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Book reviews

Peter Duren, Invitation to Classical Analysis, Pure and Applied Undergraduate Texts, Volume 17, American Mathematical Society, Providence, Rhode Island, 2012, xiii+392 pp; ISBN: 978-0-8218-6932-1.

In modern calculus textbooks, as, for instance, those by Walter Rudin or Jean Dieudonné, the focus is on the theoretical foundations of the subject, with less attention paid to concrete applications. The classical treatises on calculus, as well as those directed to physicists or engineers, contain such topics but treated in a less rigorous manner. The aim of the present book is to fill in this gap by presenting some substantial topics in classical analysis with full rigorous proofs and in historical perspective. As a thread running through the book one can mention the calculation of the sum of the series $\sum_{n=1}^{\infty} n^{-2}$. This problem posed by Johann Bernoulli, known as the *Basel problem*, was solved in 1735 by Leonard Euler who proved the remarkable equality $\sum_{n=1}^{\infty} n^{-2} = \pi^2/6$ (known as Euler's sum). Since then many proofs of this equality have been found, some of them being recorded in this book.

Although the reader is assumed to have acquired a good command of basic principles of mathematical analysis, the first chapter of the book contains a review of them, for convenience and easy reference. Chapters 2, *Special sequences*, and 3, *Power series*, contain also some standard material, along with some more special topics – the product formulae of Vieta and Wallis, Stirling's formula, a first treatment of Euler's sum, nowhere differentiable continuous functions.

In Ch. 4, *Inequalities*, beside some standard material, one can mention an elementary straightforward proof of Hilbert's inequality recently found by David Ullrich (to appear in American Mathematical Monthly). Ch. 5, *Infinite products*, is devoted to a topic less treated, or neglected at all, in modern courses of calculus. Ch. 6, *Approximation by polynomials*, contains the proofs given by Lebesgue, Landau, and Bernstein, to uniform approximation of continuous functions by polynomials, along with some refinements due to Pál, Fekete, and Müntz-Szasz. A proof of the Stone-Weierstrass approximation theorem is also included. Abel's Theorem, proved in Chapter 3, asserts that if the power series $\sum_{n=0}^{\infty} a_n x^n$ converges in $(-1,1)$ and $\sum_{n=0}^{\infty} a_n = A$, then $\lim_{x\to 1-} \sum_{n=0}^{\infty} a_n x^n = A$. Tauber's classical theorem asserts that the converse result holds under the additional hypothesis that $na_n \to 0$. The result was extended by Hardy and Littlewood to the case of boundedness of the sequence $\{na_n\}$ and for Cesàro summability, which is more general than Abel's summability. The proofs of these results, given by Karamata, are presented in the seventh chapter of the book.

Fourier series are treated in Chapter 8 at an elementary level, meaning the exclusion of Lebesgue integration and complex analysis. In spite of these restrictions some spectacular results can be derived, proving the power of Fourier analysis.

Chapters 9, *The Gamma function*, and 14, *Elliptic functions*, are dealing with these special classes of functions, while Bessel functions and hypergeometric functions are treated in Ch. 13, *Differential equations.* Ch. 11, *Bernoulli numbers*, is concerned mainly with the properties of these numbers and applications to the calculation of the sum of the series $\zeta(2k) = \sum_{n=1}^{\infty} n^{-2k}$. Applications of Riemann zeta function to number theory are considered in Ch. 10, *Topics in number theory*, along with other results in this area. Ch. 12, *The Cantor set*, is concerned with cardinal numbers, Cantor set, Cantor-Scheeffer function, space-filling curves.

The book is very well organized – detailed name and subject indexes, historical notes on the evolution of mathematical ideas and their relevance for physical applications, capsule scientific biographies of the major contributors and a gallery of portraits. It is devoted to undergraduates who learned the basic principles of analysis, and are prepared to explore substantial topics in classical analysis. It is designed for self-study, but can also serve as a text for advanced courses in calculus.

The book reflects the delight the author experienced when writing it, a delight that will be surely shared by its readers as well.

Tiberiu Trif

Peter D. Lax and Lawrence Zalcman, Complex Proofs of Real Analysis, University Lecture Notes, Volume 58, American Mathematical Society, Providence, Rhode Island, 2012, xi+90 pp; ISBN- 978-0-8218-7599-9.

