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ON THE STABILITY OF DISTRIBUTIONS OF THE COMPOSED RANDOM VARIABLE BASED ON THE STABILITY OF THE SOLUTION OF THE DIFFERENTIAL EQUATIONS FOR CHARACTERISTIC FUNCTIONS

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Abstract. In this paper we give some conditions for the stability of the distribution functions of composed random variables by considering the stability of the solutions of differential equations for characteristic functions.

1. **Introduction**

We consider a random variable (r.v.)

$$\eta = \sum_{k=1}^{\nu} \xi_k, \tag{1.1}$$

where $\xi_1, \xi_2, ...$ are i.i.d. random variables possessing the same distribution function $F(x)$ with the corresponding characteristic function $\varphi(t)$, ν is a positive valued r.v. independent of all ξ_k $(k = 1, 2, ...)$ and has the moment generating function $a(z)$.

The r.v. η is called the composed random variable of ξ_j, η has the characteristic function defined by

$$\psi(t) = a[\varphi(t)].$$

In [2], [3] and [4], we obtained the following results.

1. Suppose that ν follows the Poisson law with the parameter λ and ξ follows the exponential law with the parameter θ. If the statistic T_1 is zero-regression w.r.t

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the statistic λ_1 (T_1 and λ_1 were showed in [2]), then the characteristic function $\psi_1(t)$ of η satisfies the following equation

$$3[\psi_1^{''}(t)]^2\psi_1^2(t) - 2\psi_1^{'}(t)\psi_1^{'''}(t)\psi_1^2(t) - [\psi_1^{'}(t)]^4 = 0 \tag{1.2}$$

where $\psi_1(0) = 0; \psi_1^{'}(0) = i\lambda\theta; \psi_1^{''}(0) = -\lambda\theta^2(2+\lambda)$.

2. Assume that ν follows the Poisson law with the parameter λ, ξ follows the negative binomial distribution function with the parameters p and q. If the statistic T_2 is zero-regression with the statistic λ_1 (T_2 and λ_1 were showed in [2]) then the characteristic function $\psi_2(t)$ of η satisfies the following equation

$$[\psi_2^{'}(t)]^4 + 2\psi_2^{'''}(t)\psi_2^{'}(t)\psi_2^2(t) - 3[\psi_2^{''}(t)]^2\psi_2^2(t) - \psi_2^{'}(t)\psi_2^2(t) = 0, \tag{1.3}$$

where $\psi_2(0) = 1; \psi_2^{'}(0) = i\lambda\frac{q}{p}; \psi_2^{''}(0) = -\frac{\lambda^2 q^2}{p^2} - \frac{2\lambda q^2}{p^2}$.

3. If ν follows the Poisson law with the paramater λ, ξ follows the Normal law $N(0,1)$ and if the statistic T_3 is zero-regression with the statistic λ_1 (T_3 and λ_1 were showed in [2]) then the characteristic function $\psi_3(t)$ of η satisfies the following equation

$$\begin{gathered}\psi_3^{(4)}(t)\psi_3^{''}(t)\psi_3^4(t) - \psi_3^{(4)}(t)[\psi_3^{'}(t)]^2\psi_3^3(t) + 2\psi_3^{'}(t)\psi_3^{''}(t)\psi_3^{'''}(t)\psi_3^3(t)\\ -3[\psi_3^{''}(t)]^2\psi_3^3(t) + 6\psi_3^{''}(t)[\psi_3^{'}(t)]^2\psi_3^2(t) + 6\psi_3^{''}(t)[\psi_3^{'}(t)]^4\psi_3(t)\\ +2\psi_3^{''}(t) + 2[\psi_3^{''}(t)]^2\psi_3^4(t) - [\psi_3^{'}(t)]^4\psi_3^2(t) - [\psi_3^{'''}(t)]^2\psi_3^2(t)\\ -2\psi_3^{'}(t)\psi_3^{''}(t)\psi_3^4(t) = 0,\end{gathered}$$

where

$$\psi_3(0) = 1; \psi_3^{'}(0) = 0; \psi_3^{''}(0) = -\lambda; \psi_3^{'''}(0) = 0. \tag{1.4}$$

