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Totally supra $b-$continuous and slightly supra $b-$continuous functions

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Abstract. In this paper, totally supra b-continuity and slightly supra b-continuity are introduced and studied. Furthermore, basic properties and preservation theorems of totally supra b-continuous and slightly supra b-continuous functions are investigated and the relationships between these functions and their relationships with some other functions are investigated.

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1. Introduction and preliminaries

In 1983, A. S. Mashhour et al. [11] introduced the supra topological spaces. In 1996, D. Andrijevic [1] introduced and studied a class of generalized open sets in a topological space called b-open sets. This type of sets discussed by El-Atike [10] under the name of $\gamma-$open sets. Also, in recent years, Ekici has studied some relationships of $\gamma-$open sets [5, 6, 8, 9]. In 2010, O. R. Sayed et al. [12] introduced and studied a class of sets and a class of maps between topological spaces called supra b-open sets and supra b-continuous functions, respectively. Now we introduce the concepts of totally supra b-continuous and slightly supra b-continuous functions and investigate several properties for these concepts.

Throughout this paper (X, τ), (Y, ρ) and (Z, σ) (or simply X, Y and Z) denote topological spaces on which no separation axioms are assumed unless explicitly stated. For a subset A of (X, τ), the closure and the interior of A in X are denoted by $Cl(A)$ and $Int(A)$, respectively. The complement of A is denoted by $X - A$. In the space (X, τ), a subset A is said to be b-open [1] if $A \subseteq Cl(Int(A)) \cup Int(Cl(A))$. A subcollection $\mu \subseteq 2^X$ is called a supra topology [11] on X if X, $\phi \in \mu$ and μ is closed under arbitrary union. (X, μ) is called a supra topological space. The elements of μ

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are said to be supra open in (X, μ) and the complement of a supra open set is called a supra closed set. The supra closure of a set A, denoted by $Cl^\mu(A)$, is the intersection of supra closed sets including A. The supra interior of a set A, denoted by $Int^\mu(A)$, is the union of supra open sets included in A. The supra topology μ on X is associated with the topology τ if $\tau \subseteq \mu$.

Definition 1.1. [12] *Let (X, μ) be a supra topological space. A set A is called a supra b-open set if $A \subseteq Cl^\mu(Int^\mu(A)) \cup Int^\mu(Cl^\mu(A))$. The complement of a supra b-open set is called a supra b-closed set.*

Definition 1.2. [2] *Let (X, μ) be a supra topological space. A set A is called a supra α-open set if $A \subseteq Int^\mu(Cl^\mu(Int^\mu(A)))$. The complement of a supra α-open set is called a supra α-closed set.*

Theorem 1.3. [12]. *(i) Arbitrary union of supra b-open sets is always supra b-open.*

(ii) Finite intersection of supra b-open sets may fail to be supra b-open.

Lemma 1.4. [12] *The intersection of a supra α-open set and a supra b-open set is a supra b-open set.*

Definition 1.5. [12] *The supra b-closure of a set A, denoted by $Cl^\mu_b(A)$, is the intersection of supra b-closed sets including A. The supra b-interior of a set A, denoted by $Int^\mu_b(A)$, is the union of supra b-open sets included in A.*

Definition 1.6. [7] *A function $f : X \longrightarrow Y$ is called:*

(1) slightly γ-continuous at a point $x \in X$ if for each clopen subset V in Y containing $f(x)$, there exists a γ-open subset U of X containing x such that $f(U) \subset V$.

(2) slightly γ-continuous if it has this property at each point of X.

Definition 1.7. [3, 7] *A function $f : X \longrightarrow Y$ is called:*

(i) γ-irresolute if for each γ-open subset G of Y, $f^{-1}(G)$ is γ-open in X.

(ii) γ-open if for every γ-open subset A of X, $f(A)$ is γ-open in Y.

