BOOK REVIEWS

Joe Diestel, Jan H. Fourie and Johan Swart, ***The metric theory of tensor products - Grothendieck's Résumé revisited***, American Mathematical Society, Providence, Rhode Island 2008,x+278 pp, ISBN:978-0-8218-4440-3

More that fifty years ago Alexander Grothendieck published his famous *Résumé de la théorie metriques des produits tensoriels topologiques,* in Boll. Soc. Mat. Sao Paolo **8**(1953/1956), 1–79, revitalizing the interest in Banach space theory and tracing the way for future investigation. Among the great ideas the genius of Grothendieck (guided by "his inborn compass", as is said in the Preface) isolated, were: the study of isomorphic invariants of special Banach spaces by comparing them with other Banach spaces via the bounded linear operators between them, the importance of the nature and location of finite dimensional subspaces (the local theory of Banach spaces), the use of diagram chasing to catch the essential isomorphic characteristics of Banach spaces. In spite of the wealth of ideas contained in Grothendieck's paper, "the infamous Résumé" (as it is called called in the Preface) is very hard to read and to find (practically no proofs are given and the Sao Paolo journal is a bibliographical rarity), so that its results are not generally known even to experts in Banach space theory. In a paper published in 1968 in the Polish journal Studia Mathematica, J. Lindenstrauss and A. Pelczynski demystified the ideas of the Résumé by getting rid of their tensor product formulations, giving a new proof to the Fundamental Grothendieck's Inequality used in the study of operator ideals and opening a new fertile and rich period in the study of Banach spaces, lasting in lethargy after the glorious time of Banach and his school from the thirties.

The present book is devoted to the presentation of the fundamental ideas from the Résumé by using mathematical tools available at the time of its writing. Some parts, were subsequent developments might shorten some arguments are presented in Notes and Remarks. Also, three appendices B. *The Blaschke selection principle and compact convex sets in finite dimensional Banach spaces,* C. *A short introduction to Banach lattices,* and D. *Stonean spaces and injectivity,* present, this time in a modern language, the main tools available to Grothendieck. The first appendix, A. *The problems of the Résumé*, discusses the solutions to the open problems from the Résumé starting with the most famous of them - the approximation problem, solved negatively by Per Enflo in 1973. An Epilogue contains a brief discussion on some recent results in the theory of operator space theory (or noncommutative Banach space theory) as developed by Effros, Ruan, Blecher, Paulsen, Pisier, a.o.

The basic results of the Résumé are presented in the four chapters of the main text: 1. *Basics of tensor norms,* 2. *The role of $C(K)$-spaces and L^1-spaces,* 3. $\otimes$-*norms related to Hilbert space*, and 4. *The fundamental theorem and its consequences* (containing a proof of Grothendieck's Fundamental Inequality).

Written in an alive and entertaining style, but with detailed and rigorous proofs, the book makes available to a large audience the treasure of fundamental ideas contained in the Résumé, a landmark in the development of functional analysis.

The book can be used for advanced courses on Banach spaces, or for self-study.

S. Cobzaş

Victor G. Zvyagin and Dmitry A. Vorotnikov, ***Topological approximation methods for evolutionary problems of nonlinear hydrodynamics,*** Walter de Gruyter, Berlin 2008, xii+ 230 pp, ISBN 978-3-11-020222-9, ISSN 0941-813X

There are several methods to solve the evolutionary problems of fluid dynamics as the Faedo-Galerkin method, the iteration method, the method of evolutionary equations, and others. The authors of the present book propose another approach, based on the interpretation of the initial-boundary value problem as an operator equation in some appropriate function space. Usually the maps involved in this equation do not posses good operator properties, so that one approximates the initial equation by smoothing the nonlinear terms, or adding terms of higher order with a small parameter, allowing the study of this approximating equation in spaces with more suitable topological properties and the use of various discretization method with guaranteed convergence. The final step consists in passing to limit in the approximating equation, by letting the parameters to tend to 0 to find a solution of the original equation (usually in a topology weaker than that of the spaces where the approximating equation was studied).