4. If ν follows the binomial law with the parameters p and q, ξ follows the exponential law with the parameter θ, and if the statistic T_4 is zero-regression with the statistic λ_1 (T_4 and λ_1 were showed in [2]) then the characteristic function $\psi_4(t)$ of η satisfies the following equation

$$3n^2[\psi_4^{''}(t)]^2\psi_4^2(t) - 2n^2\psi_4^{'}(t)\psi_4^{'''}(t)\psi_4^2(t) - (n^2-1)[\psi_4^{'}(t)]^4 = 0, \tag{1.5}$$

where $\psi_4(0) = 1; \psi_4^{'}(0) = inp\theta; \psi_4^{''}(0) = -n^2\theta^2p^2 - n\theta^2p^2$.

5. If ν follows the negative binomial law with the parameters p and q, ξ follows the exponential law with the parameter θ, and if the statistic T_5 is zero-regression with the statistic λ_1 (T_5 and λ_1 were showed in [2]) then the characteristic function $\psi_5(t)$ of η satisfies the following equation

$$3[\psi_5^{''}(t)]^2 - 2\psi_5^{'}(t)\psi_5^{'''}(t) = 0, \tag{1.6}$$

where $\psi_5(0) = 1; \psi_5^{'}(0) = i\theta\frac{q}{p}; \psi_5^{''}(0) = -\frac{2\theta^2 q}{p^2}$.

6. If ν follows geometric law with the parameters α and β, ξ follows the exponential law with the parameter θ and if the statistic T_6 is zero-regression with the statistic λ_1 (T_6 and λ_1 were showed in [4]) then the characteristic function $\psi_6(t)$ of η satisfies the following equation

$$3[\psi_6^{''}(t)]^2 - 2\psi_6^{'}(t)\psi_6^{'''}(t) = 0, \tag{1.7}$$

where $\psi_6(0) = 1; \psi_6^{'}(0) = i\frac{\theta}{\alpha}; \psi_6^{''}(0) = -2(\frac{\theta}{\alpha})^2$.

7. If ν follows the Geometric law with the parameters α and β, ξ follows the negative binomial law with the parameters p and q, and if the statistic T_7 is zero-regression with the statistic λ_1 (T_7 and λ_1 were showed in [4]) then the characteristic function $\psi_7(t)$ of η satisfies the equation

$$\{[\psi_7^{''}(t)]^2 - \psi_7^{'}(t)\psi_7^{'''}(t)\}\psi_7^2(t) + 2[\psi_7^{'}(t)]^2\psi_7^{''}(t)\psi_7(t) - 2[\psi_7^{'}(t)]^4 = 0, \tag{1.8}$$

where $\psi_7(0) = 1; \psi_7^{'}(0) = \frac{iq}{p\alpha}; \psi_7^{''} = -\frac{q(1-\beta p + q)}{\alpha^2 p^2}$.

In [2] and [4] we have considered also the stability of the composed random variables and we showed that if the condition that the statistics T_i $(i = 1, 5, 6, 7)$ are zero-regression with the statistic λ_1 (T_i and λ_1 were showed in [2], [4]) is replaced by the condition that $T_i (i = 1, 5, 6, 7)$ are ϵ-zero regression with the statistic λ_1 (for some small enough number ϵ) then the characteristic functions $\psi_i(t)$ of η have to satisfy the differential equations which have the same left sides of the differential equations (1.2), (1.3),...,(1.8) but their right sides are functions $r_i(t)$ which are small enough for all t.