Definition 1.8. [7] *A space X is called γ-connected provided that X is not the union of two disjoint nonempty γ-open sets.*

Definition 1.9. [3] *A space X is said to be:*

(i) $\gamma - T_1$ if for each pair of distinct points x and y of X, there exist γ-open sets U and V containing x and y, respectively such that $y \notin U$ and $x \notin V$.

(ii) $\gamma - T_2$ (γ-Hausdorff) if for each pair of distinct points x and y of X, there exist disjoint γ-open sets U and V in X such that $x \in U$ and $y \in V$.

Definition 1.10. [3, 7] *A space X is said to be:*

(i) $\gamma-$Lindelöf if every γ-open cover of X has a countable subcover.

(ii) $\gamma-$closed-compact if every γ-closed cover of X has a finite subcover.

(iii) $\gamma-$closed-Lindelöf if every cover of X by γ-closed sets has a countable subcover.

2. Totally supra $b-$continuous functions

In this section, the notion of totally supra b-continuous functions is introduced. If A is both supra b-open and supra b-closed, then it is said to be supra b-clopen.

Definition 2.1. [12] *Let (X, τ) and (Y, ρ) be two topological spaces and μ be an associated supra topology with τ. A function $f : (X, \tau) \longrightarrow (Y, \rho)$ is called a supra b-continuous function if the inverse image of each open set in Y is supra b-open in X.*

Definition 2.2. *Let (X, τ) and (Y, ρ) be two topological spaces and μ be an associated supra topology with τ. A function $f : (X, \tau) \longrightarrow (Y, \rho)$ is called a totally supra b-continuous function if the inverse image of each open set in Y is supra b-clopen in X.*

Remark 2.3. Every totally supra b-continuous function is supra b-continuous but the converse need not be true as it can be seen from the following example.

Example 2.4. Let $X = \{a, b, c\}$ and $\tau = \{X, \phi, \{a, b\}\}$ be a topology on X. The supra topology μ is defined as follows: $\mu = \{X, \phi, \{a\}, \{a, b\}\}$. Let $f : (X, \tau) \longrightarrow (X, \tau)$ be a function defined as follows: $f(a) = a$, $f(b) = c$, $f(c) = b$. The inverse image of the open set $\{a, b\}$ is $\{a, c\}$ which is supra b-open but it is not supra b-clopen. Then f is supra b-continuous but it is not totally supra b-continuous.

Definition 2.5. *A supra topological space (X, μ) is called supra $b-$connected if it is not the union of two nonempty disjoint supra b-open sets.*

Theorem 2.6. *A supra topological space (X, μ) is supra $b-$connected if and only if X and ϕ are the only supra b-clopen subsets of X.*

Proof. Obvious. □

Theorem 2.7. *Let (X, τ) be a topological spaces and μ be an associated supra topology with τ. If $f : (X, \tau) \longrightarrow (Y, \rho)$ is a totally supra b-continuous surjection and (X, μ) is supra $b-$connected, then (Y, ρ) is an indiscrete space.*

Proof. Suppose that (Y, ρ) is not an indiscrete space and let V be a proper nonempty open subset of (Y, ρ). Since f is a totally supra b-continuous function, then $f^{-1}(V)$ is a proper nonempty supra b-clopen subset of X. Therefore $X = f^{-1}(V) \cup (X - f^{-1}(V))$ and X is a union of two nonempty disjoint supra b-open sets, which is a contradiction. Therefore X must be an indiscrete space. □

Theorem 2.8. *Let (X, τ) be a topological space and μ be an associated supra topology with τ. The supra topological space (X, μ) is supra $b-$connected if and only if every totally supra b-continuous function from (X, τ) into any T_0-space (Y, ρ) is a constant map.*

