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BOOK REVIEWS

Charles Chidume, ***Geometric Properties of Banach Spaces and Nonlinear Iterations***, Lectures Notes in Mathematics, Vol. 1965, Springer-Verlag London Limited, 2009, xvii+326 pp; ISBN: 978-1-84882-189-7, e-ISBN: 978-1-84882-190-3, DOI: 10.1007/978-1-84882-190-3.

The classical Banach Contraction Principle asserts that every contraction T on a complete metric space (X, ρ) has a unique fixed point $\bar{x}$ and the Picard iteration $x_{n+1} = Tx_n,\ n \geq 0$, converges to $\bar{x}$, for every $x_0 \in X$. This result is no longer true for nonexpansive mappings (i.e., such that $\rho(Tx, Ty) \leq \rho(x, y),\ x, y \in X$), even when X is a weakly compact subset of a Banach space E. The study of fixed points for nonexpansive mappings defined on convex subsets of Banach spaces has put in evidence strong connections to the geometric properties of the underlying Banach space - normal structure, rotundity and smoothness properties characterized in terms of various constants and moduli. Also, even if T has a fixed point, the Picard iteration could not converge to the fixed point of T. By a clever modification of Picard iteration, namely $x_{n+1} = \frac{1}{2}(x_n + Tx_n),\ n \geq 0$, Krasnoselki (1955) succeeded to obtain convergence to the fixed point in some cases. Later extensions to Kransnoselski's idea were given by Mann (1953): $x_{n+1} = (1 - \alpha_n)x_n + \alpha_n Tx_n,\ n \geq 0$, (called also the Krasnoselski-Mann iteration), and Ishikawa (1974): $y_n = (1-\beta_n)x_n + \beta_n Tx_n,\ x_{n+1} = (1 - \alpha_n)x_n + \alpha_n Ty_n,\ n \geq 0$. The hypotheses on (α_n) in Mann iteration are: (i) $\alpha_n \in (0, 1),\ \lim_n \alpha_n = 0$, and (ii) $\sum_{n=0}^{\infty} \alpha_n = \infty$, with some similar conditions in Ishikawa's method.

The aim of the present book is to give an introduction to this very active area of investigation. The basic results from the geometry of Banach spaces are presented in the first five chapters of the book: 1. *Some geometric properties of Banach spaces*, 2. *Smooth spaces*, 3. *Duality map in Banach spaces*, 4. *Inequalities in uniformly convex spaces*, and 5. *Inequalities in uniformly smooth spaces.*

The study of iterative procedures starts in Chapter 6. *Iterative methods for fixed points of nonexpansive mappings*, and continues along the rest of the chapters (there are 23) with topics as: descent methods for variational inequalities (in Ch. 7), iterative procedures for zeros of generalized accretive operators (Chapters 8 and 9), iterations for pseudo-contractive mappings (Chapters 10 to 12), iterative methods for generalized nonexpansive mappings (Ch. 14), for families of nonexpansive mappings (Chapters 15 to 17), for asymptotically nonexpansive mappings (Chapters 20 and 21) and for nonexpansive semigroups (Ch. 22). Chapter 13 is concerned with applications to Hammerstein integral equations while the last chapter, 23. *Single-valued accretive operators; Applications; Some open questions,* is concerned with continuity conditions implying the single-valuedness of set-valued accretive operators with applications to differential inclusions. This chapter contains also some open problems and recommendations for further reading.

Based on a rich bibliography (561 items) and including many original contributions of the author, the book is of great help for graduate and postgraduate students, as well as for researchers interested in fixed point theory, geometry of Banach spaces and numerical solution of various kinds of equations - operator differential equations, differential inclusions,variational inequalities.

A good companion in reading it could be the recent book by V. Berinde, *Iterative Approximation of Fixed Points*, LNM 1912, Springer, Berlin 2007, dealing with similar topics, with emphasis on the Hilbert space setting. Together, these two books cover a lot of the research work done in the last 20 years in the field of iterative procedures for fixed points.

