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# Fekete-Szegö problem for a class of analytic functions

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**Abstract.** In the present investigation, by taking $\phi(z)$ as an analytic function, sharp upper bounds of the Fekete-Szegö functional $|a_3 - \mu a_2^2|$ for functions belonging to the class $\mathcal{M}^{\alpha}_{g,h}(\phi)$ are obtained. A few applications of our main result are also discussed.

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## 1. Introduction

Let $\mathcal{A}$ be the class of analytic functions $f$ defined on the unit disk $\Delta := \{z \in \mathbb{C} : |z| < 1\}$ of the form

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n. \tag{1.1}$$

Let $\mathcal{S}$ be the subclass of $\mathcal{A}$ consisting of univalent functions. For two functions $f$ and $g$ analytic in $\Delta$ we say that $f$ is *subordinate* to $g$ or $g$ is *superordinate* to $f$, denoted by $f \prec g$, if there is an analytic function $w$ with $|w(z)| \leq |z|$ such that $f(z) = g(w(z))$. If $g$ is univalent, then $f \prec g$ if and only if $f(0) = g(0)$ and $f(\Delta) \subseteq g(\Delta)$.

A function $p(z) = 1+p_1 z+p_2 z^2+\ldots$ is said to be in the class $\mathcal{P}$ if $\operatorname{Re} p(z) > 0$. Let $\phi$ be an analytic univalent function in $\Delta$ with positive real part and $\phi(\Delta)$ be symmetric with respect to the real axis, starlike with respect to $\phi(0) = 1$ and $\phi'(0) > 0$. Ma and Minda [6] gave a unified presentation of various subclasses of starlike and convex functions by introducing the classes $\mathcal{S}^*(\phi)$ and $\mathcal{C}(\phi)$ satisfying $zf'(z)/f(z) \prec \phi(z)$ and $1 + zf''(z)/f'(z) \prec \phi(z)$ respectively, which includes several well-known classes as special case. For example, when $\phi(z) = (1 + Az)/(1 + Bz)$ $(-1 \leq B < A \leq 1)$ the class $\mathcal{S}^*(\phi)$ reduces to the class $\mathcal{S}^*[A, B]$ introduced by Janowski [3].

Ali *et al.*[1] introduced the class $\mathcal{M}(\alpha,\phi)$ of $\alpha$-convex functions with respect to $\phi$ consisting of functions $f$ in $\mathcal{A}$, satisfying

$$(1-\alpha)\frac{zf'(z)}{f(z)}+\alpha\left(1+\frac{zf''(z)}{f'(z)}\right)\prec\phi(z).$$

The class $\mathcal{M}(\alpha,\phi)$ includes several known classes namely $\mathcal{S}^*(\phi)$, $\mathcal{C}(\phi)$ and $\mathcal{M}(\alpha,(1+(1-2\alpha)z)/(1-z)) =: \mathcal{M}(\alpha)$. The class $\mathcal{M}(\alpha)$ is the class of $\alpha$-convex functions, introduced and studied by Miller and Mocanu [7]. Several coefficient problems for $p$- valent analytic functions were considered by Ali *et al.* [2].

In 1933, Fekete and Szegö proved that

$$|a_2^2-\mu a_3|\leq\begin{cases}4\mu-3 & (\mu\geq 1),\\ 1+\exp\left(-\frac{2\mu}{1-\mu}\right) & (0\leq\mu\leq 1),\\ 3-4\mu, & (\mu\leq 0)\end{cases}$$

holds for the functions $f\in\mathcal{S}$ and the result is sharp. The problem of finding the sharp bounds for the non-linear functional $|a_3-\mu a_2^2|$ of any compact family of functions is popularly known as the Fekete-Szegö problem. Keogh and Merkes [4], in 1969, obtained the sharp upper bound of the Fekete-Szegö functional $|a_2^2-\mu a_3|$ for functions in some subclasses of $\mathcal{S}$. For many results on Fekete-Szegö problems see [1, 2, 9, 10, 12, 13, 14].