Proof. $\Rightarrow$) Suppose that $f : (X, \tau) \longrightarrow (Y, \rho)$ is a totally supra b-continuous function, where (Y, ρ) is a T_0-space. Assume that f is not constant and $x, y \in X$ such that $f(x) \neq f(y)$. Since (Y, ρ) is T_0, and $f(x)$ and $f(y)$ are distinct points in Y, then there is an open set V in (Y, ρ) containing only one of the points $f(x)$, $f(y)$. We take the case $f(x) \in V$ and $f(y) \notin V$. The proof of the other case is similar. Since f is a totally supra b-continuous function, $f^{-1}(V)$ is a supra b-clopen subset of X and $x \in f^{-1}(V)$, but $y \notin f^{-1}(V)$. Since $X = f^{-1}(V) \cup (X - f^{-1}(V))$, X is a union of two nonempty disjoint supra b-open subsets of X. Thus (X, μ) is not supra $b-connected$, which is a contradiction.

$\Leftarrow$) Suppose that (X, μ) is not a supra $b-connected$ space, then there is a proper nonempty supra b-clopen subset A of X. Let $Y = \{a, b\}$ and $\rho = \{Y, \phi, \{a\}, \{b\}\}$, define $f : (X, \tau) \longrightarrow (Y, \rho)$ by $f(x) = a$ for each $x \in A$ and $f(x) = b$ for $x \in X - A$. Clearly f is not constant and totally supra b-continuous where Y is T_0, and thus we have a contradiction. □

Definition 2.9. *A supra topological space X is said to be:*

(i) supra $b - T_1$ if for each pair of distinct points x and y of X, there exist supra b-open sets U and V containing x and y, respectively such that $y \notin U$ and $x \notin V$.

(ii) supra $b - T_2$ if for each pair of distinct points x and y in X, there exist disjoint supra b-open sets U and V in X such that $x \in U$ and $y \in V$.

Theorem 2.10. *Let (X, τ) and (Y, ρ) be two topological spaces and μ be an associated supra topology with τ. Let $f : (X, \tau) \longrightarrow (Y, \rho)$ be a totally supra b-continuous injection. If Y is T_0 then (X, μ) is supra $b - T_2$.*

Proof. Let $x, y \in X$ with $x \neq y$. Since f is injection, $f(x) \neq f(y)$. Since Y is T_0, there exists an open subset V of Y containing $f(x)$ but not $f(y)$, or containing $f(y)$ but not $f(x)$. Thus for the first case we have, $x \in f^{-1}(V)$ and $y \notin f^{-1}(V)$. Since f is totally supra b-continuous and V is an open subset of Y, $f^{-1}(V)$ and $X - f^{-1}(V)$ are disjoint supra b-clopen subsets of X containing x and y, respectively. The second case is proved in the same way. Thus X is supra $b - T_2$. □

Definition 2.11. *Let (X, τ) be a topological space and μ be an associated supra topology with τ. A function $f : (X, \tau) \longrightarrow Y$ is called a strongly supra b-continuous function if the inverse image of every subset of Y is a supra b-clopen subset of X.*

Remark 2.12. Every strongly supra b-continuous function is totally supra b-continuous, but the converse need not be true as the following example shows.

Example 2.13. Let $X = \{a, b, c\}$ and $\tau = \{X, \phi\}$ be a topology on X. The supra topology μ is defined as follows: $\mu = \{X, \phi, \{a, c\}\}$. Let $f : (X, \tau) \longrightarrow (X, \tau)$ be the identity function, then f is totally supra b-continuous but it is not strongly supra b-continuous.

3. Slightly supra $b-$continuous functions

In this section, the notion of slightly supra b-continuous functions is introduced and characterizations and some relationships of slightly supra b-continuous functions and basic properties of slightly supra b-continuous functions are investigated and obtained.

Definition 3.1. *Let (X, τ) and (Y, ρ) be two topological spaces and μ be an associated supra topology with τ. A function $f : (X, \tau) \longrightarrow (Y, \rho)$ is called a slightly supra b-continuous function at a point $x \in X$ if for each clopen subset V in Y containing $f(x)$, there exists a supra b-open subset U in X containing x such that $f(U) \subseteq V$. The function f is said to be slightly supra b-continuous if it has this property at each point of X.*

Remark 3.2. Every supra b-continuous function is slightly supra b-continuous but the converse need not be true as it can be seen from the following example.

