

ON THE CONVERGENCE RATES OF PICARD, MANN AND ISHIKAWA ITERATIONS OF GENERALIZED CONTRACTIVE OPERATORS

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Abstract. The convergence rates of Picard, Mann and Ishikawa iterations have been compared by several authors for a class of quasi-contractive maps defined on an arbitrary closed convex subset of a Banach space (e.g. [1], [3] and [10]). In this paper, a comparison of the convergence rates of those iterations are studied for a more general class of operators called the generalized contractive operators.

1. Introduction

Let X be a real Banach space, and C a nonempty convex subset of X . Let T be a self map of C , and let $p_o, x_o, y_o, z_o \in C$. The Picard iteration is defined by

$$p_{n+1} = Tp_n. \quad (1)$$

The Mann iteration (see [7]) is defined by

$$x_{n+1} = (1 - \alpha_n)x_n + \alpha_nTx_n. \quad (2)$$

The Ishikawa iteration (see [6]) is defined by

$$y_{n+1} = (1 - \alpha_n)y_n + \alpha_nTz_n \quad (3)$$

$$z_n = (1 - \beta_n)y_n + \beta_nTy_n \quad (4)$$

where $\{\alpha_n\} \subset (0, 1)$, $\{\beta_n\} \subset [0, 1)$.

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Definition 1. [18]. Let (X, d) be a metric space. $T : X \rightarrow X$ will be called a *Zamfirescu operator* if there exist the real numbers a, b, c satisfying $0 < a < 1$, $0 < b, c < 1/2$ such that for each pair $x, y \in C$, at least one of the following is true:

- (i) $d(Tx, Ty) \leq ad(x, y)$;
- (ii) $d(Tx, Ty) \leq b[d(x, Tx) + d(y, Ty)]$;
- (iii) $d(Tx, Ty) \leq c[d(x, Ty) + d(y, Tx)]$.

Definition 2. [4]. Let T be a mapping of a metric space (X, d) into itself. A mapping T is called a *quasi-contraction* if for some $0 \leq k < 1$ and all $x, y \in X$,

$$d(Tx, Ty) \leq k \cdot \max\{d(x, y); d(x, Tx); d(y, Ty); d(x, Ty); d(y, Tx)\}. \quad (5)$$

Clearly a Zamfirescu operator is a quasi-contraction map. Quasi-contraction map is one of the most general contractive maps. For results on quasi-contraction maps see [4-5],[14-15] and [17].

Definition 3. [8]. Let T be a mapping of a metric space (X, d) into itself. A mapping T will be called a *generalized contractive operator* if for some $0 \leq k < 1$ and all $x, y \in X$,

$$d(Tx, Ty) \leq k \cdot \max\{d(x, y); d(x, Tx); d(y, Ty); d(x, Ty) + d(y, Tx)\}. \quad (6)$$

A *generalized contractive operator* is more general than a *quasi-contraction* as can be seen from the following example.

Example. [8]. Let $X = R$ with the usual metric. Define $T : X \rightarrow X$ by $Tx = x$. Clearly T is a generalized contractive operator. In fact, $d(x, Ty) + d(y, Tx) = 2d(x, y)$, $d(Tx, Ty) = d(x, y)$. Let $k = \frac{3}{4}$. Then $d(Tx, Ty) \leq k\{d(x, Ty) + d(y, Tx)\}$. However T is not a quasicontraction.

The Ishikawa iteration and the Mann iteration converge to a fixed point of T when T is a *Zamfirescu operator* defined on a closed convex set of a Banach space (see [2], [9]). The Picard iteration converges faster than the Mann iteration [3] while the Mann iteration converges faster than the Ishikawa iteration [1] when dealing with the same class of Zamfirescu operators defined on a closed convex subset of a Banach space. In [5] it was shown that the Ishikawa iteration converges to the fixed