The Hadamard product (or convolution) of $f(z)$, given by (1.1) and $g(z) = z+\sum_{n=2}^{\infty}g_nz^n$ is defined by

$$(f*g)(z):=z+\sum_{n=2}^{\infty}a_ng_nz^n=:(g*f)(z).$$

Recently, using the Hadamard product Murugusundaramoorthy *et al.* [8] introduced a new class $M_{g,h}(\phi)$ of functions $f\in\mathcal{A}$ satisfying

$$\frac{(f*g)(z)}{(f*h)(z)}\prec\phi(z)\quad(g_n>0,h_n>0,g_n-h_n>0),$$

where $g,h\in\mathcal{A}$ and are given by

$$g(z)=z+\sum_{n=2}^{\infty}g_nz^n\ \text{ and }\ h(z)=z+\sum_{n=2}^{\infty}h_nz^n. \tag{1.2}$$

Motivated by the work of Ma and Minda [6] and others [1, 2, 4, 8], in the present paper, we introduce a more general class $M_{g,h}^{\alpha}(\phi)$ defined using convolution and subordination and deduce Fekete-Szegö inequality for this class. Certain applications of our results are also discussed. In fact our results extend several earlier known works in [4, 6, 8].

**Definition 1.1.** *Let $g$ and $h$ are given by (1.2) with $g_n>0,h_n>0$ and $g_n-h_n>0$. A function $f\in\mathcal{A}$ given by (1.1) is said to be in the class $M_{g,h}^{\alpha}(\phi)$, if it satisfies*

$$(1-\alpha)\frac{(f*g)(z)}{(f*h)(z)}+\alpha\frac{(f*g)'(z)}{(f*h)'(z)}\prec\phi(z)\quad(\alpha\geq 0), \tag{1.3}$$

*where $\phi$ is an analytic function with $\phi(0)=1$ and $\phi'(0)>0$.*

Note that in Definition 1.1, we are not assuming $\phi(\Delta)$ to be symmetric with respect to the real axis and starlike with respect to $\phi(0) = 1$. In order to prove the class $M^{\alpha}_{g,h}(\phi)$ is non empty, consider the function $f(z) = z/(1-z)$. Assuming

$$\Phi(z) = (1-\alpha)\frac{(f*g)(z)}{(f*h)(z)} + \alpha\frac{(f*g)'(z)}{(f*h)'(z)},$$

we have $\Phi(z) = 1+(1+\alpha)(g_2-h_2)z+\cdots$. Clearly $\Phi(0) = 1$ and $\Phi'(0) = (1+\alpha)(g_2-h_2) > 0$, thus $f(z) = z/(1-z) \in M^{\alpha}_{g,h}(\phi)$.

**Remark 1.2.** For various choices of the functions $g$, $h$, $\phi$ and the real number $\alpha$, the class $M^{\alpha}_{g,h}(\phi)$ reduces to several known classes, we enlist a few of them below:

1. The class $M^{0}_{g,h}(\phi) =: M_{g,h}(\phi)$, introduced and studied by Murugusundaramoorthy *et al.* [8].
2. If we set
$$g(z) = \frac{z}{(1-z)^2}, \quad h(z) = \frac{z}{(1-z)} \tag{1.4}$$
and $\phi(z) = (1+z)/(1-z)$, then the class $M^{\alpha}_{g,h}(\phi)$ reduces to the class of $\alpha$-convex functions.
3. $M^{\alpha}_{\frac{z}{(1-z)^2},\frac{z}{(1-z)}}(\phi) =: \mathcal{M}(\alpha,\phi)$.
4. For the functions $g$ and $h$ given by (1.4), $M^{\alpha}_{g,h}((1+z)/(1-z)) =: \mathcal{M}(\alpha)$ is the class of $\alpha$-convex functions.
5. $M^{0}_{\frac{z}{(1-z)^2},\frac{z}{(1-z)}}(\phi) =: \mathcal{S}^*(\phi)$ and $M^{1}_{\frac{z}{(1-z)^2},\frac{z}{(1-z)}}(\phi) =: \mathcal{C}(\phi)$ are the well known classes of $\phi$-starlike and $\phi$-convex functions respectively.