Let us consider the following differential equations

$$F(\psi(t), \psi^{'}(t), \psi^{''}(t), ..., \psi^{(n)}(t)) = 0 \tag{1.9}$$

and

$$F(\psi(t), \psi^{'}(t), \psi^{''}(t), ..., \psi^{(n)}(t)) = r(t), \tag{1.10}$$

where $r(t) = \overline{r(t)}, r(0) = 0, |r(t)| \leqslant \epsilon$ (for some small enough number ϵ). If the function F in equations (1.9) and (1.10) satisfies the condition $\frac{\partial F}{\partial \psi^{(n)}} \neq 0$ then from (1.9) we can represent $\psi(t)$ in the form

$$\psi^{(n)}(t) = f[\psi(t), \psi^{'}(t), ..., \psi^{(n-1)}(t)], \tag{1.11}$$

where the solution $\psi_\epsilon(t)$ of equation (1.10) can be represented in the form

$$\psi_\epsilon^{(n)}(t) = f[\psi_\epsilon(t), \psi_\epsilon^{'}(t), ..., \psi_\epsilon^{(n-1)}(t)] + a(t), \tag{1.12}$$

where $|a(t)| < \epsilon$.

A problem arisen is that under which condition imposing on the function $f(x_1, x_2, ..., x_n)$, the solution of the differential equation (1.9), is stable in the following sense: there exist $T = T(\epsilon)$ such that $T(\epsilon) \to \infty$ when $\epsilon \to 0$, and $\delta = \delta(\epsilon)$, such that $\delta(\epsilon) \to 0$ when $\epsilon \to 0$ such that

$$|\psi_\epsilon(t) - \psi(t)| < C\delta(\epsilon), \text{for all } \ t, |t| \leqslant T(\epsilon),$$

where C is a constant independent of ϵ.

2. Stability theorem of the solution of the differential equations

Let us consider the differential equations (1.9) and (1.10) with solutions satisfying the equations (1.11) and (1.12).

Theorem 2.1. *If the function $f(x_1, x_2, ..., x_n)$ is continuous, differentiable in variables and satisfies the Lipschitz's condition, that means there exists a positive constant N, such that*

$$|f(x_1, x_2, ..., x_n) - f(y_1, y_2, ..., y_n)| \leqslant N \sum_{i=1}^{n} |x_i - y_i|$$

for all $(x_1, x_2, ..., x_n) \in R^n$ *and* $(y_1, y_2, ..., y_n) \in R^n$ *and if* $\psi(t)$ *is a bounded function and satisfies the conditions:*

$$\exists M \in \mathbb{R}^1, 0 < M < +\infty, |\psi^{(k)}(t)| < M, \textit{for all } k = 1, 2, ..., n; \textit{ for all } t,$$

then, for every small enough positive number ϵ*, there exists a positive number* $T = T(\epsilon)$*,* $T(\epsilon) \to \infty$ *when* $\epsilon \to 0$ *and a positive number* $\delta, 0 < \delta < 1$*, such that*

$$|\psi_\epsilon(t) - \psi(t)| < C\epsilon^{1-\delta}, \textit{for all } t, |t| \leqslant T(\epsilon),$$

where C *is a constant independent of* ϵ*.*

Lemma 2.1. *Suppose that all eigenvalues of a constant matrix* A *have negative real parts, then there exist constants* $\alpha > 0$ *and* $\beta > 0$*, such that*

$$||e^{At}|| \leqslant \beta e^{-\alpha t}. \tag{2.1}$$

where $||\cdot||$ *denotes the norm in the space of the square matrics and* $e^{At} = \sum_{k=0}^{\infty} \dfrac{(At)^k}{k!}$.

Lemma 2.2. *Suppose that* $u(t)$ *and* $f(t)$ *are integrable nonegative real functions on* $[t_0, t_0 + T]$ *and* $K(t, s)$ *is a nonegative real function, bounded on* $[t_0, t_0 + T]$.

