Stud. Univ. Babeş-Bolyai Math. 58(2013), No. 4, 539–543

# Book reviews

**Will H H. Moore & David A. Siegel, A Mathematical Course for Political & and Social Research**, Princeton University Press, Princeton and Oxford, 2013, xix+430 pp, ISBN 978-0-691-15995-9 (hardback), ISBN 978-0-691-15917-1 (paperback).

The aim of the present book is to introduce the political scientists to some mathematical tools used in their discipline. In spite of its abstract character, mathematics helps them to develop rigorous theories based on the observed data and phenomena and, at the same time, gives them rigorous tests on the implications of developed theories. As the book is addressed to an audience with little prior knowledge of mathematics (usually at the high school level), the formal mathematics rigor is sacrificed in the favor of intuition, compensated by some footnotes and comments providing formal formalism. The book is divided into five main parts: I. *Building blocks*, II. *Calculus in one dimension*, III. *Probability,* IV. *Linear algebra*, and V. *Multivariate calculus and optimization.*

The first part is concerned with basic tools used in mathematics: notation, basic results on computation in arithmetic and algebra, functions, relations and utility (closely related to the subject matter of the book). This part ends with some notions from calculus – sequences and series, continuous functions – completed in the second part with differentiation and integration of functions of one variable, with applications to extrema. The third part of the book is concerned with an important topic for political sciences, namely probability theory. Although statistics is an essential tool in political sciences, the stretch is here on probability, including only a brief discussion on statistical inference. The fourth part discusses vector, matrices, vector spaces, and ends with a brief discussion of some more advanced topics – eigenvalues and Markov chains. The last part of the book contains some results on calculus in several variables with applications to optimization – unconstrained, and constrained both with equality and inequality constraints.

The text is completed with many worked examples, exercises, and applications to various topics in political and social sciences. Written in an intuitive and accessible way, this book can be used as a primer for math novices in the social sciences as well as a handy reference for the researchers in this area.

Nicolae Popovici

**Igor Kriz and Aleš Pultr, Introduction to Mathematical Analysis**, Birkhäuser-Springer, Basel, 2013, ISBN 978-3-0348-0635-0; ISBN 978-3-0348-0636-7 (eBook); DOI 10.1007/978-3-0348-0636-7, xx+510 pp.

As the authors mention in the Preface, their aim is "to write a book which the students may want to keep after the course is over, and which could serve them as a bridge to higher mathematics". With this aim in mind the authors included in their book some topics from topology, calculus of real functions of one and several real variables, elements of complex analysis, some differential and Riemannian geometry, elements of functional analysis, as well as some applications.

Some basic tools from topology, viewed as a background of the whole analysis (understood in a large sense), are treated in Chapters 2 and 7, *Metric and topological spaces*, I and II, respectively. Calculus of real functions of one or several real variables is treated in chapters 1. *Preliminaries*, 3. *Multivariable differential calculus,* 4. *Integration I: Multivariable Riemann integral and basic ideas toward the Lebesgue integral,* 5. *Integration II: Measurable functions, measure and the techniques of Lebesgue integration,* 8. *Line integrals and Green's theorem.* The authors treat first Riemann's integral and then the Lebesgue integral is introduced via Daniel's method. The basics of complex analysis are developed in Chapters 10. *Complex analysis I: Basic concepts,* and 13. *Complex analysis II: Further topics.* The chapters concerned with differential geometry are: 12. *Smooth manifolds, differential forms and Stokes' theorem,* and 14. *Tensor calculus and Riemannian geometry.* Two final chapters are devoted to some results from functional analysis: 16. *Banach and Hilbert spaces: Elements of functional analysis,* and 17. *A few applications of Hilbert spaces,* (including a Hilbert space proof of the Radon-Nikodym theorem). Other applications included in the book are to differential equations, in Chapters 6. *Systems of ordinary differential equations,* and 7. *Systems of linear differential equations,* and to calculus of variations in Chapter 14. *Calculus of variations and the geodesic equation.*

Some supplementary material is included in Chapter 11. *Multilinear algebra* (tensor products and the exterior Grassmann algebra are presented by the means of homological algebra), and in two appendices, A. *Linear algebra I: Vector spaces*, and B. *Linear algebra II: More about matrices.*

Each chapter ends with a set of well chosen exercises completing the main text. Treating in a unified and coherent way several topics from mathematical analysis, both real and complex, differential and Riemannian geometry, functional analysis, and their applications, the present well written book is a valuable addition to the existing ones on similar topics. It can be used by graduate students in mathematics and researchers in mathematics and other areas (physics, chemistry, economics) to find a rigorous foundations and details on several topics in analysis. The instructors can recommend the book as a supplementary material for their courses.

S. Cobzaş

**Niels Lauritzen, Undergraduate Convexity – From Fourier and Motzkin to Kuhn and Tucker**, World Scientific, London - Singapore - Beijing, 2013, xiv + 283 pages, ISBN: 978-981-4412-51-3 and 978-981-4452-76-2 (pbk).

As the author says in the Preface – "Convexity is a key concept in modern mathematics with rich applications in economics and optimization". The aim of this book is to present at an elementary level (the prerequisites are some familiarity with calculus and linear algebra) the basic results on convexity in the finite dimensional setting, i.e. in the space $\mathbb{R}^n$. The book can be divided into three main parts – Chapters 1–6 are devoted to convex sets, Chapters 7–9 to convex functions, and the last chapter, Chapter 10, *Convex optimization*, to applications. In spite of its elementary level some consistent applications are included as well. A special attention is paid to the algorithmic questions as, e.g., to find whether a point belongs to the convex hull of a finite set of points.

The first two chapters of the book present some results on Fourier-Motzkin elimination method (a generalization of Gauss' method) to solve systems of linear inequalities and some results on affine spaces and subspaces, affine independence, affine hulls.