In order to make the book as self-contained as possible, the authors have included (mostly with full proofs) the basic results on Sobolev function spaces, degree theory and operator equations. This is done in Chapters 2. *Basic function spaces.Embedding and compactness theorems*, 3. *Operator equations in Banach spaces* (including a section on Leray-Schauder degree), and 4. *Attractors for evolutionary equations in Banach spaces.*

The preliminary material from rheology, required for the understanding of the considered models, is presented (from a mathematician's point of view) in Chapter 1. *Non-Newtonian flows.*

Chapters 5. *Strong solutions for equations of motion of viscoelastic medium*, 6. *Weak solutions for equations of motion of viscoelastic medium*, and 7. *The regularized Jeffreys model*, are dedicated to the application of the developed methods to the equations describing the motion of viscoelastic media. Since the problem of the existence of global strong solutions is open in the general case, after presenting some particular cases of the existence of strong solutions in the fifth chapter, the authors concentrate on in the rest of the book on the existence of the weak solutions.

The book is clearly written, in a didactic manner, providing the reader with a good mathematical introduction to the operator methods for the solution of initial-boundary value problems for the equations of viscoelastic fluid mechanics.

The book will be useful both for mathematicians interested in nonlinear operator equations as well as for those working in fluid mechanics.

Mirela Kohr

J.W.P. Hirschfeld, G. Korchmáros, F. Torres,*Algebraic Curves over a Finite Field*, Princeton University Press (Princeton Series in Applied Mathematics), 2008, Hardback, 696 pp., ISBN: 978-0-691-09679-7.

The theory of algebraic curves over finite fields has become of great importance, both for its own sake and for the many applications that it has in number theory, finite geometry, coding theory and cryptography as well. Being given the enormous progress in this subject, an encyclopedia-like book devoted to it, like the book published by Professors Hirschfeld, Korchmáros and Torres, is welcome in the mathematical community.

The book is a self-contained introduction to this subject. It contains a huge amount of material, consisting of elementary, classical results, but also of current research topics. The exposition is divided into three parts.

The first part deals with the general theory of algebraic curves over an algebraically closed field of arbitrary characteristic. One defines first a plane algebraic curve as the zero locus of a (homogeneous) polynomial and then one develops the theory in a classical, geometric way. All the important projective invariants (namely the degree, inflexion, k-fold point, ordinary singularity, intersection number, bitangents) are introduced and studied, starting from the very beginning of the book.

Because many problems on curves can be reduced to investigating their intersection, the intersection number plays a central role. In Chapter 2, using elimination theory, its usual definition is extended in such a way that a sort of Bezout's theorem could work for intersections of plane curves, not only for the intersection of a plane curve with a line.

In working with curves their singularity is always important. One could also look for methods to eliminate some kinds of singularities of a certain curve (unfortunately projective transformations are not enough). This is done in the third chapter, where, among other useful results concerning singularities of curves, it is proved that every plane curve can be transformed by locally quadratic transformations to one with only ordinary singularities. In this sort of analysis the notion of the branch of a plane curve is essential. Chapter 4 treats the theory of branches, using formal power series. An idea that turns out to be useful in many contexts is that a plane curve needs to be considered as the set of its branches rather than the set of its points.

In the fifth chapter there are studied the effects of birational transformations on plane curves and the birational invariants of such curves, particularly their genus. Other birational invariants are the order and dimension of linear series, extensively studied in the next chapter based on the idea of adjoint curves. Here we come across to the Riemann-Roch theorem, which, besides giving an alternative definition of the genus, has several applications in algebraic geometry.

Even if we use birational transformations, an irreducible plane curve cannot always be transformed into a non-singular plane curve. In order to accomplish this space curves must be considered (a space curve is the image of an irreducible plane curve under a birational transformation), which are studied in Chapter 7. In this chapter the theory of non-classical curves is also presented.

The second part of the book is the central one. It develops extensively the theory of algebraic curves defined on the algebraic closure of a finite field. In chapter eight there are laid down the foundations of this particular topic. The important Stöhr-Voloch theorem is presented, followed by an elementary proof of the Stöhr-Voloch Bound for non-classical plane curves. The latter provides an accurate estimate of the number of F_q-rational branches for large families of curves.

In the following chapter it it deduced the famous Hasse-Weil Bound from the Riemann hypothesis for function fields over finite fields. There are also discussed some far-reaching consequences of the Hasse-Weil Theorem concerning curves over finite fields.