point of T when T is a *quasi-contraction* map defined on a closed convex set of a Banach space. Also the Picard iteration converges faster to the fixed point of T than the Mann iteration [10] while the Mann iteration converges faster than the Ishikawa iteration [11] when T is a *quasi-contraction*. That answers the question posed in [3]. In this paper, we investigate the convergence rate of the Picard, Mann and Ishikawa iteration when dealing with a more general class of operators called the *generalized contractive operators* (6). It was proved that the Picard iteration converges to the fixed point of T faster than the Mann iteration and the Mann iteration converges faster than the Ishikawa iteration when T is a *generalized contractive operator*. It should be noted that the Picard iteration converges to the fixed point of T when T is a *generalized contractive operator* [13] while both the Ishikawa and consequently the Mann iterations of this class of maps also converges to the fixed point of T [12].

The definitions and the methodology of Berinde [3], also used in [1] and [10], will be adopted .

Definition 4. [3]. Let $\{a_n\}_{n=0}^{n=\infty}$ and $\{b_n\}_{n=0}^{n=\infty}$ be two sequences of real numbers that converge to a and b respectively, and assume there exists

$$l = \lim_{n \rightarrow \infty} \frac{|a_n - a|}{|b_n - b|}.$$

If $l = 0$, then we say that $\{a_n\}_{n=0}^{n=\infty}$ converges faster to a than $\{b_n\}_{n=0}^{n=\infty}$ to b .

Definition 5. [3]. Let $\{u_n\}_{n=0}^{n=\infty}$ and $\{v_n\}_{n=0}^{n=\infty}$ be two fixed point iteration procedures that converge to the same fixed point p on a normed space X such that the error estimates

$$\|u_n - p\| \leq a_n, \quad n = 0, 1, 2, \dots \tag{7}$$

and

$$\|v_n - p\| \leq b_n, \quad n = 0, 1, 2, \dots \tag{8}$$

are available, where $\{a_n\}_{n=0}^{n=\infty}$ and $\{b_n\}_{n=0}^{n=\infty}$ are two sequences of positive numbers (converging to zero). If $\{a_n\}_{n=0}^{n=\infty}$ converges faster than $\{b_n\}_{n=0}^{n=\infty}$, then we say that $\{u_n\}_{n=0}^{n=\infty}$ converges faster to p than $\{v_n\}_{n=0}^{n=\infty}$.

2. The main results

Theorem 1. *Let K be a nonempty closed convex subset of a Banach space X and let $T : K \rightarrow K$ be a generalized contractive operator (6). Then*

- 1) *T has a unique fixed point p in X ;*
- 2) *The Picard iteration $\{p_n\}$ defined by $Tp_n = p_{n+1}$ converges to p for any $p_o \in K$;*
- 3) *The Mann iteration $\{x_n\}$, defined by $x_{n+1} = (1-\alpha_n)x_n + \alpha_nTx_n$, $n=1,2,\dots$ such that $\sum \alpha_n = \infty$, converges strongly to p for any $x_o \in K$;*
- 4) *The Picard iteration converges to p faster than Mann iteration.*

Proof. For the proofs of 1) and 2) see ([13]). The Ishikawa iteration of T converges to p [12]. By setting $\beta_n = 0$ for all n , it is clear that the Mann iteration converges too.

We now proof (4). Since T is a generalized contractive operator (6), then,

$$\begin{aligned} \|Ty - Tx\| &\leq k \max\{\|y - x\|, \|x - Tx\|, \|y - Ty\|, \\ &\|x - Ty\| + \|y - Tx\|\}. \end{aligned}$$