Example 3.3. Let R and N be the real numbers and natural numbers, respectively. Take two topologies on R as $\tau = \{R, \phi\}$ and $\rho = \{R, \phi, R - N\}$ and μ be the associated supra topology with τ defined as $\mu = \{R, \phi, N\}$. Let $f : (R, \tau) \longrightarrow (R, \rho)$ be an identity function. Then, f is slightly supra b-continuous, but it is not supra b-continuous.

Remark 3.4. Since every totally supra b-continuous function is supra b-continuous then every totally supra b-continuous function is slightly supra b-continuous but the converse need not be true. The function f in Example 3.3 is slightly supra b-continuous but it is not totally supra b-continuous.

Remark 3.5. Since every strongly supra b-continuous function is totally supra b-continuous then every strongly supra b-continuous function is slightly supra b-continuous but the converse need not be true. The function f in Example 2.13 is slightly supra b-continuous but it is not strongly supra b-continuous.

Theorem 3.6. *Let (X, τ) and (Y, ρ) be two topological spaces and μ be an associated supra topology with τ. The following statements are equivalent for a function $f : (X, \tau) \longrightarrow (Y, \rho)$:*

(1) f is slightly supra b-continuous;
(2) for every clopen set $V \subseteq Y$, $f^{-1}(V)$ is supra b-open;
(3) for every clopen set $V \subseteq Y$, $f^{-1}(V)$ is supra b-closed;
(4) for every clopen set $V \subseteq Y$, $f^{-1}(V)$ is supra b-clopen.

Proof. (1) $\Rightarrow$ (2): Let V be a clopen subset of Y and let $x \in f^{-1}(V)$. Since f is slightly supra b-continuous, by (1) there exists a supra b-open set U_x in X containing x such that $f(U_x) \subseteq V$; hence $U_x \subseteq f^{-1}(V)$. We obtain that $f^{-1}(V) = \cup\{U_x : x \in f^{-1}(V)\}$. Thus, $f^{-1}(V)$ is supra b-open.

(2) $\Rightarrow$ (3): Let V be a clopen subset of Y. Then $Y - V$ is clopen. By (2) $f^{-1}(Y - V) = X - f^{-1}(V)$ is supra b-open. Thus $f^{-1}(V)$ is supra b-closed.

(3) $\Rightarrow$ (4): It can be shown easily.

(4) $\Rightarrow$ (1): Let $x \in X$ and V be a clopen subset in Y with $f(x) \in V$. Let $U = f^{-1}(V)$. By assumption U is supra b-clopen and so supra b-open. Also $x \in U$ and $f(U) \subseteq V$. □

Corollary 3.7. [7] *Let (X, τ) and (Y, ρ) be topological spaces. The following statements are equivalent for a function $f : X \longrightarrow Y$:*

(1) f is slightly γ-continuous;
(2) for every clopen set $V \subset Y, f^{-1}(V)$ is γ-open;

(3) for every clopen set $V \subset Y, f^{-1}(V)$ is γ-closed;
(4) for every clopen set $V \subset Y, f^{-1}(V)$ is γ-clopen.

Theorem 3.8. *Every slightly supra b-continuous function into a discrete space is strongly supra b-continuous.*

Proof. Let $f : X \longrightarrow Y$ be a slightly supra b-continuous function and Y be a discrete space. Let A be any subset of Y. Then A is a clopen subset of Y. Hence $f^{-1}(A)$ is supra b-clopen in X. Thus f is strongly supra b-continuous. □

Definition 3.9. *Let (X, τ) and (Y, ρ) be two topological spaces and μ, η be associated supra topologies with τ and ρ, respectively. A function $f : (X, \tau) \to (Y, \rho)$ is called a supra b-irresolute function if the inverse image of each supra b-open set in Y is a supra b-open set in X.*

Theorem 3.10. *Let (X, τ), (Y, ρ) and (Z, σ) be topological spaces and μ, η be associated supra topologies with τ and ρ, respectively. Let $f : (X, \tau) \to (Y, \rho)$ and $g : (Y, \rho) \to (Z, \sigma)$ be functions. Then, the following properties hold:*

(1) If f is supra b-irresolute and g is slightly supra b-continuous, then gof is slightly supra b-continuous.