The third part contains several advanced results on curves over finite fields and on automorphism groups of curves. The major result is the finiteness of the K-automorphism group of the function field of irreducible plane curves of genus greater than one. It also collects the most important families of curves over finite fields. We could mention the maximal curves (curves for which the Hasse-Weil upper bound is attained), which are naturally used in algebraic-geometry codes. In the last chapter there are presented some applications of curves in coding theory and in the combinatorics of finite projective spaces.

The book is clearly written. Besides its 13 chapters, it contains an appendix, presenting the necessary background on field theory and group theory. The book is very well documented: the bibliography has an impressive number of 520 titles, giving a rough idea of the breadth of the subject and of the enormous documenting work done by the authors.

The publishing of this book written by professors Hirschfeld, Korchmáros and Torres is certainly a welcome and waited event. I am sure that every mathematician (graduate student, professor or researcher) working in the subject of algebraic curves over finite fields finds it indispensable.

Daniel Arnold Moldovan

Luca Capogna, Donatella Danielli, Scott D. Pauls and Jeremy T. Tyson, ***An Introduction to the Heisenberg Group and the Sub-Riemannian Isoperimetric Problem***, Progress in Mathematics (series editors: H. Bass, J. Oesterlé, A. Weinstein), vol 259, Birkhäuser Verlag, Basel-Boston-Berlin, 2007, 223 pp; ISBN-13: 978-3-7643-8132-5, e-ISBN-13: 978-3-7643-8133-2.

Sub-Riemannian (also known as Carnot-Carathéodory) spaces are spaces whose metric structure may be viewed as a constrained geometry, where motion is possible only along a given set of directions, changing from point to point. They play a central role in the general program of analysis on metric spaces, while simultaneously continuing to figure prominently in applications from other scientific disciplines ranging from robotic control and planning problems to MRI function, to new models of neurobiological visual processing and digital image reconstruction.

The book is divided in nine chapters.
The first chapter, *The isoperimetric Problem in Euclidean Space*, contains a short presentation of the isoperimetric problem and its solution in Euclidean space, indicating a few proofs for the sharp isoperimetric inequality in the plane arising from diverse areas such as complex analysis, differential geometry, geometric measure theory, nonlinear evolution PDEs (curvature flow), and integral geometry.

The second chapter, *The Heisenberg group and Sub-Riemannian Geometry* is concerned with the presentation of the Heisenberg group, the sub-Riemannian structure of it, and the Riemannian approximants to Heisenberg and Carnot groups.

Chapter 3, *Applications of Heisenberg group*, contains a selection of pure and applied mathematical models which feature aspects of Heisenberg geometry: CR geometry, Gromov hyperbolic spaces, jet spaces, path planning for nonholonomic motion, and the functional structure of the mammalian visual cortex.

The fourth chapter, *Horizontal Geometry of Submanifolds*, discusses invariance of the Sub-Riemannian metric with respect to Riemannian extension, the second fundamental form and horizontal geometry of hypersurfaces in $\mathbb{H}^N$.

Chapter 5, *Sobolev and* **BV** *Spaces*, contains the sub-Riemannian Green's formula and the fundamental solutions of the Heisenberg Laplacian, and embedding theorems for the Sobolev and BV-spaces, in particular Sobolev-Gagliardo-Nirenberg inequality, and the compactness of the embedding $BV \hookrightarrow L^1$ on the John domains.

In Chapter 6, *Geometric Measure Theory and Geometric Function Theory*, there are presented area and co-area formulas, Pansu-Rademacher theorem, first variation of the perimeter and the quasiconformal mapping on $\mathbb{H}$.

Chapters 7 and 8 are ample study of isoperimetric inequality in $\mathbb{H}$ and isoperimetric profile of $\mathbb{H}$. These chapters contain the isoperimetric inequality in Hadamard manifold, Pansu's proof of the isoperimetric inequality in $\mathbb{H}$, Pansu's conjecture, C^2 and convex isoperimetric profile in $\mathbb{H}$. Also, here is presented the Riemannian approximation approach, the horizontal mean curvature, the isoperimetric problem in the Grushin plane and the classification of symmetric CMC surfaces in $\mathbb{H}^n$.

The last chapter, Chapter 9, *Best Constants for other Geometric Inequalities on the Heisenberg group*, contains the L^2-Sobolev embedding theorem, Moser-Trudinger and Hardy inequalities.

The book is very well written and it is a nice introduction to the theory of sub-Riemannian differential geometry and geometric analysis in the Heisenberg group. I warmly recommend the book to researchers in sub-Riemannian geometry, and to those interested in PDEs, calculus of variations and its applications.