A famous saying of Poncelet asserts that "*Between two truths of the real domain, the easiest and shortest path quite often passes through the complex domain*". Since this dictum was endorsed and popularized by Hadamard, it is usually attributed to him. The present book is a brilliant illustration of this claim - the authors show how the method of complex analysis can be used to provide quick and elegant proofs of a wide variety of results in various areas of analysis. Beside analysis, a proof of the Prime Number Theorem ($\lim_{x\to 0} \pi(x)/\left(\frac{x}{\log x}\right) = 1$) based on contour integration of Riemann zeta function is included. Other earlier results obtained by the methods of complex analysis concern evaluations of the values of $\zeta(2k) = \sum_{n=1}^{\infty} n^{-2k}$, a proof of the fundamental theorem of algebra, and applications to approximation theory – uniform weighted approximation in $C_0(\mathbb{R})$ and Müntz's theorem.

The core of the book is formed by the applications to operator theory, in Chapter 3, and to harmonic analysis, in Chapter 4. Among the applications to operator theory we mention Rosenblum's elegant proof of Fuglede-Putnam theorem, Toeplitz operators and their inversion, Beurling's characterization of invariant subspaces of the unilateral shift on the Hardy space H^2, Szegö's theorem in prediction theory, Riesz-Thorin convexity theorem with applications to the boundedness of the Hilbert transform on $L^p(\mathbb{R}^n)$, $1 < p < \infty$.

The applications to harmonic analysis include D. J. Newman's proof of the uniqueness of the Fourier transform in $L^1(\mathbb{R})$, uniqueness and nonuniqueness results for the Radon transform, Paley-Wiener theorem and Titchmarsh convolution theorem, Hardy's theorem on the Fourier transform.

The fifth chapter of the book contains a proof of the famous Kahane-Gleason-Želazko theorem giving conditions for a linear functional on a commutative Banach algebra with unit to be multiplicative. The Fatou-Julia theorem in complex dynamics is treated in the sixth chapter. A coda deals briefly with two unusual applications of complex analysis – to fluid dynamics and to statistical mechanics (the stochastic Loewner evolution equation).

The book is fairly self-contained, the prerequisites being a standard course in complex analysis and familiarity with some results in functional analysis and Fourier transform. Some more specialized topics from complex analysis, as Liouville's theorem in Banach spaces, the Borel-Crathéodory inequality, Phragmen-Lindlöf theorem, and some results on normal families, are presented in Appendices. Relevant historical remarks and bibliographical references accompany each topic included in the book.

Written in a lively and entertaining style, but mathematically rigorous at the same time, the book is addressed to all mathematicians interested in elegant proofs of some fundamental results in mathematics. The instructors of complex analysis can use it as a source to enrich their lectures with nice examples.

Gabriela Kohr

Martin Väth, Topological Analysis – From the basics to the triple degree for nonlinear Fredholm inclusions, Series in Nonlinear Analysis and Applications, Vol. 16, ix+490 pp, Walter de Gruyter, Berlin - New York, 2012, ISBN: 978-3-11-027722-7, e-ISBN: 978-3-11-027733-3, ISSN: 0941-813X.

The degree theory, originally developed by Leray and Schauder in the thirties of the last century for a rather restricted class of equations, turned to be one of the most powerful tools of nonlinear analysis. Since then it has been successively extended to encompass much larger classes of equations, including even those involving noncompact or multivalued maps. The aim of the present monograph is to give a self-contained introduction to the whole area, culminating with a general degree theory for function triples, the first monograph treatment of this notion in such generality.

The book is divided into three parts: I. *Topology and multivalued maps*, II. *Coincidence degree for Fredholm maps*, and III. *Degree for function triples.*

The presentation is given in logical order, meaning from general results to particular ones, rather that in a didactic order. For instance, many results concerning single-valued continuous functions, developed in the second chapter devoted to topology, are obtained as particular cases of those referring to multivalued maps - mean-value results for continuous functions, some results on proper maps. This chapter contains also a a detailed treatment of separation axioms, including two less known – T_5 (every subset is T_4) and T_6 (perfectly normal spaces). Some specific results to metric spaces, as measures of noncompactness and condensing maps, embedding and extension results, are treated in the third chapter. The fourth chapter deals with homotopies, retracts, ANR and AR spaces, while the last one of this part (Ch. 5) is concerned with some advanced topological tools – covering space theory, dimension theory, Vietoris map.