(2) If f is slightly supra b-continuous and g is continuous, then gof is slightly supra b-continuous.

Proof. (1) Let V be any clopen set in Z. Since g is slightly supra b-continuous, $g^{-1}(V)$ is supra b-open. Since f is supra b-irresolute, $f^{-1}(g^{-1}(V)) = (gof)^{-1}(V)$ is supra b-open. Therefore, gof is slightly supra b-continuous.

(2) Let V be any clopen set in Z. By the continuity of g, $g^{-1}(V)$ is clopen. Since f is slightly supra b-continuous, $f^{-1}(g^{-1}(V)) = (gof)^{-1}(V)$ is supra b-open. Therefore, gof is slightly supra b-continuous. □

Corollary 3.11. *Let (X, τ), (Y, ρ) and (Z, σ) be topological spaces and μ, η be associated supra topologies with τ and ρ, respectively. If $f : (X, \tau) \to (Y, \rho)$ is a supra b-irresolute function and $g : (Y, \rho) \to (Z, \sigma)$ is a supra b-continuous function, then gof is slightly supra b-continuous.*

Corollary 3.12. [7] *Let $f : X \to Y$ and $g : Y \to Z$ be functions. Then, the following properties hold:*

(1) If f is γ-irresolute and g is slightly γ-continuous, then $gof : X \to Z$ is slightly γ-continuous.

(2) If f is γ-irresolute and g is γ-continuous, then $gof : X \to Z$ is slightly γ-continuous.

Definition 3.13. *A function $f : (X, \tau) \to (Y, \rho)$ is called a supra b-open function if the image of each supra b-open set in X is a supra b-open set in Y.*

Theorem 3.14. *Let (X, τ), (Y, ρ) and (Z, σ) be topological spaces and μ, η be associated supra topologies with τ and ρ, respectively. Let $f : (X, \tau) \to (Y, \rho)$ be a supra b-irresolute, supra b-open surjection and $g : (Y, \rho) \to (Z, \sigma)$ be a function. Then g is slightly supra b-continuous if and only if gof is slightly supra b-continuous.*

Proof. $\Rightarrow$) Let g be slightly supra b-continuous. Then by Theorem 3.10, gof is slightly supra b-continuous.

$\Leftarrow$) Let gof be slightly supra b-continuous and V be clopen set in Z. Then $(gof)^{-1}(V)$ is supra b-open. Since f is a supra b-open surjection, then $f((gof)^{-1}(V)) = g^{-1}(V)$ is supra b-open in Y. This shows that g is slightly supra b-continuous. □

Corollary 3.15. [7] *$f : X \to Y$ be surjective, γ-irresolute and γ-open and $g : Y \to Z$ be a function. Then $gof : X \to Z$ is slightly γ-continuous if and only if g is slightly γ-continuous.*

Theorem 3.16. *Let (X, τ) be a topological space and μ be an associated supra topology with τ. If $f : (X, \tau) \longrightarrow (Y, \rho)$ is a slightly supra b-continuous function and (X, μ) is supra $b -$ connected, then Y is connected.*

Proof. Suppose that Y is a disconnected space. Then there exist nonempty disjoint open sets U and V such that $Y = U \cup V$. Therefore, U and V are clopen sets in Y. Since f is slightly supra b-continuous, $f^{-1}(U)$ and $f^{-1}(V)$ are supra b-open in X. Moreover, $f^{-1}(U)$ and $f^{-1}(V)$ are disjoint and $X = f^{-1}(U) \cup f^{-1}(V)$. Since f is surjective, $f^{-1}(U)$ and $f^{-1}(V)$ are nonempty. Therefore, X is not supra $b-connected$. This is a contradiction and hence Y is connected. □