If $\|Ty - Tx\| \leq k\|y - Ty\|$, then

$$\|Ty - Tx\| \leq k\{\|y - x\| + \|x - Tx\| + \|Tx - Ty\|\}$$

and so,

$$\|Ty - Tx\| \leq \frac{k}{1-k}\{\|y - x\| + \|x - Tx\|\}. \quad (9)$$

If $\|Ty - Tx\| \leq k\{\|x - Ty\| + \|y - Tx\|\}$, then,

$$\|Ty - Tx\| \leq k\{\|x - Tx\| + \|Tx - Ty\| + \|y - x\| + \|x - Tx\|\}$$

which, after computing, gives

$$\|Tx - Ty\| \leq \frac{k}{1-k}\{\|y - x\| + 2\|x - Tx\|\}. \quad (10)$$

Denote $\delta = \max\{k, \frac{k}{1-k}\} = \frac{k}{1-k}$. Then in view of (9) and (10), inequality (6) gives

$$\|Ty - Tx\| \leq \delta\{\|y - x\| + 2\|x - Tx\|\}. \quad (11)$$

Suppose p is a fixed point of T , then, if $x = p$ and $y = p_n$, from (11) we obtain

$$\|Tp_n - p\| \leq \delta\|p_n - p\|. \quad (12)$$

If we assume Picard approximation technique in (11) by assuming that $Tp_n = p_{n+1}$ for all n , we obtain

$$\|p_{n+1} - p\| \leq \delta\|p_n - p\|$$

which inductively gives

$$\|p_{n+1} - p\| \leq \delta^n\|p_1 - p\|, \quad n \geq 0. \quad (13)$$

Following the same procedure in proving (10), it can be shown that

$$\|Tx - Ty\| \leq \delta\|x - y\| + 2\delta\|y - Tx\| \quad (14)$$

for all $x, y \in K$ where $\delta = \frac{k}{1-k}$.

Let $\{x_n\}_{n=0}^{\infty}$ be the Mann iteration as defined in the Theorem and $x_0 \in K$ arbitrary. Then

$$\begin{aligned} \|x_{n+1} - p\| &= \|(1 - \alpha_n)x_n + \alpha_nTx_n - (1 - \alpha_n + \alpha_n)p\| \\ &= \|(1 - \alpha_n)(x_n - p) + \alpha_n(Tx_n - p)\| \\ &\leq (1 - \alpha_n)\|x_n - p\| + \alpha_n\|Tx_n - p\|. \end{aligned} \quad (*)$$

If $x = p$ and $y = x_n$ in (14) we obtain

$$\|Tx_n - p\| \leq \delta\|x_n - p\| + 2\delta\|x_n - p\| = 3\delta\|x_n - p\|$$

and therefore by (*) we obtain

$$\|x_{n+1} - p\| \leq [1 - \alpha_n + 3\delta\alpha_n]\|x_n - p\| \leq [1 + 3\delta\alpha_n + \delta]\|x_n - p\|, \quad n = 0, 1, 2, \dots$$

which implies that

$$\|x_{n+1} - p\| \leq \prod_{k=1}^n [1 + 3\delta\alpha_k + \delta]\|x_1 - p\|, \quad n = 0, 1, 2, \dots \quad (15)$$

In order to compare $\{p_n\}$ and $\{x_n\}$ we must compare δ^n and $\prod_{k=1}^n [1 + 3\delta\alpha_k + \delta]$.

We first note that $\delta < 1 + 3\delta\alpha_k + \delta$ for each k . Therefore $\frac{\delta}{1+3\delta\alpha_k+\delta} < 1$ for each k . Hence

$$\lim_{n \rightarrow \infty} \frac{\delta^n}{\prod_{k=1}^n [1 + 3\delta\alpha_k + \delta]} \rightarrow 0.$$

This shows that the Picard iteration converges faster than the Mann iteration.

Corollary 1. [10]. *Let K be a nonempty closed convex subset of a Banach space X and let $T : K \rightarrow K$ be a quasi-contraction map (5). Then*

- 1) T has a unique fixed point p in X ;
- 2) The Picard iteration $\{p_n\}$ defined by $Tp_n = p_{n+1}$ converges to p for any $p_o \in K$;
- 3) The Mann iteration $\{x_n\}$, defined by $x_{n+1} = (1-\alpha_n)x_n + \alpha_nTx_n$, $n=1,2,\dots$ such that $\sum \alpha_n = \infty$, converges strongly to p for any $x_o \in K$;
- 4) The Picard iteration converges to p faster than Mann iteration.