The second part of the book contains some results from linear functional analysis – linear bounded operators on Banach spaces, Fredholm operator and an orientation theory for families of Fredholm operators based on determinants associated with them – and basic nonlinear functional analysis – Gâteaux and Fréchet differentiation, inverse and implicit function theorems, orientation for families of nonlinear Fredholm maps on Banach manifolds, a brief but fairly complete treatment of Brouwer degree with applications. This part ends with an introduction to Benevieri-Furi degrees, based on a definition of orientation by which the degree theory in infinite dimensional setting reduces to the finite dimensional case. The Leray-Schauder degree (the infinite dimensional version of Brouwer degree) is postponed to Chapter 13, where it is obtained as a particular case of a more general notion.

The highlight of the book is the third part where a very general degree theory, which unifies a lot of known degrees theories, is developed. It is concerned with problems of the form: (1) $F(x) \in \varphi(\Phi(x))$, where (a) F is a nonlinear Fredholm operator of index 0; (b) Φ is a multivalued mapping with acyclic values $\Phi(x)$; (c) φ is continuous , and (d) the composition $\varphi \circ \Phi$ is, roughly speaking, "more compact that F is proper". The key notion throughout the book is that of orientation with is gradually extended from linear Fredholm operators on Banach spaces to nonlinear Fredholm maps on Banach manifolds and on Banach bundles. The book contains also an account on Banach manifolds, which are the basic tools of the degree theory.

By the choice of material this excellent book can be used for several purposes. First as supplementary material for various courses in topology or functional analysis. Even in the standard part of the book, concerning topology and functional analysis, some shorter and elegant proofs to known results are given. Also the author pays a special attention to foundation – the necessity of the Axiom of Choice for various results is carefully checked.

Second, as a self-contained introduction to the degree theory in various of its hypostases, starting with Brouwer degree in Eucildean spaces and on manifolds and culminating with the degree theory for function triples, a very active and important domain of research with many applications in various areas of mathematics.

Radu Precup

Anne Greenbaum and Timothy P. Chartier, Numerical Methods. Design, Analysis, and Computer Implementation of Algorithms, Princeton University Press, 2012, 470 pp.; ISBN: 978-0-691-15122-9.

This book is a concise and modern exposition of both standard (Solution of nonlinear equations – Chapter 3, Numerical Linear Algebra – Chapters 7 and 12, Floating-point arithmetic – Chapter 5, Condition and stability of algorithms – Chapter 6, Interpolation – Chapter 8, Numerical Differentiation – Chapter 9, Numerical

Integration – Chapter 10, Ordinary Differential Equations – Chapters 11 and 13, Partial Differential Equations) and nontraditional (Mathematical modeling – Chapter 1, Monte Carlo Methods – Chapter 3, fractals – section 4.6, Markov chains – section 12.1.5) topics on numerical analysis.

The good balance between mathematical rigor, practical and computational aspect of numerical methods, and computer programs offers the instructor the flexibility to focuss on different aspects of numerical methods, depending on the aim of the course, the background and the interests of students.

Biographical information about mathematicians and short discussions on the history of numerical methods humanize the text.

The book contains extensive examples, presented in an intuitive way with high quality figure (some of them quite spectacular) and useful MATLAB codes. MATLAB exercises and routines are well integrated within the text, and a concise introduction into MATLAB is given in Chapter 2. The emphasis is on program's numerical and graphical capabilities and its applications, not on its syntax. The usage of MATLAB Toolbox `chebfun` facilitates presentation of recent results on interpolation at Chebyshev points and provides symbolic capabilities at speed of numeric procedures. A large variety of problems graded by the difficulty point of view are included. Applications are modern and up to date (e.g. information retrieval and animation, classical applications from physics and engineering).

Appendices on linear algebra and multivariate Taylor's theorem help the understanding of theoretical results in the text. A reach and comprehensive list of references is attached at the end of the book. Supplementary materials are available online.

I am sure this text will become a great title for the subject.

Intended audience: especially graduate students in mathematics and computer science, but also useful to applied mathematicians, computer scientists, engineers and physicists interested in applications of numerical analysis.

Radu Trîmbiţaş

Darryl D Holm, Geometric Mechanics, 2nd Edition, Imperial College Press, London, 2011.
Part I: **Dynamics and Symmetry**, xxiv + 441 pp, ISBN: 13 978-1-84816-774-2.
Part II: **Rotating, Translating and Rolling**, xx + 390 pp, ISBN: 13 978-1-84816-777-3.

