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Book reviews

Francis Clarke, Functional Analysis, Calculus of Variations and Optimal Control, Graduate Texts in Mathematics 264, Springer, London - Heidelberg - New York - Dordrecht, 2013, ISBN 978-1-4471-4819-7; ISBN 978-1-4471-4820-3 (eBook); DOI 10.1007/978-1-4471-4820-3, xiv+591 pp.

The book consists of four parts: I. *Functional Analysis*, II. *Optimization and Nonsmooth Analysis*, III. *Calculus of Variations*, and IV. *Optimal Control*. Treating apparently disparate topics, these parts are, in fact, strongly interconnected, each topic being treated in a way that makes it of use for the remaining, and making use of the preceding ones.

The first part can be considered as an introduction to some topics in functional analysis (although some familiarity with basic functional analysis and real analysis is assumed), mainly those useful in the next three parts. To this end, besides some standard material on normed spaces, continuous linear functionals and operators, some topics useful in optimization and calculus of variations, as derivatives, tangents and normals, convex functions and convex analysis, perturbed minimization principles (Ekeland's variational principle) and regularity, are included. Besides this, some proofs are given using optimization methods. For instance, the open mapping theorem is proven via a Decrease Principle, which, in its turn, is a consequence of Ekeland Variational Principle. Some regularity problems are discussed as well, including Graves-Lyusternik theorem. As application, the direct method of the calculus of variations is illustrated by a result on the existence of minima of convex lsc functions defined on convex subsets of reflexive Banach spaces. The direct method will be systematically exploited in the next chapters to prove the existence of solutions in calculus of variations and optimal control problems. Lebesgue spaces $L^p(\Omega)$, $\Omega \subset \mathbb{R}^n$, are treated in detail - duality and reflexivity (via uniform rotundity in the case $1 < p < \infty$ and directly for $p = 1$), weak compactness. Measurable multifunctions, measurable selections and semicontinuity properties of integral functionals, and some weak closure results in L^1 related to differential inclusions are considered as well. This part ends with a short introduction to Hilbert spaces, including a smooth minimization principle and proximal subdifferentials, with applications to dense Fréchet differentiability of convex functions and to Moreau-Yosida approximation.

The second part starts with a discussion, motivated by examples, on the main problems in optimization theory and on various techniques used for their solving ((mainly in the convex case) - deductive and inductive methods, multipliers and Lagrange type results, Kuhn-Tucker theorem. All this discussion leads to the conclusion of the need for a nonsmooth calculus whose development starts in Chapter 10 with the study of generalized gradients for locally Lipschitz functions defined on Banach spaces, the corresponding subdifferentials and the calculus rules. The notion of generalized gradient was introduced by the author and exposed in the book, F. H. Clarke, *Optimization and Nonsmooth Analysis*, Wiley-Interscience, New York, 1983 (republished as vol. 5 of Classics in Applied Mathematics, SIAM, 1990), a breakthrough in the study of optimization problems. For further developments, see F. H. Clarke, Yu. S. Ledyaev, R. J. Stern, and P. R.Wolenski, *Nonsmooth Analysis and Control Theory*, GMT vol. 178, Springer-Verlag, New York, 1998. This study continues in the next chapter with the consideration of proximal subgradients and proximal calculus, Dini and viscosity subdifferentials. For both of these two types of subdifferentials one proves multiplier rules. The second part ends with a chapter, Ch. 12, *Invariance and monotonicity,* dealing with flow invariance for differential inclusions of the form $x'(t) \in F(x(t)),$ a.e. $t \in [a, b]$, where $x : [a, b] \to \mathbb{R}^n$ is absolutely continuous and F is a multifunction from $\mathbb{R}^n$ to $\mathbb{R}^n$.

The third part is devoted to the calculus of variations, modeled on the study of the minimization of the functional $J(x) = \int_a^b \Lambda(t, x(t), v(t))dt$ over a convenable class of functions. In the first chapter of this part, Ch. 14, *The classical theory*, one supposes that $\Lambda : \mathbb{R}^3 \to \mathbb{R}$ is twice differentiable and $x \in C^2[a, b]$. The first chapter of this part (Ch. 14, *The classical theory*) is concerned mainly with necessary conditions, a subject that is over than three hundred years old which is presented in historical perspective, emphasizing the contributions of some great mathematicians - Euler, Lagrange, Legendre, Jacobi, Tonelli, and others. The theory is extended in the next chapter to the class of Lipschitz functions, and to absolutely continuous functions (the natural framework to study the problem) in Chapter 16. A general multiplier rule is proved in Chapter 17 with application to the isoperimetric problem. One considers also nonsmooth Lagrangians and Hamilton-Jacobi methods. In the last chapter of this part, Ch. 20, *Multiple integrals*, the interval $[a, b]$ is replaced by a nonempty open bounded subset Ω of $\mathbb{R}^n$, and the solutions are discussed first in the classical context (that is for $x \in C^2(\bar{\Omega})$), and then one considers Lipschitz solutions and solutions in Sobolev space.