Corollary 3.17. [7] *If $f : X \to Y$ is slightly γ-continuous surjective function and X is γ-connected space, then Y is a connected space.*

Corollary 3.18. *The inverse image of a disconnected space under a slightly supra b-continuous surjection is supra $b -$ disconnected.*

Recall that a space X is said to be (1) locally indiscrete if every open set of X is closed in X, (2) 0-dimensional if its topology has a base consisting of clopen sets.

Theorem 3.19. *Let (X, τ) be a topological space and μ be an associated supra topology with τ. If $f : (X, \tau) \longrightarrow (Y, \rho)$ is a slightly supra b-continuous function and Y is locally indiscrete, then f is supra b-continuous.*

Proof. Let V be any open set of Y. Since Y is locally indiscrete, V is clopen and hence $f^{-1}(V)$ are supra b-open in X. Therefore, f is supra b-continuous. □

Theorem 3.20. *Let (X, τ) be a topological space and μ be an associated supra topology with τ. If $f : (X, \tau) \longrightarrow (Y, \rho)$ is a slightly supra b-continuous function and Y is 0-dimensional, then f is supra b-continuous.*

Proof. Let $x \in X$ and $V \subseteq Y$ be any open set containing $f(x)$. Since Y is 0-dimensional, there exists a clopen set U containing $f(x)$ such that $U \subseteq V$. But f is slightly supra b-continuous then there exists a supra b-open set G containing x such that $f(x) \in f(G) \subseteq U \subseteq V$. Hence f is supra b-continuous. □

Corollary 3.21. [7] *If $f : X \to Y$ is slightly γ-continuous and Y is 0-dimensional, then f is γ-continuous.*

Theorem 3.22. *Let (X,τ) be a topological space and μ be an associated supra topology with τ. Let $f:(X,\tau)\longrightarrow(Y,\rho)$ be a slightly supra b-continuous injection and Y is 0-dimensional. If Y is T_1 (resp. T_2), then X is supra $b-T_1$ (resp. supra $b-T_2$).*

Proof. We prove only the second statement, the prove of the first being analogous. Let Y be T_2. Since f is injective, for any pair of distinct points $x,y\in X$, $f(x)\neq f(y)$. Since Y is T_2, there exist open sets V_1, V_2 in Y such that $f(x)\in V_1$, $f(y)\in V_2$ and $V_1\cap V_2=\phi$. Since Y is 0-dimensional, there exist clopen sets U_1, U_2 in Y such that $f(x)\in U_1\subseteq V_1$ and $f(y)\in U_2\subseteq V_2$. Consequently $x\in f^{-1}(U_1)\subseteq f^{-1}(V_1)$, $y\in f^{-1}(U_2)\subseteq f^{-1}(V_2)$ and $f^{-1}(U_1)\cap f^{-1}(U_2)=\phi$. Since f is slightly supra b-continuous, $f^{-1}(U_1)$ and $f^{-1}(U_2)$ are supra b-open sets and this implies that X is supra $b-T_2$. □

Definition 3.23. *A space X is said to be:*

(i) clopen T_1 [4,7] if for each pair of distinct points x and y of X, there exist clopen sets U and V containing x and y, respectively such that $y\notin U$ and $x\notin V$.

(ii) clopen T_2 (clopen Hausdorff or ultra-Hausdorff) [13] if for each pair of distinct points x and y in X, there exist disjoint clopen sets U and V in X such that $x\in U$ and $y\in V$.