The following lemmas are required in order to prove our main results. Lemma 1.3 of Ali *et al.* [2], is a reformulation of the corresponding result for functions with positive real part due to Ma and Minda [6].

Let $\Omega$ be the class of analytic functions $w$, normalized by the condition $w(0) = 0$, satisfying $|w(z)| < 1$.

**Lemma 1.3.** [2] *If $w \in \Omega$ and $w(z) := w_1 z + w_2 z^2 + \cdots (z \in \Delta)$, then*

$$|w_2 - t w_1^2| \le \begin{cases} -t & (t \le -1), \\ 1 & (-1 \le t \le 1), \\ t & (t \ge 1). \end{cases}$$

*For $t < -1$ or $t > 1$, equality holds if and only if $w(z) = z$ or one of its rotations. For $-1 < t < 1$, equality holds if and only if $w(z) = z^2$ or one of its rotations. Equality holds for $t = -1$ if and only if $w(z) = z(\lambda+z)/(1+\lambda z)$ $(0 \le \lambda \le 1)$ or one of its rotations, while for $t = 1$, equality holds if and only if $w(z) = -z(\lambda+z)/(1+\lambda z)$ $(0 \le \lambda \le 1)$ or one of its rotations.*

**Lemma 1.4.** [4] *(see also* [11]*) If $w \in \Omega$, then, for any complex number $t$,*

$$|w_2 - t w_1^2| \le \max\{1; |t|\}$$

*and the result is sharp for the functions given by $w(z) = z^2$ or $w(z) = z$.*

## 2. Fekete-Szegö problem

We begin with the following result:

**Theorem 2.1.** *Let $\phi(z) = 1 + B_1 z + B_2 z^2 + \cdots$. If $f(z)$ given by (1.1) belongs to the class $M^{\alpha}_{g,h}(\phi)$, then, for any real number $\mu$,*

$$|a_3 - \mu a_2^2| \leq \begin{cases} \frac{B_1 A}{(1+2\alpha)(g_3-h_3)} & (\mu \leq \sigma_1), \\ \frac{B_1}{(1+2\alpha)(g_3-h_3)} & (\sigma_1 \leq \mu \leq \sigma_2), \\ \frac{B_1 A}{(1+2\alpha)(h_3-g_3)} & (\mu \geq \sigma_2), \end{cases} \tag{2.1}$$

*where*

$$A = \frac{B_2}{B_1} - \frac{[(1+3\alpha)(h_2^2 - h_2 g_2) + \mu(1+2\alpha)(g_3 - h_3)]B_1}{(1+\alpha)^2(g_2-h_2)^2},$$

$$\sigma_1 := \frac{(B_2 - B_1)(1+\alpha)^2(g_2-h_2)^2 - (1+3\alpha)(h_2^2 - h_2 g_2)B_1^2}{(1+2\alpha)(g_3-h_3)B_1^2}$$

*and*

$$\sigma_2 := \frac{(B_2 + B_1)(1+\alpha)^2(g_2-h_2)^2 - (1+3\alpha)(h_2^2 - h_2 g_2)B_1^2}{(1+2\alpha)(g_3-h_3)B_1^2},$$

*and for any complex number $\mu$*

$$|a_3 - \mu a_2^2| \leq \frac{B_1}{2(1+2\alpha)(g_3-h_3)} \max\{1; |t|\}, \tag{2.2}$$

*where*

$$t := \frac{[(1+3\alpha)(h_2^2 - h_2 g_2) + \mu(1+2\alpha)(g_3-h_3)]B_1^2 - B_2(1+\alpha)^2(g_2-h_2)^2}{(1+\alpha)^2(g_2-h_2)^2 B_1}.$$

