first inequality in (2.3) we have

$$\left|\frac{f(x)}{x^r}-\frac{f(t)}{t^r}\right| \leq \frac{1}{|r|}\left\|f^{\prime} \ell-r f\right\|_{\infty}\left|\frac{1}{x^r}-\frac{1}{t^r}\right|, \tag{3.11}$$

for any $t, x \in[a, b]$, provided $f^{\prime} \ell-r f \in L_{\infty}[a, b]$.

Integrating over $t \in[a, b]$ we get

$$\begin{aligned} \left|\frac{f(x)}{x^r}(b-a)-\int_a^b \frac{f(t)}{t^r} dt\right| & \leq \int_a^b\left|\frac{f(x)}{x^r}-\frac{f(t)}{t^r}\right| dt \\ & \leq \frac{1}{|r|}\left\|f^{\prime} \ell-r f\right\|_{\infty} \int_a^b\left|\frac{1}{x^r}-\frac{1}{t^r}\right| dt\end{aligned} \tag{3.12}$$

for $r \in \mathbb{R}$, $r \neq 0$.

For $r \in (0, \infty) \setminus \{1\}$ we have

$$\begin{aligned}
&\int_a^b \left| \frac{1}{x^r} - \frac{1}{t^r} \right| dt \\
&= \int_a^x \left(\frac{1}{t^r} - \frac{1}{x^r} \right) dt + \int_x^b \left(\frac{1}{x^r} - \frac{1}{t^r} \right) dt \\
&= \frac{x^{1-r} - a^{1-r}}{1-r} - \frac{1}{x^r}(x-a) + \frac{1}{x^r}(b-x) - \frac{b^{1-r} - x^{1-r}}{1-r} \\
&= \frac{2x^{1-r} - a^{1-r} - b^{1-r}}{1-r} + \frac{1}{x^r}(b+a-2x)
\end{aligned}$$

for any $x \in [a, b]$.

For $r < 0$, we also have

$$\int_a^b \left| \frac{1}{x^r} - \frac{1}{t^r} \right| dt = \frac{a^{1-r} + b^{1-r} - 2x^{1-r}}{1-r} + \frac{1}{x^r}(2x - a - b)$$

for any $x \in [a, b]$.

For $r = 1$ we have

$$\int_a^b \left| \frac{1}{x} - \frac{1}{t} \right| dt = 2 \left(\ln \frac{x}{\sqrt{ab}} + \frac{\frac{a+b}{2} - x}{x} \right)$$

for any $x \in [a, b]$, and the inequality (3.10) is obtained.

The sharpness of the constant 2 follows as in the proof of Theorem 3.1 and the details are omitted. □

Remark 3.4. If we take $x = A$ in Theorem 3.3, then we we have

$$\begin{aligned}
&\left| \frac{f(A)}{A^r}(b-a) - \int_a^b \frac{f(t)}{t^r} dt \right| && (3.13) \\
&\leq \frac{2}{|r|} \|f'\ell - rf\|_\infty \begin{cases} \frac{A^{1-r} - A\left(a^{1-r}, b^{1-r}\right)}{1-r}, & r \in (0, \infty) \setminus \{1\}, \\[2ex] \frac{A\left(a^{1-r}, b^{1-r}\right) - A^{1-r}}{1-r}, & \text{if } r < 0. \end{cases}
\end{aligned}$$

Also, for $r = 1$, we have

$$\left| \frac{f(A)}{A}(b-a) - \int_a^b \frac{f(t)}{t} dt \right| \leq 2 \|f'\ell - f\|_\infty \ln \frac{A}{G}. \tag{3.14}$$

Remark 3.5. The interested reader may obtain other similar results in terms of the p-norms $\|f'\ell - rf\|_p$ with $p \geq 1$. However, since some calculations are too complicated, the details are not presented here.

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