If the following inequality holds:

$$u(t) \leqslant f(t) + \int_{t_0}^{t} K(t, s)u(s)ds, \tag{2.2}$$

then

$$u(t) \leqslant h(t), \textit{for all } t, \quad t_0 \leqslant t \leqslant t_0 + T, \tag{2.3}$$

where $h(t)$ *is the solution of the equation*

$$h(t) = f(t) + \int_{t_0}^{t} K(t, s)h(s)ds. \tag{2.4}$$

Proof of the theorem 2.1. At first, we consider $t \geq 0$, (the case $t \leqslant 0$ is carried out similarly).

Putting $x_1 = \psi(t), x_2 = \psi^{'}(t), ..., x_n = \psi^{(n-1)}(t)$, then the differential equation (1.11) can be written in the form

$$\begin{cases} \dfrac{dx_1}{dt} = x_2 \\ \dfrac{dx_2}{dt} = x_3 \\ \cdots \\ \dfrac{dx_n}{dt} = f(x_1, x_2, ..., x_n). \end{cases} \tag{2.6}$$

Let us denote $X = (x_1, x_2, ..., x_n)^T \in \mathbb{R}^n$

$$A = \begin{bmatrix} -n & 1 & 1 & \cdots & 1 \\ 1 & -n & 1 & \cdots & 1 \\ . & . & . & \cdots & . \\ 1 & 1 & 1 & \cdots & -n \end{bmatrix}$$

$$G(X) = \begin{bmatrix} +nx_1 - x_3 & \cdots - x_n \\ -x_1 + nx_2 & \cdots - x_n \\ .. & \cdots \\ -x_1 - x_2 & \cdots + nx_{n-1} \\ -x_1 - x_2 & \cdots - x_{n-1} + nx_n + f(x_1, x_2, ..., x_n) \end{bmatrix}.$$

Then the differential equation (2.6) reduces to the equation:

$$\frac{dX}{dt} = AX + G(X). \tag{2.7}$$

By a similar way, the differential equation (1.10) can be rewritten as follows

$$\frac{dY}{dt} = AY + G(Y) + a(t), \tag{2.8}$$

where $Y = (y_1, y_2, ..., y_n)^T \in \mathbb{R}^n, y_1 = \psi_\epsilon(t), ..., y_n = \psi_\epsilon^{(n-1)}(t)$ and $a(t)$ is given in (1.12).

Since $f(x_1, x_2, ..., x_n)$ is continuous and differentiable function in variables and satisfies the Lipshitz condition, there exists a positive constant l, such that

$$||G(X) - G(Y)|| \leqslant l||X - Y|| \text{ for all } X, Y \in \mathbb{R}^n. \tag{2.9}$$

On the other hand, we have

$$det(A - \lambda E) = (\lambda + 1)(\lambda + n + 1)^{n-1}, \tag{2.10}$$

so, the eigenvalues of matrix A are

$$\lambda_1 = -1, \lambda_2 = -(n+1) = \lambda_3 = ... = \lambda_n.$$

We see that the eigenvalues of matrix A have negative real parts.

According to the Lemma 2.1, there exists constants $\alpha, \beta > 0$, such that

$$||e^{At}|| \leqslant \beta e^{-\alpha t}. \tag{2.11}$$

From (2.7) and (2.8) we get

$$X(t) = e^{At}X(0) + \int_0^t e^{A(t-s)}G[X(s)]ds \tag{2.12}$$

$$Y(t) = e^{At}Y(0) + \int_0^t e^{A(t-s)}G[Y(s)]ds + \int_0^t e^{A(t-s)}a(s)ds. \tag{2.13}$$

Since $X(0) = Y(0)$,

$$||X(t) - Y(t)|| \leqslant \int_0^t ||e^{A(t-s)}||.||G[X(s)] - G[Y(s)]||ds + \int_0^t ||e^{A(t-s)}||.|a(s)|ds.$$

Using the estimations $(2.9), (2.11)$ and by (1.12) we have

$$||X(t) - Y(t)|| \leqslant \beta e^{-\alpha t}\int_0^t le^{\alpha s}||X(s) - Y(s)||ds + \beta e^{-\alpha t}\epsilon \int_0^t e^{\alpha s}ds.$$