Corollary 2. [3, Theorem 4]. *Let X be a Banach space, K a closed convex subset of X , and $T : K \rightarrow K$ a Zamfirescu operator. Then*

- 1) T has a unique fixed point p in X ;
- 2) The Picard iteration $\{p_n\}$ defined by $Tp_n = p_{n+1}$ converges to p for any $p_o \in K$;
- 3) The Mann iteration $\{x_n\}$, defined by $x_{n+1} = (1-\alpha_n)x_n + \alpha_nTx_n$, $n=1,2,\dots$ such that $\sum \alpha_n = \infty$, converges strongly to p for any $x_o \in K$;
- 4) Picard iteration converges faster than Mann iteration.

Observe that Corollary 1 is more general than Corollary 2 which is the main result in [3].

Theorem 2. *Let K be a nonempty closed convex subset of a Banach space X and let $T : K \rightarrow K$ be a generalized contraction map (6). Let $\{x_n\}$ and $\{y_n\}$ be the Mann and Ishikawa iterations respectively defined by (2) and (3)-(4) for $x_o, y_o \in K$ with $\{\alpha_n\}$ and $\{\beta_n\}$ real sequences such that $0 \leq \alpha_n, \beta_n \leq 1$ and $\sum \alpha_n = \infty$. Then $\{x_n\}$ and $\{y_n\}$ converge strongly to the unique fixed point of T , and moreover, the Mann iteration converges to the fixed point of T faster than the Ishikawa iteration.*

Proof. The Ishikawa iteration (3)-(4) converges strongly to the unique fixed point of T (e.g. see [12]). Consequently, if $\beta_n = 0$ for all n , the Mann iteration converges strongly to the unique fixed point of T . Since the fixed point of T is unique [13], then both iterations must converge to the same fixed point which we denote by p .

It is not difficult to see that the quasi-contraction map satisfies the following inequalities

$$\|Tx - Ty\| \leq \delta\{\|x - y\| + 2\|x - Tx\|\} \quad (16)$$

$$\|Tx - Ty\| \leq \delta\{\|x - y\| + 2\|y - Tx\|\} \quad (17)$$

for all $x, y \in K$ where $\delta = \max\{k, \frac{k}{1-k}\} = \frac{k}{1-k}$.

Let $\{x_n\}$ be the Mann iteration associated with T , then, in view of (2), we have

$$\|x_{n+1} - p\| \leq (1 - \alpha_n)\|x_n - p\| + \alpha_n\|Tx_n - p\|. \quad (18)$$

Suppose $x = p$ and $y = x_n$, (16) becomes

$$\|Tx_n - p\| \leq \delta\|x_n - p\|. \quad (19)$$

In view of (18) and (19), we have

$$\|x_{n+1} - p\| \leq (1 - \alpha_n)\|x_n - p\| + \alpha_n\delta\|x_n - p\| = [1 - \alpha_n(1 - \delta)]\|x_n - p\|. \quad (20)$$

Hence

$$\|x_{n+1} - p\| \leq \prod_{k=1}^n [1 - \alpha_k(1 - \delta)]\|x_1 - p\|, \quad n = 0, 1, 2, \dots \quad (21)$$

It is clear that

$$1 - \alpha_k(1 - \delta) > 0 \quad \forall k = 0, 1, 2, \dots \quad (22)$$

Similarly, let $\{y_n\}$ be the Ishikawa iteration defined in (3)-(4), then, we have

$$\|y_{n+1} - p\| \leq (1 - \alpha_n)\|y_n - p\| + \alpha_n\|Tz_n - p\|. \quad (23)$$

If $x = p$ and $y = z_n$ in (17), we have

$$\|Tz_n - p\| \leq \delta\|z_n - p\| + 2\delta\|z_n - p\| = 3\delta\|z_n - p\|. \quad (24)$$

If $x = p$ and $y = y_n$ in (17), we have

$$\|Ty_n - p\| \leq \delta\|y_n - p\| + 2\delta\|y_n - p\| = 3\delta\|y_n - p\|. \quad (25)$$