S. Cobzaş

Leon A. Takhtajan, ***Quantum mechanics for mathematicians***, Graduate Studies in Mathematics, Vol. 95, American Mathematical Society, Providence, Rhode Island 2008,x+278 pp, ISBN:978-0-8218-4630-8

From the ancient times, the development of physics stimulated the research in mathematics, leading to the discovery of new areas in mathematics. In this sense, the most known is the creation by L. Schwartz and S. L. Sobolev of distribution theory, as a response to some curious manipulation (at least for mathematicians) of some strange functions, the best known being the Dirac delta function. Up to the early 20th century classical physics, especially classical mechanics, was an integral part of curricula for students in mathematics. The situation is totally different with quantum physics, which uses very advanced mathematical tools, but has never been a part of the curriculum of graduate students in mathematics. In trying to fill this gap the author taught, for more that 14 years, a course on quantum mechanics for students in mathematics at Stony Brook University. In fact, mathematics and physics are strongly interconnected, some topics as, for instance, string theory being chapters of mathematics and physics as well.

The inspiration for this course came for a similar one taught by L. D. Faddeev at Leningrad (now Sankt Petersburg) State University. The AMS published a course by L. D. Faddeev, *Elementary introduction to quantum field theory. Quantum fields and strings: a course for mathematicians,* AMS 1999, and the translation of an another one: L. D. Faddeev and O. A. Yakubovskiĭ, *Lectures on Quantum Mechanics for Mathematics Students* with an appendix by Leon Takhtajan, AMS, 2009, originally published in Russian with Leningrad University Publishers, Leningard 1980. The first part of the book is based mainly on these course, but using more advanced mathematical tools, complete rigorous proofs and going beyond the topics presented there.

The book is divided into two parts: 1. *Foundations,* and 2. *Functional methods and supersymmetry.* The first part starts with a chapter, 1. *Classical mechanics,* where the exposition follows the traditional line, starting with the Lagrangian formalism and introducing the Hamiltonian through the Legendre transform as, for instance, in the classical treatise by V. I. Arnold.

The study of quantum mechanics starts in Chapter 2. *Basic principles of quantum mechanics,* based on the Dirac-von Neumann axioms: an infinite dimensional separable complex Hilbert space $\mathcal{H}$, called the space of states, the family of observables formed of the space of self-adjoint operators on $\mathcal{H}$, and the states $\mathcal{S}$ which are the positive trace class operators M on $\mathcal{H}$ with Tr $M = 1$. Pure states are the projection operators onto the one-dimensional spaces, all of the other states being called mixed states.

The first part continues with Chapters 3. *Schrödinger equation,* and 4. *Spin and identical particles.*

The second part contains the chapters 5. *Path integral formulation of quantum mechanics* (the Feynman approach to quantum mechanics), 6. *Integration in function spaces* (Wiener integration theory, the Feynman-Kac formula), 7. *Fermion systems* (anticommutativity relations, Grassmann algebra), and 8. *Supersymmetry.*

There are a lot of problems spread throughout the book. Some of these require to fill in some proofs, only sketched in the main text, while others refer to supplementary topics, not treated in the main text. For these last ones, exact references are given.

Each chapter ends with a section of *Notes and references,* containing bibliographical references and recommendations for further reading.

The bibliography contains a list of carefully chosen monographs, both classical and modern, on mathematics and physics, survey papers and some fundamental research papers, all being referred in the Notes section of the chapters.

By a cleaver selection of the material and the clear way of exposing it, the book is recommended for graduate students in mathematics looking for applications in physics, as well as for student in physics desiring to be acquainted, in a rigorous but, at the same time, quick and accessible manner, with the basic mathematical tools used in quantum mathematics. The devoted readers can follow the paths indicated by the author in problems and notes, to acquire a master introduction to the area.