The two volumes of Darryl Holm's *Geometric Mechanics* offer an attractive introduction to the tools and language of modern geometric mechanics. These volumes are designed for advanced undergraduates and beginning graduate students in mathematics, physics and engineering. The minimal prerequisite for reading *Geometric Mechanics* is a working knowledge of linear algebra, multivaluable calculus and some familiarity with Hamilton's principle, Euler-Lagrange variational principles and canonical Poisson brackets in classical mechanics, at the beginning undergraduate level.

In this second edition the author preserves the organization of the first edition (2008). However, the substance of the text has been rewritten throughout to improve the flow and to enrich the development of the material. In particular, the role of

Noether's theorem about the implications of Lie group symmetries for conservation laws of dynamical systems has been emphasized throughout, with many applications. Many worked examples on adjoint and coadjoint actions of Lie groups on smooth manifolds have been added. The enhanced coursework examples have been expanded.

The first volume contains six chapters: Fermat's ray optics; Newton, Lagrange, Hamilton and the rigid body; Lie, Poincaré, Cartan: Differential forms; Resonances and S^1 reduction; Elastic spherical pendulum; Maxwell–Bloch laser-matter equations, and two appendixes: Enhanced coursework and Exercises for review and further study.

The chapters of the second volume are: Galileo; Newton, Lagrange, Hamilton and the rigid body; Quaternions; Adjoint and coadjoint actions; The special orthogonal group $SO(3)$; Adjoint and coadjoint semidirect-product group actions; Euler–Poincaré and Lie–Poisson equations on $SE(3)$; Heavy-top equations; The Euler–Poincaré theorem; Lie–Poisson Hamiltonian form of a continuum spin chain; Momentum maps; Round, rolling rigid bodies. This volume ends with four appendices: Geometrical structure of classical mechanics; Lie groups and Lie algebras; Enhanced coursework; Poincaré's 1901 paper.

The two volumes of the second edition of Holm's *Geometric Mechanics* are ideal for classroom use, student projects and self-study.

Ferenc Szenkovits

Jaroslaw Kurzweil, Generalized Ordinary Differential equations – Non Absolutely Continuous Solutions, Series in Real Analysis, Vol. 11, World Scientific, London - Singapore - Beijing, 2012, ix + 197 pages, ISBN: 13 978-981-4324-02-1 and 10 981-4324-02-7.

Using a Riemann type approach, the author of the present book discovered in the fifties a new kind of integral, called non-absolutely convergent integral, which is more general than Lebesgue's integral and agrees with it in case of absolute integrability. Since R. Henstock independently arrived at the same conclusions, approximatively at the same time, the integral is known as the Henstock-Kurzweil (HK) integral. A good presentation of this integral, along with other types of integral, is given in the book by D. S. Kurtz and Ch. W. Swartz *Theories of Integration - The integrals of Riemann, Lebesgue, Henstock-Kurzweil and Mc Shane*, World Scientific, London-Singapore-Beijing, 2012, as well as in two previous books by the author – Teubner, Leipzig, 1980 (in German) and World Scientific 2000. The main point of the HK integral is the validity of a very general form of the fundamental theorem of calculus $\int_a^b f' = f(b) - f(a)$ for any differentiable function f. One of the main application of this new integral, given in a paper published by the author in 1957, was to the existence of generalized solutions and continuous dependence on the parameter for generalized ordinary differential equations (GODE), where solutions of infinite variation can occur. The aim of the present book is to give a systematic development of this subject. Since the main motivation came from Kapitza's pendulum equation studied by P. Kapitza in 1951, the first part of the book (Chapters 2-4) is devoted to this equation.

The second part of the book (Chapters 5-13) is concerned with strong Riemann solutions of the differential equation $\frac{d}{dt}x = D_tG(x,\tau,t)$.

The existence and properties of strong Henstock-Kurzweil (SKH) solutions of the above equations are studied in the third part (Chapters 14-18), based on some averaging techniques. The fourth part (Chapters 19-24) deals with SKH solutions of the equation $\frac{d}{dt}x = F(x,\tau,t)$, where $F : X \times [a,b]^2 \to X$, X a Banach space, is a mapping satisfying some natural conditions. The fifth (and the last, Chapters 25-27) part of the book is concerned with GODEs of the form $\frac{d}{dt}x = D_tG(x,\tau,t)$, $\frac{d}{dt}x = D_tG^\circ(x,\tau,t)$, where $G^\circ(x,t) = (SR)\int_a^t D_tG(x,\sigma,s)$.

The book is an important contribution to the area of differential equations, proving the power and versatility of the generalized integral discovered by Henstock and Kurzweil.

Valeriu Anisiu