We know by (4) that

$$\|z_n - p\| \leq (1 - \beta_n)\|y_n - p\| + \beta_n\|Ty_n - p\|. \quad (26)$$

In view of (23)-(26), we have

$$\begin{aligned} \|y_{n+1} - p\| &\leq (1 - \alpha_n)\|y_n - p\| + 3\delta\alpha_n\|z_n - p\| \\ &\leq (1 - \alpha_n)\|y_n - p\| + 3\delta\alpha_n[(1 - \beta_n)\|y_n - p\| \\ &\quad + \beta_n\|Ty_n - p\|] \\ &= (1 - \alpha_n)\|y_n - p\| + 3\delta\alpha_n(1 - \beta_n)\|y_n - p\| \\ &\quad + 3\delta\alpha_n\beta_n\|Ty_n - p\| \\ &= [(1 - \alpha_n) + 3\delta\alpha_n(1 - \beta_n) + 9\alpha_n\beta_n\delta^2]\|y_n - p\| \\ &= [1 - \alpha_n(1 - 3\delta + 3\beta_n\delta - 9\beta_n\delta^2)]\|y_n - p\| \\ &= [1 - \alpha_n(1 - 3\delta)(1 + 3\beta_n\delta)]\|y_n - p\|. \quad (**) \end{aligned}$$

Since $(1 - 3\delta)(1 + 3\beta_n\delta) < 1 - 9\delta^2 \leq 1$, it is clear that

$$1 - \alpha_n(1 - 3\delta)(1 + 3\beta_n\delta) > 0 \quad \forall n = 0, 1, 2, \dots \quad (27)$$

We consider the following two cases.

Case (1). Let $\delta \in (0, 1/3]$. Hence

$$1 - \alpha_n(1 - 3\delta)(1 + 3\beta_n\delta) \leq 1 \quad \forall n = 0, 1, 2, \dots \quad (28)$$

(**) then becomes

$$\|y_{n+1} - p\| \leq \|y_n - p\| \quad \forall n \quad (29)$$

and hence

$$\|y_{n+1} - p\| \leq \|y_1 - p\| \quad \forall n. \quad (30)$$

If we compare the coefficients of (21) and (30), and using Definition 5 so that

$$a_n = \prod_{k=1}^n [1 - \alpha_k(1 - \delta)] \quad \text{and} \quad b_n = 1, \quad (31)$$

we have $\lim_{n \rightarrow \infty} (a_n/b_n) = 0$

Case (ii). Let $\delta > 1/3$. In this case we have

$$1 - \alpha_n(1 - 3\delta)(1 + 3\beta_n\delta) \leq 1 - \alpha_n(1 - 9\delta^2) \quad (32)$$

and so (**) becomes

$$\|y_{n+1} - p\| \leq [1 - \alpha_n(1 - 9\delta^2)]\|y_n - p\| \quad \forall n. \quad (33)$$

Hence

$$\|y_{n+1} - p\| \leq \prod_{k=1}^n [1 - \alpha_k(1 - 9\delta^2)]\|y_1 - p\|. \quad (34)$$

Comparing (21) and (34) and using Definition 5, we have

$$a_n = \prod_{k=1}^n [1 - \alpha_k(1 - \delta)] \text{ and } b_n = \prod_{k=1}^n [1 - \alpha_k(1 - 9\delta^2)]. \quad (35)$$

Clearly, $a_n \geq 0$ and $b_n \geq 0 \quad \forall n$ and $\frac{a_n}{b_n} = \prod_{k=1}^n \frac{1 - \alpha_k(1 - \delta)}{1 - \alpha_k(1 - 9\delta^2)}$. Also

$$\frac{\min[1 - \alpha_k(1 - \delta), k = 1, 2, \dots, n]}{\max[1 - \alpha_k(1 - 9\delta^2), k = 1, 2, \dots, n]} < 1.$$

Since $\prod_{k=1}^n \frac{1 - \alpha_k(1 - \delta)}{1 - \alpha_k(1 - 9\delta^2)} < \left(\frac{\min[1 - \alpha_k(1 - \delta), k=1, 2, \dots, n]}{\max[1 - \alpha_k(1 - 9\delta^2), k=1, 2, \dots, n]}\right)^n$ then $\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = 0$.