The last part of the book is devoted to optimal control, a natural generalization of the calculus of variations, having as a central topic Pontryagin's maximum principle. This very important result is presented gradually and from different angles, starting with the classical Maximum Principle proved by Pontryagin a.o. around 1960. The author presents several variants and extensions of this principle, culminating with a very general one involving differential inclusions, from which one deduces all previous principles, but which has an intrinsic interest too. No easy proof of this principle is known. Although, not easy too, this new approach gives a full, more streamlined, more unified and self contained treatment of this difficult matter. Existence and regularity and the use of inductive methods to check presumably solutions of control problems are discussed as well.

There are a lot of exercises spread throughout the book, completing the main text with further results and examples. Besides these, each part ends with a set of additional, more demanding exercises (most of them original). Full, partial, or hints only solutions for some of them are given in the endnotes.

Each of the four parts the book, or selections of chapters, can be used for courses on different topics. Some variants, experienced by the author himself, are suggested in the Preface.

Written by an expert in the field, with outstanding contributions to nonsmooth analysis, calculus of variations and optimal control, the present book, written in a live but rigorous style, will help the interested people to a smooth approach and a better understanding of this difficult subject in mathematics, both pure and applied, which is optimal control.

S. Cobzaş

Mikhail Popov and Beata Randrianantoanina, Narrow Operators on Function Spaces and Vector Lattices, Studies in Mathematics, Vol. 45, xiii + 319 pp, Walter de Gruyter, Berlin - New York, 2013, ISBN: 978-3-11-026303-9, e-ISBN: 978-3-11-026334-3, ISSN: 0179-0986.

By an F-space one understands a complete metric linear space. For an atomless finite measure space (Ω, Σ, μ) one denotes by $L_0(\mu)$ the space of equivalence classes of real- or complex-valued μ-measurable functions and let $\Sigma^+ = \{A \in \Sigma : \mu(A) > 0\}$. A Köthe F-space is a subspace E of $L_0(\mu)$ such that $(\mathrm{K}_i)\;\; y \in E$ and $|x| \leq |y| \;\Rightarrow\; y \in E$ and $\|x\| \leq \|y\|$, and $(\mathrm{K}_{ii})\;\; 1_\Omega \in E$. If further E is Banach and $(\mathrm{K}_{iii})\;\; E \subset L_1(\mu)$, then E is called a Köthe-Banach space. A *sign* is a measurable function on Ω taking only the values $0, \pm 1$. A sign x is called mean zero if $\int_\Omega x d\mu = 0$. If supp $(x) = A \in \Sigma$, then x is called a sign on A. A continuous linear operator T form a Köthe F-space E to an F-space X is called narrow if for every $A \in \Sigma^+$ and every $\varepsilon > 0$ there exists a sign x of mean 0 on A such that $\|Tx\| < \varepsilon$. Although narrow operators were considered and used under different names by J. Bourgain (1981) and H. P. Rosenthal (1981-1984), they were formally defined and systematically studied by A. Plichko and M. Popov in Dissertationes Mathematicae vol. 306 (1990). Narrow operators turned to be a very important class of continuous linear operators with applications to the study of Banach and quasi-Banach spaces and non-locally convex spaces. The aim of the present book is to give a comprehensive presentation of the modern theory of narrow operators, including very recent results (some available only as preprints, or published for the first time), defined on function spaces and vector lattices. The authors with their collaborators have important contributions to the domain which are included in the book.