Radu Precup

Stephen Lynch, *Dynamical Systems with Applications using MATHEMATICA®*, xvi+484 pp, Birkhäuser, Boston -Basel - Berlin, 2007, ISBN-13: 978-0-8176-4482-6

The book is a good introduction to dynamical systems theory. In the first part, after a short introduction to MATHEMATICA®, differential equations and dynamical systems are considered, with examples taken from mechanical systems, chemical kinetics, electric circuits, interacting species and economics. The second part is devoted to discrete dynamical systems, with many advanced examples from electromagnetic waves, optical resonators, chaos, fractals, neurodynamics.This book presents an original, cheap and powerful solution to the problem of analysis of large data sets.

The theory and applications are presented with the aid of the MATHEMATICA® package. Throughout the book, MATHEMATICA® is viewed as a tool for solving systems or producing exciting graphics. Each chapter contains a subsection with "Mathematica Commands in Text Format". The author suggests that the reader should save the relevant example programs. These programs can then be edited accordingly when attempting to solve the exercises at the end of each chapter. The solution combines C language, data base query and management, statistics and data visualization. The text is aimed at graduate students and working scientists in various branches of applied mathematics, natural sciences and engineering. The material is intelligible to readers with a general mathematical background. Fine details and theorems with proof are kept at a minimum. This book is informed by the research interests of the author which are nonlinear ordinary differential equations, nonlinear optics and fractals. Some chapters include recently published research articles and provide a useful resource for open problems in nonlinear dynamical systems.The book intends to be an alternative to classical statistical books: it does not separate descriptive and inferential An efficient tutorial guide to MATHEMATICA® is included. The knowledge of a computer language would be beneficial but not essential. The MATHEMATICA® programs are kept as simple as possible and the author's experience has shown that this method of teaching using MATHEMATICA® works well with computer laboratory class of small sizes.statistics, simple models are combined into more complex model in a hierarchical way and it is computer oriented. recommend "Dynamical Systems with Applications using MATHEMATICA®" as a good handbook for a diverse readership, for graduates and professionals in mathematics, physics, science and engineering. This book could be considered as a new and extended version of the following books, also written by the same author:

Dynamical Systems with Applications using MATLAB®, ISBN 978-0-8176-4321-8, and *Dynamical Systems with Applications using* MAPLE, ISBN 978-0-8176-4150-4.

Damian Trif

Dean Corbae, Maxwell B. Stinchcombe and Juraj Zeman, *An Introduction to Mathematical Analysis for Economic Theory and Econometrics*, Princeton University Press, Princeton and Oxford 2009, xxi+671 pp; ISBN: 978-0-691-11867-3,

The book covers a broad spectrum of topics from analysis, functional analysis and measure theory, needed for economic theory and econometrics. Its aim is to bridge the gap existing between the basic mathematical economics tools (calculus, linear algebra, constrained optimization) and the advanced economics texts, as, for instance, N. L. Stokey and R. E. Lucas, *Recursive Methods in Economic Dynamics*, Harvard University Press, 1989, which assume a working knowledge of functional analysis, measure theory and probability. In fact, one of the motivations to write such a textbook comes from the difficulties encountered by the students of the one of the authors to understand the above mentioned book. The present book contains a choice of topics from various areas of mathematics, as lattices, convex analysis, functional analysis, measure theory, probability, that are widely used in economics and econometrics. The book is self-contained in what concerns the mathematical part - almost any theorem used in proving some result is itself proved as well. Another feature of the book is the wealth of examples from economic theory and economics (whose understanding requires an undergraduate basic ground in economics), providing intuition and motivation for grasping the difficult mathematical ideas developed in the book.

The first part of this book (Chapters 1 to 6) was taught in the first-semester Ph.D. core sequence at the University of Pittsburgh and the University of Texas. The second part (Chapters 7 to 11) was taught as a graduate mathematical economics class. Chapters 1 to 6 cover basic mathematics for economics: 1. *Logic*, 2. *Set theory* (including lattices and Tarski's fixed point theorem with application to stable matchings), 3. *The space of real numbers*, 4. *The finite-dimensional metric space of real vectors*, 5. *Finite-dimensional convex analysis* (dealing with finite dimensional normed spaces, Kuhn-Tucker theorem, Lagrange multipliers, etc), and 6. *Metric spaces* (with applications to the space of probabilistic distribution functions on $\mathbb{R}$ equipped with Levy's metric). The set of real numbers is introduced *via* equivalence classes of Cauchy sequences of rational numbers. This construction as well as the notion of completeness (with different meanings in different contexts) form the red thread of the presentation along the book. The logic properties and set operations are presented in parallel, as paradigms of the same idea.