*Proof.* If $f \in \mathcal{M}^{\alpha}_{g,h}(\phi)$, then there exits an analytic function $w(z) = w_1 z + w_2 z^2 + \cdots \in \Omega$ such that

$$(1-\alpha)\frac{(f*g)(z)}{(f*h)(z)} + \alpha\frac{(f*g)'(z)}{(f*h)'(z)} = \phi(w(z)). \tag{2.3}$$

A computation shows that

$$\frac{(f*g)(z)}{(f*h)(z)} = 1 + a_2(g_2-h_2)z + [a_3(g_3-h_3) + a_2^2(h_2^2 - h_2 g_2)]z^2 + \cdots, \tag{2.4}$$

$$\frac{(f*g)'(z)}{(f*h)'(z)} = 1 + 2a_2(g_2-h_2)z + [3a_3(g_3-h_3) + 4a_2^2(h_2^2 - h_2 g_2)]z^2 + \cdots \tag{2.5}$$

and

$$\phi(w(z)) = 1 + B_1 w_1 z + (B_1 w_2 + B_2 w_1^2)z^2. \tag{2.6}$$

From (2.3), (2.4), (2.5) and (2.6), we have

$$(1+\alpha)(g_2-h_2)a_2 = B_1 w_1 \tag{2.7}$$

and

$$(1+2\alpha)(g_3-h_3)a_3 + (1+3\alpha)(h_2^2 - h_2 g_2)a_2^2 = B_1 w_2 + B_2 w_1^2. \tag{2.8}$$

A computation using (2.7) and (2.8) give

$$|a_3 - \mu a_2^2| = \frac{B_1}{(1+2\alpha)(g_2-h_2)}[w_2 - t w_1^2], \tag{2.9}$$

where

$$t := -\frac{B_2}{B_1} + \frac{[(1+3\alpha)(h_2^2 - h_2 g_2) + \mu(1+2\alpha)(g_3 - h_3)]B_1}{(1+\alpha)^2(g_2 - h_2)^2}. \tag{2.10}$$

Now the first inequality (1.3) is established as follows by an application of Lemma 1.3.

If

$$-\frac{B_2}{B_1} + \frac{[(1+3\alpha)(h_2^2 - h_2 g_2) + \mu(1+2\alpha)(g_3 - h_3)]B_1}{(1+\alpha)^2(g_2 - h_2)^2} \leq -1,$$

then

$$\mu \leq \frac{(B_2 - B_1)(1+\alpha)^2(g_2 - h_2)^2 - (1+3\alpha)(h_2^2 - h_2 g_2)B_1^2}{(1+2\alpha)(g_3 - h_3)B_1^2} := \sigma_1$$

and Lemma 1.3, gives

$$|a_3 - \mu a_2^2| \leq \frac{B_1 A}{(1+2\alpha)(g_3 - h_3)}.$$

For

$$-1 \leq -\frac{B_2}{B_1} + \frac{[(1+3\alpha)(h_2^2 - h_2 g_2) + \mu(1+2\alpha)(g_3 - h_3)]B_1}{(1+\alpha)^2(g_2 - h_2)^2} \leq 1,$$

we have $\sigma_1 \leq \mu \leq \sigma_2$, where $\sigma_1$ and $\sigma_2$ are as given in the statement of theorem. Now an application of Lemma 1.3 yields

$$|a_3 - \mu a_2^2| \leq \frac{B_1}{(1+2\alpha)(g_3 - h_3)}.$$

For

$$-\frac{B_2}{B_1} + \frac{[(1+3\alpha)(h_2^2 - h_2 g_2) + \mu(1+2\alpha)(g_3 - h_3)]B_1}{(1+\alpha)^2(g_2 - h_2)^2} \geq 1,$$

we have $\mu \geq \sigma_2$ and it follows from Lemma 1.3 that

$$|a_3 - \mu a_2^2| \leq \frac{B_1 A}{(1+2\alpha)(h_3 - g_3)}.$$

Now the second inequality (2.2) fallows by an application of Lemma 1.4 as follows:

$$\begin{aligned} |a_3 - \mu a_2^2| &= \frac{B_1}{(1+2\alpha)(g_2 - h_2)}[w_2 - t w_1^2] \\ &\leq \frac{B_1}{(1+2\alpha)(g_3 - h_3)} \max\{1; |t|\}, \end{aligned}$$

where $t$ is given by (2.10). □

**Remark 2.2.** If we set $\alpha = 1$, $g$ and $h$ are as given by (1.4), then Theorem 2.1 reduces to the result [6, Theorem 3] of Ma and Minda. When $\alpha = 0$, Theorem 2.1 reduces to the result [8, Theorem 2.1], proved by Murugusundaramoorthy *et al.* Note that there were few typographical errors in the assertion of the result [8, Theorem 2.1] and it is rectified in the following corollary:

**Corollary 2.3.** [8, Theorem 2.1] *Let* $\phi(z) = 1 + B_1 z + B_2 z^2 + \cdots$. *If* $f(z)$ *given by* (1.1) *belongs to the class* $M_{g,h}(\phi)$, *then for any real number* $\mu$,

$$|a_3 - \mu a_2^2| \leq \begin{cases} \frac{B_1}{g_3-h_3}\left(\frac{B_2}{B_1} - \frac{[(h_2^2-h_2g_2)+\mu(g_3-h_3)]B_1}{(g_2-h_2)^2}\right) & (\mu \leq \sigma_1), \\ \frac{B_1}{g_3-h_3} & (\sigma_1 \leq \mu \leq \sigma_2), \\ \frac{B_1}{g_3-h_3}\left(\frac{[(h_2^2-h_2g_2)+\mu(g_3-h_3)]B_1}{(g_2-h_2)^2} - \frac{B_2}{B_1}\right) & (\mu \geq \sigma_2), \end{cases}$$

*where*

$$\sigma_1 := \frac{(B_2 - B_1)(g_2 - h_2)^2 - (h_2^2 - h_2 g_2)B_1^2}{(g_3 - h_3)B_1^2}$$

*and*

$$\sigma_2 := \frac{(B_2 + B_1)(g_2 - h_2)^2 - (h_2^2 - h_2 g_2)B_1^2}{(g_3 - h_3)B_1^2}.$$

Here below, we discuss some applications of Theorem 2.1:

**Corollary 2.4.** *Let* $\phi(z) = 1 + B_1 z + B_2 z^2 + \cdots$ . *Assume that*

$$g(z) = z + \sum_{n=2}^{\infty} \frac{n\Gamma(n+1)\Gamma(2-\delta)}{\Gamma(n-\delta+1)} z^n \quad \text{and} \quad h(z) = z + \sum_{n=2}^{\infty} \frac{\Gamma(n+1)\Gamma(2-\delta)}{\Gamma(n-\delta+1)} z^n.$$

*If* $f(z)$ *given by* (1.1) *belongs to the class* $M^{\alpha}_{g,h}(\phi)$, *then for any real number* $\mu$,

$$|a_3 - \mu a_2^2| \leq \begin{cases} \frac{(2-\delta)(3-\delta)B_1}{12(1+2\alpha)}\left(\frac{B_2}{B_1} - \frac{[12\mu(1+2\alpha)(2-\delta)-4(3-\delta)(1+3\alpha)]B_1}{4(3-\delta)(1+\alpha)^2}\right) & (\mu \leq \sigma_1), \\ \frac{(2-\delta)(3-\delta)B_1}{12(1+2\alpha)} & (\sigma_1 \leq \mu \leq \sigma_2), \\ \frac{(2-\delta)(3-\delta)B_1}{12(1+2\alpha)}\left(\frac{[12\mu(1+2\alpha)(2-\delta)-4(3-\delta)(1+3\alpha)]B_1}{4(3-\delta)(1+\alpha)^2} - \frac{B_2}{B_1}\right) & (\mu \geq \sigma_2), \end{cases}$$

*where*

$$\sigma_1 := \frac{(3-\delta)[(B_1 - B_2)(1+\alpha)^2 + (1+3\alpha)B_1^2]}{3(2-\delta)(1+2\alpha)B_1^2}$$