Csaba Varga

Vasile Staicu (Editor), ***Differential Equations, Chaos and Variational Problems***, Progress in Nonlinear Differential Equations and Their Applications Vol. 75, Birkhäuser, Basel, 2007, ISBN 978-3-7643-8481-4.

The book under review is a collection of original papers and state-of-the-art contributions written by leading mathematicians in honor of Arrigo Cellina and James A. Yorke on the occasion of their 65^{th} anniversary and introduced at the Conference Views on ODEs (VODE2006), June 21 - 24, 2006, Aveiro, Portugal. Arrigo Cellina and James A. Yorke were born in the same day of August, 3^{rd} 1941. Their outstanding contributions deeply influenced the scientific developments of many younger mathematicians. A short presentation of their lives and work is contained in the Editorial Introduction. The volume contains 32 contributed papers by distinguished mathematicians from all over the world covering topics related to the work of Cellina and Yorke - differential equations, delay-differential equations, variational problems , differential inclusions, Young measures, control theory, dynamical systems, chaotic systems and their relations with physical systems. Among the contributors (some of them co-workers of the celebrated) we mention: P. Agarwal, Z. Arstein, J.-P. Aubin, A. Bessan, H. Frankowska, A. Cellina, F. Clarke, C. Corduneanu, J.Mahwin, B. S. Mordukhovich, J. Myjak, D. O'Reagan, N. S. Papageorgiu, B. Ricceri.

Covering a lot of research areas, both pure and applied, this collection of wonderful papers will be of interest to a large audience, including mathematicians, physicists and engineers. No doubtably that it will be included in many libraries all over the world.

Marian Mureşan

Luigi Ambrosio, Nicola Gigli and Giuseppe Savaré, ***Gradient Flows in Metric Spaces and in the Space of Probability Measures***, Lectures in Mathematics, ETH Zürich, 2nd Edition, Birkhäuser Verlag, Basel-Boston-Berlin, 2008, vii+334 pp; ISBN: 978-3-7643-8721-1, e-ISBN3: 978-3-7643-8722-8.

The present book is formed of two parts: I. *Gradient flow in metric spaces,* and II. *Gradient flow in the space of probability measures.* Apparently independent, the inclusion of them in the same book is motivated by the fact that the space of probability measures, treated in the second part of the book, is one of the main field of application of the general theory of analysis in metric spaces. The last years were marked by an intense research activity in this field, one of the leading schools being that from the Scuola Normale Superiore di Pisa headed by Luigi Ambrosio, one of the authors of the present book.

The first part contains four chapters 1. *Curves and gradients in metric spaces*, 2. *Existence of curves of maximal slope*, 3. *Proofs of the convergence theorems*, and 4. *Generation of contraction semigroups.*

The main idea used in the first part is that of maximal slope of a curve in a complete (or at least Polish) metric space $(\mathcal{S}, d)$, meaning an absolutely continuous mapping $v : (a, b) \to \mathcal{S}$ such that $d(v(s), v(t)) \leq \int_s^t m(r)dr, \ \forall s, t, \ a < s, t < b,$ for some $m \in L^p(a, b)$, where (a, b) is an interval in $\mathbb{R}$ (possibly unbounded). One proves that there exists a minimal function $m = |v'|$, called the maximal slope of v, given by $|v'|(t) = \lim_{s \to t} d(v(s), v(t))/|s - t|$, $\mathcal{L}^1$-a.e. $t \in (a, b)$. In the case when $\mathcal{S}$ is a Banach or Hilbert space, this allows to extend some results on Fréchet differentiable curves. The main result of this part is the convergence of an Euler type discretization method for finding a curve ϕ of maximal slope such that $u_0 \in D(\phi)$ and $u(0+) = u_0$. The convergence theorem is enounced in the second chapter, while Chapter 3 is dedicated to the long and delicate proof of this theorem.

The second part of the book is concerned with spaces of probability measures endowed with the Kantorovich-Rubinstein-Wasserstein distnce, called here briefly the Wasserstein distance, one of the main illustrating realizations of the theory developed in the first part. This theory is closely related to the optimal transportation problem presented in Chapter 6. *The optimal transportation problem.* The first chapter of the second part, Chapter 5. *Preliminary notions on measure theory,* contains a survey, mostly without proofs, on the measure theory on separable metric spaces. The rest of the chapters of this part are headed as follows: 7. *The Wasserstein distance and its behaviour along geodesics*; 8. *A. C. curves and the continuity equation*; 9. *Convex functionals on* $\mathcal{P}_p(X)$; 10. *Metric slope and subdifferential calculus in* $\mathcal{P}_p(X)$.