The book is divided into twelve chapters. The first one contains preliminary results on F-spaces, Köthe function spaces, vector lattices, measure theory (Maharam theorem), the definition of narrow operators and their initial properties. The term "small" used in Chapter 2, *Each "small" operator is narrow*, refers to compact or AM-operators (send order bounded sets to compact ones), Dunford-Pettis operators, strictly singular operators (an operator $T \in \mathcal{L}(E, X)$ is called Z-strictly singular if it does not preserve an isomorphic copy of any subspace X_0 of E isomorphic to Z). Every narrow operator is strictly singular, but a still open problem, posed by Plichko and Popov in the mentioned paper, is whether or not is every strictly singular operator narrow. Chapter 3 contains some applications of narrow operators to operators on non-locally convex spaces (as e.g., the isomorphic classification of a class of Köthe F-spaces, quotients of $L_p(\mu)$, $0 < p < 1$). In Chapter 4, *Noncompact narrow operators*, it is shown that there exists spaces with non compact narrow operators and one gives a characterization on narrow expectation operators. In Chapter 5 spectral properties and numerical radii for narrow operators are studied, as well as some ideal properties of them - ST is narrow if T is but it could be not narrow if only S is narrow, so the narrow operators form only a left ideal. Chapter 6 is concerned with Daugavet-type properties of narrow operators T (i.e. the equality $\|I + T\| = 1 + \|T\|$) acting on Lebesgue or Lorentz spaces. Plichko and Popov, *loc cit*, have shown that any narrow operator has the Daugavet property. Here one gives an extension of this result.

Chpater 7, *Strict singularity versus narrowness*, is concerned with the problem mentioned above of narrowness of strictly singular operators. This chapter contains some very deep results on the narrowness of some classes of strictly singular operators: ℓ_1- as well L_1-strictly singular operators on L_1 (Bourgain and Rosenthal, (1983) and Rosenthal (1984)), L_p-strictly singular operators on L_p, $1 < p < 2$, (Johnson, Maurey, Schechtman, and Tzafriri, Memoirs AMS, 1979).

In Chapter 8, *Weak embeddings of* L_1, one discusses several types of embeddings – semi-embeddings, G_δ-embeddings and sign-embeddings. The important result of Talagrand, giving a negative answer to the three-space problem for isomorphic embeddings of L_1, which asserts that there exists a subspace Z of L_1 such that neither Z nor L_1/Z contains an isomorph of L_1, is included.

Chapter 9, *Spaces X for which every operator* $T \in \mathcal{L}(L_p, X)$ *is narrow,* contains characterizations of these spaces (for instance, in terms of ranges of vector measures) and some particular spaces for which this is true are emphasized as, for instance, $\mathcal{L}(E, c_0(\Gamma))$, $\mathcal{L}(L_p, L_r)$ for $1 \leq p < 2$ and $p < r < \infty$. In Section 9.5 one gives a partial answer to the problem concerning strictly singular operators – every ℓ_2-strictly singular operator from $\mathcal{L}(L_p, X)$ with $1 < p < \infty$ and X Banach with an unconditional basis is narrow.

Chapter 10, *Narrow operators on vector lattices,* is concerned with an extension of narrowness to vector lattices as given in a paper by Maslyuchenko, Mykhaylyuk and Popov, Positivity (2009). In Chapter 11, some variants of the notion of narrowness are briefly discussed. Among them, one used by V. Kadets, R. Shvidkoy and D. Werner in the study of Daugavet property.

There are 28 research problems spread through the book. For the convenience of the reader these are collected in the last chapter, Chapter 12, with references to the places where they occurred and bibliographical references. The book ends with a complete bibliography of 142 titles, a Name Index and a Subject Index.

The book is clearly written with full proofs, most of which, in spite of the fact that some have been simplified by the authors or by colleagues of them, still remain long and involved. The book includes topics which are of great interest for researchers in functional analysis, mainly in Banach and quasi-Banach spaces and operator theory, making available for the first time in book form many results scattered through various publications. Undoubtedly it will become a standard reference in the area.

S. Cobzaş

Glen Van Brummelen, The Mathematics of the Heavens and the Earth - The Early History of Trigonometry, Princeton University Press, Princeton and Oxford, 2009, xvii+329 pp, Hardbound, ISBN 978-0-691-12973-0.

The book under review is written by a well known expert in the field of history of mathematics. It is - as its subtitle emphasizes - a concise history of the early plane and spherical trigonometry from the dawn of civilization to the Middle Ages (1550).

In the first chapter *Precursors*, the author enumerates the earliest trigonometric results from Egypt, Babylon and Ancient Greece, like finding the slope of a pyramid, arc measurement and the 360° circle or Aristarchus and Archimedes reasoning for determining the shape of Earth, Moon and Sun and the ratio of the distances to the Moon and to the Sun.