Hence

$$||X(t) - Y(t)||e^{\alpha t} \leqslant \beta\epsilon \int_0^t e^{\alpha s}ds + \int_0^t \beta l||X(s) - Y(s)||e^{\alpha s}ds. \tag{2.14}$$

If we put $||X(t) - Y(t)||e^{\alpha t} = u(t), f(t) = \beta\epsilon \int_0^t e^{\alpha s}ds, K(s,t) = l\beta$ and $t_0 = 0$, and by the lemma 2.2, we have the following estimation

$$u(t) \leqslant f(t) + \int_0^t \beta l u(s)ds. \tag{2.15}$$

It follows from the Lemma 2.2 that

$$u(t) \leqslant \psi(t),$$

where $\psi(t)$ is the solution of equation

$$\psi(t) = f(t) + \int_0^t \beta l \psi(s) ds.$$

Therefore we have

$$\begin{aligned}\psi(t) &= e^{\int_0^t \beta l ds}[f(0) + \int_0^t f^{'}(s) e^{-\int_0^s \beta l ds} ds] \\ &= e^{\beta l t} \int_0^t \beta \epsilon e^{\alpha s - \beta l s} ds \\ &= \frac{\beta \epsilon}{\alpha - \beta l}(e^{\alpha t} - e^{\beta l t}).\end{aligned}$$

So we obtain

$$||X(t) - Y(t)|| \leqslant \frac{\beta}{\alpha - \beta l} \epsilon (1 - e^{\beta l t - \alpha t}). \tag{2.16}$$

If $\alpha - \beta l > 0$ then $||X(t) - Y(t)|| \leqslant \dfrac{\beta}{\alpha - \beta l}\epsilon$ for all t.

If $\alpha - \beta l < 0$, then

$$\frac{\beta}{\alpha - \beta l}\epsilon(1 - e^{\beta l t - \alpha t}) \leqslant \frac{\beta}{|\alpha - \beta l|}\epsilon e^{(\beta l - \alpha)t}.$$

Now if we choose $T(\epsilon) = \dfrac{1}{\beta l - \alpha}\ln(\dfrac{1}{\epsilon})^{\delta}$, where $0 < \delta < 1$, then

$$T(\epsilon) \to \infty, \quad \text{when} \quad \epsilon \to 0.$$

So, for all t, $0 < t \leqslant T(\epsilon)$ we get the estimation:

$$||X(t) - Y(t)|| \leqslant \frac{\beta}{|\alpha - \beta l|}\epsilon^{1-\delta} = C\epsilon^{1-\delta},$$

where C is a constant independent of ϵ.

3. **Stability theorems for the distribution of the composed random variable**

Let us consider the composed random variable η in (1.1)

$$\eta = \sum_{k=1}^{\nu} \xi_k.$$

Suppose that $(X_1, X_2, ..., X_n)$ is n independent observations on η and that the absolute moments $E(|\eta|^k)$ for $k = 1, 2, 3, 4$ are finite.

We put

$$\lambda_k = \sum_{i=1}^{n} X_i^k \quad (k = 1, 2, 3, 4).$$

• $T_1 = A_1\lambda_4 + 3B_1\lambda_2^2 + 2C_1\lambda_1\lambda_3 + 6\lambda_2\lambda_1^2 - \lambda_1^4$, where

$$A_1 = n(5-n); B_1 = n^2 - 5n + 5; C_1 = (n^2 - 5n + 10) \tag{3.1}$$

• $T_2 = A_2\lambda_4 + 3B_2\lambda_2^2 - C_2\lambda_1\lambda_3 + 6\lambda_2\lambda_1^2 - \lambda_1^4 + (n-2)(n-3)(\lambda_1^2 - \lambda_2)$