The book is based on a NachDiplom course taught by the first author at ETH Zürich in the fall of 2001, but the material was substantially enlarged by the contributions of the second and the third authors, mainly in what concerns the error estimates in the first part and the generalized convexity properties in the second part.

This second edition of the book generally agrees with the first one, modulo some corrections and an updated bibliography. By the detailed presentation of the subject the book can be used as a textbook, but by some results never published elsewhere it is a research book as well.

Radu Precup

T.V. Panchapagesan, *The Bartle-Dunford-Schwartz integral,* Monografie Matematyczne (New Series), Vol. 69, Birkhäuser Verlag, Boston-Basel-Berlin, 2008, xv+301 pp, ISBN: 978-3-7643-8601-6 and e-ISBN: 978-3-7643-2431-5

The present book is concerned with the integration of scalar functions (real or complex) with respect to vector measures taking values in a Banach or, more generally, in a locally convex Hausdorff space (lcHs for short). The author develops a theory of integration for vector measures defined on more general structures than σ-algebras - usually δ- or σ-rings. The theory has its origins in a famous paper by Grothendieck (Canadian Math. Bull. **5** (1953), 129-173) where he showed that there is a bijection between the weakly compact linear operators $u : C(K) \to F$, K a compact Hausdorff space and F a complete lcHs, and the σ-additive F-valued vector measures, but he did not develop any integration theory to represent these operators. This was done in

1955 by Bartle, Dunford and Schwartz who developed a theory of integration for σ-additive Banach-valued measures and used it to represent weakly compact operators $u : C(K) \to X$, K a compact Hausdorff space and X a Banach space. To honor them the author calls this type of integral the BDS-integral.

About fifteen years later, Lewis developed a Pettis weak type integral of scalar functions with respect to a σ-additive vector measure $\mathbf{m}$ with range in a lcHs X. Since this kind of integral was considered also by Kluwanek, the author call it the KL-integral. Lewis proved that if $\mathbf{m}$ is defined on a σ-algebra and X is Banach, then the BDS and KL integrals agree. The author fills in some essential details lacking from Lewis' proof.

Other important spaces considered in the book are the space $\mathcal{K}(T)$ of all continuous functions with compact support defined on a locally compact Hausdorff space T, equipped with the inductive limit locally convex topology, the space $C_c(T)$ of all continuous functions with compact support with the supremum norm $\|\cdot\|_T$, and its completion $(C_0(T), \|\cdot\|_T)$ of all functions vanishing at infinity. For a lcHs X, a linear continuous operator $u : \mathcal{K}(T) \to X$ is called a Radon operator. If further, $u : C_c(T) \to X$ is continuous and its extension to $(C_0(T), \|\cdot\|_T)$ is weakly compact, then u is called a weakly compact bounded Radon operator. A representation theory for these type of operators, as well as for another class of operators, called prolongable, in the case of a quasicomplete lcHs X, is developed in Chapters 5 and 6, dedicated to integration on locally compact Hausdorff spaces.

The first chapter, 1. *Preliminaries,* has an expository character, while chapters 2. *Basic properties of the Bartle-Dunfors-Schwartz integral,* and 3. $\mathcal{L}_p$*-spaces,* $1 \le p \le \infty$, are devoted to integration of Banach-valued measures defined on δ- or σ-rings.

The main concern of the last chapter of the book, 7. *Complements to the Thomas theory,* is to extend the integration and representation results obtained by E. Thomas, Ann. Inst. Fourier (Grenoble), **20** (1970), 55-191, from the real space $\mathcal{K}(T, \mathbb{R})$ to the complex one $\mathcal{K}(T)$.

The book contains very general results about the integration of scalar functions with respect to measures with values in locally convex spaces with applications to the representation of weakly compact operators on spaces of continuous functions, completing the program initiated by A. Grothendieck. The author contributed essentially to this domain and his results are incorporated in the book.

The book is a worthy working tool for researchers in functional analysis, interested in vector measures and operator theory.

V. Anisiu