Therefore in both cases $\{a_n\}$ converges faster than $\{b_n\}$ and hence the Mann iteration converges faster than the Ishikawa iteration to the fixed point p of T .

In view of Theorems 1 and 2, we have the following results.

Corollary 3. *Let K be a nonempty closed convex subset of a Banach space X and let $T : K \rightarrow K$ be a generalized contractive map (6). Then*

- 1) T has a unique fixed point p in X ;
- 2) The Picard iteration $\{p_n\}$ defined by $Tp_n = p_{n+1}$ converges to p for any $p_o \in K$;

3) The Picard iteration converges faster to the fixed point of T than Mann iteration (2); and the Mann iteration converges faster than the Ishikawa iteration (3)-(4).

Corollary 4. *Let K be a nonempty closed convex subset of a Banach space X and let $T : K \rightarrow K$ be a quasi-contraction (5). Then*

- 1) T has a unique fixed point p in X ;

2) The Picard iteration $\{p_n\}$ defined by $Tp_n = p_{n+1}$ converges to p for any $p_o \in K$;

3) The Picard iteration converges faster to the fixed point of T than Mann iteration (2); and the Mann iteration converges faster than the Ishikawa iteration (3)-(4).

Corollary 5. ([1],[3]). Let K be a nonempty closed convex subset of a Banach space X and let $T : K \rightarrow K$ be a Zamfirescu operator. Then

1) T has a unique fixed point p in X ;

2) The Picard iteration $\{p_n\}$ defined by $Tp_n = p_{n+1}$ converges to p for any $p_o \in K$;

3) The Picard iteration converges faster to the fixed point of T than Mann iteration; and the Mann iteration converges faster than the Ishikawa iteration (3)-(4).

Remarks. 1. The technique of our proofs is due to [3] and has been used by several authors, e.g. see [16].

2. Ishikawa iteration has two parameters, $\{\alpha_n\}$ and $\{\beta_n\}$; the Mann iteration has only one parameters $\{\alpha_n\}$ while the Picard iteration has none. It appears that the more the parameters for an iteration process, the slower the rate of convergence. At least this is true in the case of Picard, Mann and the Ishikawa iterations when applied to generalized contraction maps. It is therefore an open problem whether this conjecture is true for other known iteration procedures and for a more general class of operators.

3. A *generalized contraction* map (see [14-15]) is a map satisfying the inequality

$$\|Tx - Ty\| \leq Q(M(x, y)), \tag{36}$$

where Q is a real-valued function satisfying

(a) $0 < Q(s) < s$ for each $s > 0$ and $Q(0) = 0$,

(b) Q is non-decreasing on $(0, \infty)$,

(c) $g(s) = s/(s - Q(s))$ is non-increasing on $(0, \infty)$,

$$M(x, y) = \max\{\|x - y\|, \|x - Tx\|, \|y - Ty\|, \|x - Ty\|, \|y - Tx\|\}. \tag{37}$$

The Mann and the Ishikawa iterations are equivalent when dealing with generalized contraction maps [15] i.e. if the Mann iteration converges to the fixed point of T , then the Ishikawa iteration converges to the fixed point of T and if the Ishikawa iteration converges, then the Mann iteration converges to the fixed point of T . It is still an open problem as to which of the iterations converges faster when T is a generalized contraction map. Suppose (37) is replaced with $M(x, y) = \max\{\|x - y\|, \|x - Tx\|, \|y - Ty\|, \|x - Ty\| + \|y - Tx\|\}$, will the Mann and the Ishikawa iterations still be equivalent? Will the Mann iteration still converge faster than the Ishikawa iteration to the unique fixed point of T ?

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