The study of convexity starts in the third chapter, 3. *Convex subsets*, with some elemenatray properties, convex hulls (Carathéodori's theorem), faces, extreme points, and a presentation of an algorithm, based on Carathéodori's theorem and on Bland's rule from the simplex method, to decide if a point is in the convex hull of a finite subset of $\mathbb{R}^d$.

Chapter 4, *Polyhedra*, is devoted to this important class of convex sets. Applications are given to Farkas' lemma and Gordan's theorem, Markov chains, doubly stochastic matrices, and to Hall's marriage problem. This study is continued in Chapter 5, *Computation with polyhedra*, where two important algorithms – the double description method for polyhedra and the simplex algorithm – are presented. Other properties of the convex subsets of $\mathbb{R}^d$, as the existence and characterization of projections onto closed convex sets, supporting hyperplanes, separation of convex sets, are treated in the sixth chapter, *Closed convex subsets and separating hyperplanes*, the last of the first part.

The study of convex functions begins in Chapter 7, *Convex functions*, with convex functions of one variable (continuity and differentiability properties, local minima), the case of functions of several variables being postponed to Chapter 9, *Convex functions of several variables.* For reader's convenience a chapter, 8. *Differentiable functions of several variables,* contains a presentation (with full proofs) of basic results from the differential calculus for vector functions. Note that the corresponding results for functions of one variable were proved in the seventh chapter as well. Chapter 9 contains characterizations of the convexity of differentiable functions of several variables in terms of the monotony of the first differential and positivity of the second differential. For this last result nice and simple proofs of Sylvester's criterium of the positive definiteness of matrices, as well as of the more delicate question of positive semidefiniteness, are included . This chapter contains also a discussion on the spectral properties of symmetric matrices and a study of quadratic forms (Sylvester's law of inertia).

As we yet mentioned the last chapter of the book is dedicated to applications – Karush-Kuhn-Tuccker optimality conditions, Lagrangians and saddle points, duality.

Two appendices, A. *Analysis,* and B. *Linear (in)dependence and the rank of a matrix,* complete the main text with some useful notions and results.

The book, based on one quarter courses, *Konvekse Mængder* and *Konvekse Funktioner*, taught for several years to undergraduate students in mathematics, economics and computer science at Aarhus University, is didactically written in a pleasant and alive style, with careful motivation of the considered notions, illuminating examples and pictures, and relevant historical remarks. The front cover contains a picture of Johan Ludvig William Valdemar Jensen, the creator of convex functions, some quotations from his papers being included in the book. As a matter of fact, Jensen worked as a telephone engineer in København and never acquired a formal degree in mathematics or held an academic position, and has done mathematics for its beauty and his own enjoyment. The author dedicates this book to him "as a tribute to the joyful and genuine pursuit of mathematics". All in all, this is a remarkable book, a readable and attractive introduction to the multi-faced domain of convexity and its applications.

Nicolae Popovici

**Ioannis Farmakis and Martin Moskowitz, Fixed Point Theorems and Their Applications**, World Scientific, London - Singapore - Beijing, 2013, xi + 234 pages, ISBN: 978-981-4458-91-7.

The book presents some classical fixed point theorems, with emphasis on those with an algebraic or geometric flavor and their applications. For instance, Brouwer's fixed point theorem, proved in the first chapter via Milnor's approach, is applied to the existence of positive eigenvalues and positive proper vectors of positive matrices (the Perron-Frobenius theorem) and to a glimpse of Google research engine. Similarly, in Chapter 2, *Fixed point theorems in analysis,* Schauder-Tychonoff's fixed point theorem is applied to Peano's existence theorem for differential equations. This chapter contains also a proof of the Markov-Kakutani fixed point theorem for families of affine mappings, with applications to Lie and amenable groups.

Chapter 3, *The Lefcshetz fixed point theorem,* presents the algebraic-topological fixed point theorem of S. Lefschetz, including a brief discussion on manifolds, Lie groups, transversality and a proof of the Atiyah and Singer fixed point theorem that led them to the Fields Medal index theorem for elliptic operators.

Chapter 4, *Fixed point theorems in geometry,* is devoted to the fixed point theorem of E. Cartan on compact groups of isomorphisms on Hadamard manifolds. and to the theorems of Preissmann and Weinstein on fixed points on manifolds with negative, respectively positive, curvature.

Chapter 5, *Fixed point theorems of volume preserving maps,* starts with a proof of Poincaré's recurrence theorem for volume preserving maps and includes also fixed point theorem in symplectic geometry, a discussion on Arnold's conjecture on the number of fixed points of maps on such manifolds, Poincaré's last geometric theorem on fixed points on tori, and a study of Anosov diffeomorphisms. The chapter concludes with a presentation of Lefschetz zeta function and its applications. Chapter 6, *Borel's fixed point theorem in algebraic geometry*, treats Borel's fixed point theorem for solvable algebraic groups acting on a complex projective variety.

The seventh chapter, *Miscellaneous fixed point theorems*, is concerned with applications to number theory (little Fermat's theorem and Fermat's two square theorem), Jordan's theorem on fixed points of finite groups of transformations and a fixed point for the holomorphic mappings in the unit disc, due to the second named author. The last chapter of the book, Chapter 8, *A fixed point theorem in set theory*, contains a proof of Knaster-Tarski theorem on fixed points of order preserving functions on Banach lattices with application to Schröder-Cantor-Bernstein theorem from set theory.

The book presents interest mainly by some more special fixed point theorems in algebraic topology, algebraic geometry, and differential and symplectic geometry, as well as by the interesting applications of fixed point results to various areas of mathematics. Written in a way that the chapters can be used independently, it appeals to a large audience.

Adrian Petruşel