$$A_2 = n(5-n); B_2 = n^2 - 5n + 5; C_2 = n^2 - 5n + 10 \tag{3.2}$$

• $T_3 = A_3\lambda_6 + B_3\lambda_3\lambda_1 + C_3\lambda_4\lambda_2 + E_3\lambda_1 + F_3\lambda_3 + G_3\lambda_1\lambda_2\lambda_3 + 2\lambda_1^6$

$$+(n-4)(n-5)[M_3\lambda_4 + N_3\lambda_1\lambda_2 + P_3\lambda_1^2 + \lambda_1\lambda_2 - \lambda_1^4]$$

$$+H_3\lambda_3\lambda_1^2 + K_3\lambda_2\lambda_1^2 + L_3\lambda_2\lambda_1^4,$$

where

$$A_3 = -4n(n-1)(n-2); B_3 = 24(n^2 - 3n + 2)$$

$$C_3 = n^4 - 6n^3 - 5n^2 - 60n - 120$$

$$D_3 = -n^3 + 6n^2 - 65n + 1; F_3 = 3(-n^3 + 10n^2 - 35n + 40)$$

$$E_3 = -n^4 + 12n^2 - 35n + 20; G_3 = 5(n^2 - 3n + 5), L_3 = -6n$$

$$M_3 = n(5-n); N_3 = 2(-n^2 + 5n - 13); P_3 = 3(n^2 - 5n + 7) \tag{3.3}$$

• $T_4 = A_4\lambda_4 + 3B_4\lambda_2^2 + 2C_4\lambda_1\lambda_3 + 6\lambda_2\lambda_1^2 - \lambda_1^4$, where

$$A_4 = \frac{-n^4 + 5n^3 - 6}{n^2 - 1}; B_4 = \frac{n^4 - 5n^3 + 5n^2 + 1}{n^2 - 1}; C_4 = \frac{-n^4 + 5n^3 - 10n^2 + 4}{n^2 - 1} \tag{3.4}$$

• $T_5 = 3\lambda_2^2 - 2\lambda_1\lambda_3 - \lambda_4$ (3.5)

• $T_6 = 3\lambda_2^2 - 2\lambda_1\lambda_3 - \lambda_4$ (3.6)

• $T_7 = A_7\lambda_3\lambda_1 + B_7\lambda_2^2 + C_7\lambda_1^2\lambda_2 + H_7\lambda_4 + 2\lambda_1^4$, where

$$A_7 = n^2 - n + 10; B_7 = -n^2 + 7n - 6; C_7 = -2(n+3); H_7 = -4n \tag{3.7}$$

(notice that the statistics T_1, T_5 are considered in [2] and the statistics T_6, T_7 are considered in [4]).

Definition 3.1. *Let X and Y be two random variables with $EY < \infty$. Y is said to be ϵ- zero regression with respect to X if*

$$|E(Y/X)| \leqslant \epsilon. \tag{3.8}$$

Definition 3.2. *The composed r.v. η with the distribution $\Psi_\epsilon(x)$) is called r.v. with the $\epsilon-$ approximate distribution function $\Psi_0(t)$ if $\lambda(\Psi_\epsilon; \Psi_0) \leqslant \epsilon$, where metric $\lambda(.;.)$ is defined as follows*

$$\lambda(\mathbf{\Psi}_\epsilon; \mathbf{\Psi_0}) = \min_{\mathbf{T>0}} \quad \max\{\max_{|\mathbf{t}|\leqslant \mathbf{T}} \ \frac{\mathbf{1}}{\mathbf{2}}|(\psi_\epsilon(\mathbf{t}) - \psi_{\mathbf{0}}(\mathbf{t}))|; \frac{\mathbf{1}}{\mathbf{T}}\}, \tag{3.9}$$

where $\psi_0(t), \psi_\epsilon(t)$ are the characteristic functions corresponding to the distributions $\Psi_0(x), \Psi_\epsilon(x)$ respectively.