The second part of the book contains the chapters: 7. *Measure spaces and probability* (including convergence in distribution and Skorohod theorem), 8. *The $L^p(\Omega, \mathcal{F}, P)$ and ℓ^p spaces, $p \in [1, \infty]$* (with applications to regression analysis), 9. *Probabilities on metric spaces* (Polish metric spaces, Polish measure spaces, stochastic processes, a proof of the central limit theorem), 10. *Infinite-dimensional convex analysis* (containing an introduction to topological vector spaces and locally convex spaces, including compactness, Alaoglu-Bourbaki theorem, separation and Krein-Milman theorem, Schauder fixed point theorem), and 11. *Expanded spaces* (dealing with the basic constructions of nonstandard analysis).

Each chapter ends with a set of exercises and recommendation for further reading. These refer mainly to books where the topics of the corresponding chapter are treated at large.

By exposing in a self-contained, rigorous but accessible manner a lot of essential results from analysis, functional analysis, measure theory and probability used in economic theory and econometrics, the book is a very useful tool for students specializing in these disciplines. Even students in mathematics and researchers will find the results collected by the authors very useful as well.

S. Cobzaş

Wassim M. Haddad and Vijay Sekhar Chellaboina, ***Nonlinear Dynamical Systems and Control, A Lyapunov-Based Approach*** Princeton University Press, Princeton and Oxford 2008, xvi+948 pp; ISBN: ,

The book under review is a graduate-level textbook which presents and develop an extensive treatment of stability analysis and control design of nonlinear dynamical systems using the Lyapunov methods.

The book is structured in 14 chapters. The authors introduce the definition of dynamical systems and present a systematic development of the theory of nonlinear differential equations. There are presented the qualitative theory of existence, uniqueness, continuous dependence of solutions on the initial conditions for nonlinear differential equations, the stability theory for nonlinear dynamical systems generated by these nonlinear differential equations, Lyapunov stability theorems for time-invariant nonlinear dynamical systems, invariant set stability theorems, converse Lyapunov theorems and Lyapunov instability theorems. A chapter in advanced stability theory is also included. There are described the partial stability, stability theory for time-varying systems, Lagrange stability, boundedness and ultimate boundedness, input-to-state stability, finite-time stability, semistability and stability theorems via vector Lyapunov functions. The book continues with a chapter regarding the dissipative theory for nonlinear dynamical systems and a chapter regarding the stability and optimality of feedback dynamical systems. The input -output technique for dynamical systems is a tool used in the study of infinite-dimensional systems. The authors present the concept of input-output stability and then establish connections between input-output stability and Lyapunov stability. The nonlinear optimal control problem and the stability and optimality results for backstepping control problems are given in the next chapter. Extension to disturbance rejection and robust control of nonlinear dynamical systems are presented. The last two chapters contain the discrete-time extension of the aforementioned topics.

This book is an excellent textbook addressed to graduate students of applied mathematics, control theorists and engineers studying the stability theory of dynamical systems and controls. It is a rich source of materials for researchers interested in systems theory.

Marcel-Adrian Şerban

Alexander H.W. Schmitt, ***Geometric Invariant Theory and Decorated Principal Bundles***, European Mathematical Society, 2008, 389 pp, ISBN 978-3-03719-065-4.

Classifying the objects of a category is a fundamental mathematical problem which allows, whenever it is solved, the control and the manipulation of the objects of that category easily in various purposes. This type of problems is equally important and difficult, in most of the important categories being obtained just partial results. For instance the finitely generated Abelian groups, the one and two dimensional compact manifolds are completely classified, all these results being now classical. For *topological categories*, this problem might be even more difficult than it is for *algebraic* *categories*, due to some properties of objects and morphisms of algebraic nature, which are rather absent in the case of the objects and morphisms of topological nature. However, one can expect advances in the case of the topological categories, thanks to Homotopy Theory and Algebraic Topology, whenever the classification problem advances in the case of some algebraic categories.