Theorem 3.24. *Let (X,τ) be a topological space and μ be an associated supra topology with τ. Let $f:(X,\tau)\longrightarrow(Y,\rho)$ be a slightly supra b-continuous injection and Y is clopen T_1, then X is supra $b-T_1$.*

Proof. Suppose that Y is clopen T_1. For any distinct points x and y in X, there exist clopen sets V and W such that $f(x)\in V$, $f(y)\notin V$ and $f(y)\in W$, $f(x)\notin W$. Since f is slightly supra b-continuous, $f^{-1}(V)$ and $f^{-1}(W)$ are supra b-open subsets of X such that $x\in f^{-1}(V)$, $y\notin f^{-1}(V)$ and $y\in f^{-1}(W)$, $x\notin f^{-1}(W)$. This shows that X is supra $b-T_1$. □

Corollary 3.25. [7] *If $f:X\to Y$ is slightly γ-continuous injection and Y is clopen T_1, then X is $\gamma-T_1$.*

Theorem 3.26. *Let (X,τ) be a topological space and μ be an associated supra topology with τ. Let $f:(X,\tau)\longrightarrow(Y,\rho)$ be a slightly supra b-continuous injection and Y is clopen T_2, then X is supra $b-T_2$.*

Proof. For any pair of distinct points x and y in X, there exist disjoint clopen sets U and V in Y such that $f(x)\in U$ and $f(y)\in V$. Since f is slightly supra b-continuous, $f^{-1}(U)$ and $f^{-1}(V)$ are supra b-open subsets of X containing x and y, respectively. Therefore $f^{-1}(U)\cap f^{-1}(V)=\phi$ because $U\cap V=\phi$. This shows that X is supra $b-T_2$. □

Definition 3.27. [13] *A space X is said to be mildly compact (resp. mildly Lindelöf) if every clopen cover of X has a finite (resp. countable) subcover.*

Definition 3.28. *A supra topological space (X,μ) is called supra b-compact (resp. supra b-Lindelöf) if every supra b-open cover of X has a finite (resp. countable) subcover.*

Theorem 3.29. *Let (X, τ) be a topological space and μ be an associated supra topology with τ. Let $f : (X, \tau) \longrightarrow (Y, \rho)$ be a slightly supra b-continuous surjection, then the following statements hold:*

(1) if (X, μ) is supra b-compact, then Y is mildly compact.

(2) if (X, μ) is supra b-Lindelöf, then Y is mildly Lindelöf.

Proof. We prove (1), the proof of (2) being entirely analogous.

Let $\{V_\alpha : \alpha \in \Delta\}$ be a clopen cover of Y. Since f is slightly supra b-continuous, $\{f^{-1}(V_\alpha) : \alpha \in \Delta\}$ is a supra b-open cover of X . Since X is supra b-compact, there exists a finite subset Δ_0 of Δ such that $X = \cup\{f^{-1}(V_\alpha) : \alpha \in \Delta_0\}$. Thus we have $Y = \cup\{V_\alpha : \alpha \in \Delta_0\}$ which means that Y is mildly compact. □

Definition 3.30. *A supra topological space (X, μ) is called supra b-closed compact (resp. supra b-closed Lindelöf) if every cover of X by supra b-closed sets has a finite (resp. countable) subcover.*

Theorem 3.31. *Let (X, τ) be a topological space and μ be an associated supra topology with τ. Let $f : (X, \tau) \longrightarrow (Y, \rho)$ be a slightly supra b-continuous surjection, then the following statements hold:*

(1) if (X, μ) is supra b-closed compact, then Y is mildly compact.

(2) if (X, μ) is supra b-closed Lindelöf, then Y is mildly Lindelöf.

Proof. It can be obtained similarly as Theorem 3.29. □

Corollary 3.32. [7] *Let $f : X \to Y$ be a slightly γ-continuous surjection. Then the following statements hold:*

(1) if X is γ-Lindelöf, then Y is mildly Lindelöf.

(2) if X is γ-compact, then Y is mildly compact.

(3) if X is γ–closed-compact, then Y is mildly compact.

(4) if X is γ-closed-Lindelöf, then Y is mildly Lindelöf.

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