In the second chapter *Alexandrian Greece*, the author follows the development of trigonometry in the major works of Hipparchus, Theodosius from Bithynia, Menelaus and Claudius Ptolemy. Their results were meant to explain the observed motion of the Sun, to solve other problems from Astronomy - like motion of the planets, timekeeping, or from Geography - like construction of the latitude arcs on a map.

In the following chapter *India*, the author describes how the first trigonometric results arose in this part of the world. The *chord* function used by the Greek scholars is transformed in a function proportional to the today *sine* function. The tables of this function were computed with increased accuracy using higher degree interpolation schemes for its approximation. This tables were used to solve spherical astronomy problems - like finding the right ascension of a point on the ecliptic, or other astronomical issues - like planetary equation.

The next chapter *Islam* trails the development of spherical and plane trigonometry in the area occupied by the Islamic world stretching from the borders of India through middle East across northern Africa and Spain. In this new world born in early 7^{th} century we find traces of Indian learning, but the Muslin scholars developed their own techniques to solve astronomical problems. For their religious practice they needed *qibla* - the direction in which they have to be oriented to face Mecca during their daily prayers. Moreover, they used trigonometry to design astronomical instruments like *horary quadrants*, which let them find the time of the day by the altitude of the Sun.

The last chapter *The West to 1550* is devoted to the application of trigonometry in navigation over seas. During Middle Ages and Renaissance in Europe flourished this new activity which benefited from trigonometry results. In the mean time, plane trigonometry was developed by scholars like Regiomontanus, Werner, Copernic, Rheticus, Otho and Pitiscus. The author emphasized their struggle to build precise tables of trigonometric functions (mainly sine and cosine), from 0° to 90°, with increments smaller than one degree.

The book includes a number of excerpts of translation from the original texts written by the scholars mentioned above. These are meant to give the reader of today the opportunity to experience directly what the ancient authors wrote and judge their reasoning. These old writings could be obscure for the today reader, the reason why the author provides after each text an explanation which allows a better understanding of the content.

After few *Concluding Remarks*, the book ends with an extensive bibliography which contains almost all important works for the history of trigonometry. The reference sources used to write this book are cited in *Preface*. Beside the excerpts from the old texts, the book contains reproductions from old printed works.

Being devoted to the history of mathematics, the book could be used by the teachers who want to make their lessons more attractive. Most of the excerpts from the ancient texts stand on their own and are ready to be used to illustrate trigonometric and astronomical concepts by providing historical context.

I highly recommend the book to all those interested in the way in which the ancient people solve their practical problems and hope that the next volume of this interesting history of spherical and plane trigonometry will appear soon.

Cristina Blaga

Bernard Dacorogna and Chiara Tanteri, Mathematical Analysis for Engineers, x+359 pp, Imperial College Press, World Scientific, Singapore, 2012, ISBN: 13 978-1-84816-912-8, ISBN: 10 1-84816-912-4.

This is the translation of the third French edition of a successful book destined to engineering students at l'Ecole Polythechnique Fédérale de Lausanne, but it can be also profitably used by students in mathematics and physics. The prerequisites are a basic course in analysis - differential and integral calculus.

The book is concerned with three main topics: I. *Vector analysis*, II. *Complex analysis*, an III. *Fourier analysis.* The first part contains exercises on the differential operators of mathematical physics (divergence, gradient, curl, Laplace operator), line integrals and gradient vector fields, surface integrals and Stokes theorem.

The second part is devoted to basic results in complex analysis: holomorphic functions and Cauchy-Riemann equations, complex integration, residues and their applications, conformal mappings.

The third part is concerned with Fourier series and Fourier transform, Laplace transform, and applications to ordinary and to partial differential equations.

What makes the book particularly useful are the theoretical results (definitions, basic theorems and formulae) and the examples preceding each section. These results are stated with mathematical rigor, without comments or proofs, but with references to precise pages of some books from the bibliography. Also an appendix to the first part contains some complementary results on topology, function spaces, curves and surfaces (with examples of some important curves and surfaces). The majority of the exercises help the students to master the concepts and techniques of the field, but there are several, marked by *, which present some further theoretical developments. The detailed solutions to exercises are given in the fourth part of the book.

This very well organized book will be useful to students in engineering, but also to those in mathematics and physics, as a complementary material to courses in analysis (real and complex), Fourier analysis and differential equations (both ordinary and partial).

Damian Trif