The classification of algebraic varieties up to an isomorphism has two faces, one of them consists in classifying the smooth or mildly singular complex projective varieties up to an isomorphism and the second one consists in classifying all closed subvarieties of a certain complex projective space up to projective equivalences. These classification problems are either treated by means of various numerical invariants, such as *the Hilbert polynomial* of polarized varieties or by means of *moduli spaces* (i.e. algebraic varieties whose points are in natural one-to-one correspondence to the set of isomorphism classes of objects with fixed numerical invariants), which generated Mumford's Geometric Invariant Theory as a tool to construct such moduli spaces.

In this book the author constructs more general moduli spaces for semistable projective *ϱ-bumps* of a given topological type as well as moduli spaces for semistable affine pairs with prescribed topological type for the first component.

The book is structured in two large chapters with the following content:

The first chapter deals with Geometric Invariant Theory as developed by Mumford and starts with actions of reductive linear groups on vector spaces or projective spaces. The reductivity of the groups $GL_n(\mathbb{C})$, $SL_n(\mathbb{C})$, $GL_{n_1}(\mathbb{C})\times\cdots\times GL_{n_l}(\mathbb{C})$ is shown at the end of the first section. Next, one parameterizes the space of orbits of an algebraic group action by affine algebraic varieties and formulate the basic properties of such quotients. In section 1.3 one studies the classification of projective hypersurfaces of degree d in the projective space $\mathbb{P}_{n-1}$ up to projective equivalences, while section 1.4 is devoted to the fundamental concepts of Geometric Invariant Theory, which starts by studying *good* and *geometric quotients* and continues with linearizations of group actions. Finally, the last two sections of this first chapter deal with the Hilbert-Mumford criterion and a certain refinement of it, on the existence of one parameter subgroups $\lambda : \mathbb{C}^* \longrightarrow G$ of some reductive group G, linearly represented, such that $\lim_{z\to\infty} \lambda(z)\cdot\nu = 0$, for some special $\nu \in V$. Another problem is the existence of such a one parameter subgroup with fastest possible convergence as well as the uniqueness of such a subgroup.

The second chapter starts with an overview on principal bundles which is necessary to state the classification problem developed within the rest of the chapter and continues with a review on vector bundles on complex algebraic curves. The classification of topological vector bundles on smooth projective curves and the Riemann-Roch theorem for coherent sheaves are presented alongside a discussion on bounded families of vector bundles. The classification problem is stated in terms of projective and affine *ϱ-bumps/swamps*, but is shown how the ϱ-swamp describes a family of hypersurfaces of degree d and an isomorphism is a relative version of projective equivalence, for a particular choice of the representation, namely the general classification problem specializes to the classification of some algebraic varieties. In section 2.4 one obtains, by using *decorated bundles*, the moduli space of semistable principal G-bundles with connected reductive group structure. In section 2.5 one deals with the structure group $G := GL_{r_1}(\mathbb{C})\times\cdots\times GL_{r_t}(\mathbb{C})$ by choosing a faithfull representation $\chi : G \longrightarrow GL(W)$ which allows to reduce the problem of constructing moduli spaces to the case of decorated bundles. One also study the asymptotic behavior of the semistability concept and specialize the abstract obtained results to some concrete situations which allows to see a generalization of a well-known result by King on moduli spaces of quiver representations. Therefore some steps towards the classification problems of principal G bundles with arbitrary reductive group structure have been done and this classification is finalized within the last section 2.7. The itinerary for this purpose is similar to that exposed in previous sections for the classification problem of more particular reductive structure groups, but developed at a superior level in which the role of decorated bundles, for instance, is played by *decorated pseudo-G-bundles*.

The book is very well written and uses the powerful modern mathematical languages of representations, bundles and schemes to treat a very important classification problem with serious implications to the classification problem of algebraic varieties.

Cornel Pintea