Now we obtain the following stability theorems:

Theorem 3.1. *If the statistic $T_i (i = 1, 2, 3, 4, 5, 6, 7)$ is ϵ-zero regression with respect to the λ_1 then the $\psi_{i\epsilon}(i = 1, 2, 3, 4, 5, 6, 7)$ satisfies the following differential equations with the same left sides as in (1.2), (1.3), (1.4), (1.5), (1.6), (1.7) but their right sides are the following functions:*

$$\frac{r(t)}{i^4(n-1)(n-2)(n-3)n}, \quad for (1.2) \tag{3.10}$$

where $r(0) = 0, r(t) = \overline{r(-t)}, |r(t)| \leqslant \epsilon \quad \forall t,$

$$\frac{r(t)}{i^4(n-1)(n-2)(n-3)n}, \quad for (1.3) \tag{3.11}$$

where $r(0) = 0, r(t) = \overline{r(-t)}, |r(t)| \leqslant \epsilon \quad \forall t,$

$$\frac{r(t)}{i^4 n(n-1)(n-2)(n-3)(n-4)(n-5)}, \quad for (1.4) \tag{3.12}$$

where $r(0) = 0, r(t) = \overline{r(-t)}, |r(t)| \leqslant \epsilon \quad \forall t,$

$$\frac{(n+1)r(t)}{(n-2)(n-3)n}, \quad for (1.5) \tag{3.13}$$

where $r(0) = 0, r(t) = \overline{r(-t)}, |r(t)| \leqslant \epsilon \quad \forall t,$

$$\frac{r(t)}{i^4(n-1)n}, \quad for (1.6) \tag{3.14}$$

where $r(0) = 0, r(t) = \overline{r(-t)}, |r(t)| \leqslant \epsilon \quad \forall t,$

$$\frac{r(t)}{i^4(n-1)n}, \quad for(1.7) \tag{3.15}$$

where $r(0) = 0, r(t) = \overline{r(-t)}, |r(t)| \leqslant \epsilon \quad \forall t,$

$$\frac{r(t)}{i^4 n(n-1)(n-2)(n-3)}, \quad for(1.8) \tag{3.16}$$

where $r(0) = 0, r(t) = \overline{r(-t)}, |r(t)| \leqslant \epsilon \quad \forall t.$

Theorem 3.2. *Let* $\Psi_i(x)(i = 1, 2, 4, 5, 6, 7)$ *be distribution functions with respect to the characteristic functions* $\psi_i(t)$ *of the composed ramdom variable where* $\psi_i(t)$ *satisfy the differential equation* $(1.2), (1.3), (1.5), (1.6), (1.7), (1.8)$.

Suppose that $\Psi_{i\epsilon}(x)$ *is distribution of the composed random variable corresponding to the characteristic function* $\psi_{i\epsilon}(t)$, *which is the solution of the equation with the same left side of the equation* (1.2), (1.3), (1.5), (1.6), (1.7), (1.8), *respectively, but with the right side defined by* (3.10), (3.11), (3.13), (3.14), (3.15), (3.16), *respectively, then the distribution function of composed random variable* $\Psi_{i\epsilon}(x)$ *is* $C\gamma(\epsilon)$*-approximate* $\Psi_i(x)$ *respectively, where* $\gamma(\epsilon) = \max\{\epsilon^{1-\delta}, \frac{1}{\delta \ln(\frac{1}{\epsilon})}\}$ *for* ϵ *is small enough positive number and* C *is a constant independent of* ϵ, $(0 < \delta < 1)$.

The proof of the theorem 3.1 is carried out similarly as in proof of the theorem in [4] by using definition 3.1.

The conclusion of the theorem 3.2 follows directly from the above definition 3.2 and Theorem 2.1 and with notice that $f(x_1, x_2, ..., x_n)$ in (1.11) and (1.12) to be continuous and piecewise smooth on its domain, therefore it satisfies the Lipschitz's condition.

Notice that the differential equation with the left side (1.4) and right side (3.12) does not satisfy the condition (1.19), therefore the theorem 3.2 is not valid for this case.

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