*and*

$$\sigma_2 := \frac{(3-\delta)[(B_1 + B_2)(1+\alpha)^2 + (1+3\alpha)B_1^2]}{3(2-\delta)(1+2\alpha)B_1^2}.$$

**Remark 2.5.** Taking $\alpha = 8/\pi^2, B_2 = 16/3\pi^2$ and $\delta = 1$ in Corollary 2.4, we have the result of Ma and Minda [5, Theorem 2]. When $\alpha = 0$, the above Corollary 2.4 reduces to [8, Corollary 3.2] of Murugusundaramoorthy *et al.* Note that there were few typographical errors in the assertion of [8, Corollary 3.2] and the following result is the corrected one:

**Corollary 2.6.** [8, Corollary 3.2] *Let* $\phi(z) = 1 + B_1 z + B_2 z^2 + \cdots$ . *If* $f(z)$ *given by* (1.1) *belongs to the class* $M_{g,h}(\phi)$, *then, for any real number* $\mu$,

$$|a_3 - \mu a_2^2| \leq \begin{cases} \frac{(2-\delta)(3-\delta)B_1}{12}\left(\frac{B_2}{B_1} - \frac{[12\mu(2-\delta)-4(3-\delta)]B_1}{4(3-\delta)}\right) & (\mu \leq \sigma_1), \\ \frac{(2-\delta)(3-\delta)B_1}{12} & (\sigma_1 \leq \mu \leq \sigma_2), \\ \frac{(2-\delta)(3-\delta)B_1}{12}\left(\frac{[12\mu(2-\delta)-4(3-\delta)]B_1}{4(3-\delta)} - \frac{B_2}{B_1}\right) & (\mu \geq \sigma_2), \end{cases}$$

*where*

$$\sigma_1 := \frac{(3-\delta)[B_1 - B_2 + B_1^2]}{3(2-\delta)B_1^2}$$

*and*

$$\sigma_2 := \frac{(3-\delta)[B_1 + B_2 + B_1^2]}{3(2-\delta)B_1^2}.$$

Putting $\phi(z) = (1+z)/(1-z)$, $g$ and $h$ are as given by (1.4) in Theorem 2.1, we deduce the following result:

**Corollary 2.7.** *Let $f(z)$ is given by (1.1) belongs to the class $\mathcal{M}(\alpha)$, then, for any real number $\mu$,*

$$|a_3 - \mu a_2^2| \le \begin{cases} \frac{(\alpha^2+8\alpha+3)-4\mu(1+2\alpha)}{(1+\alpha)^2(1+2\alpha)} & (\mu \le \sigma_1), \\ \frac{1}{(1+2\alpha)} & (\sigma_1 \le \mu \le \sigma_2), \\ \frac{4\mu(1+2\alpha)-(\alpha^2+8\alpha+3)}{(1+\alpha)^2(1+2\alpha)} & (\mu \ge \sigma_2), \end{cases}$$

*where $\sigma_1 := \frac{1+3\alpha}{2(1+2\alpha)}$ and $\sigma_2 := \frac{\alpha^2+5\alpha+2}{2(1+2\alpha)}$.*

Note that for $\alpha = 0$, Corollary 2.7 reduces to a result in [4] (see also [14]). By taking $\phi(z) = (1+z)/(1-z)$, $g$ and $h$, given by (1.4) in second result of Theorem 1.3, we have the following result:

**Corollary 2.8.** *Let $f(z)$ is given by (1.1) belongs to the class $\mathcal{M}(\alpha)$, then for any complex number $\mu$*

$$|a_3 - \mu a_2^2| \le \frac{1}{1+2\alpha} \max\left\{1; \left|\frac{4\mu(1+2\alpha) - (\alpha^2+8\alpha+3)}{(1+\alpha)^2}\right|\right\}.$$

**Remark 2.9.** For $\alpha = 1$, the above Corollary 2.8 reduces to the result [4, Corollary 1] of Keogh and Merkes.

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