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DIFFERENT TYPES OF PARALLEL APPROACHES FOR SOLVING NONLINEAR EQUATIONS BY NUMERICAL METHODS

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Abstract. The purpose of this paper is to give and to compare different parallel approaches to some numerical methods for solving nonlinear equations on $\mathbb{R}$, such as the secant method and the Dekker's method.

1. Introduction

The solving of a nonlinear equation is a main part of Numerical Analysis. Lots of methods are known and many papers dealing with them were written. E.g., see [6] for the secant method, [1] for Dekker's method, [7] for Steffensen's method, all these methods are considered classical ones. In order to improve the rate of convergence, several other methods have been constructed, e.g. Brent's method (see [2]), Halley's method (see [8]), and so on.

All these methods have many serial implementations, used by different software packages.

Due to the requirement of improving the speed-up of execution, several parallel approaches of these methods were also developed. For instance, for the secant method, in [3], [4], for Steffensen's method, in [5], where the technique of nonlinear multisplitting is used.

In this paper we compare some possible parallel approaches for these methods, by using parallelism at the data level, or at the execution level.

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2. **A parallel approach for the secant method**

Assuming that $f \in C^2[a,b]$ and that there exists a number $x^* \in [a,b]$ with $f(x^*) = 0$, and $f'(x^*) \neq 0$, the well known secant formula for approximating the root x^* can be written as follows:

$$x_{k+1} = x_k - \frac{x_k - x_{k-1}}{f(x_k) - f(x_{k-1})} f(x_k) \tag{1}$$

for $k = 1, 2, \ldots$, with x_0 and x_1 two initial approximations.

Remark 1. The proof of the corresponding theorem can be found in [6].

Remark 2. The chord method is derived from (1), if we keep all the time one fixed initial approximation.

In order to make (1) appropriate for parallel calculus, and observing that it is a nonlinear recurrence relation, we have to linearize it. The technique is the following (see [4]): at every step $k = 1, 2, \ldots$ we denote by

$$y_k = \frac{f(x_1) - f(x_0)}{f(x_1)} \cdot \frac{f(x_2) - f(x_1)}{f(x_2)} \cdot \ldots \cdot \frac{f(x_k) - f(x_{k-1})}{f(x_k)} \tag{2}$$

and consider $y_0 = 1$.

Then, multiplying (1) by (2), at every step $k = 1, 2, \ldots$, we get the following relation:

$$x_{k+1} \cdot y_k = x_k \cdot y_k - (x_k - x_{k-1}) \cdot y_{k-1} \tag{3}$$

which is a linear recurrence relation with two terms.

Rearranging the terms, (3) becomes:

$$x_{k+1} = x_k \left(1 - \frac{y_{k-1}}{y_k}\right) + x_{k-1} \cdot \frac{y_{k-1}}{y_k} \tag{4}$$

which can be written in matricial form as:

$$\begin{bmatrix} x_{k+1} \\ x_k \end{bmatrix} = \begin{bmatrix} 1 - \dfrac{y_{k-1}}{y_k} & \dfrac{y_{k-1}}{y_k} \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x_k \\ x_{k-1} \end{bmatrix} \tag{5}$$

or, denoting by $z_k = \begin{bmatrix} x_{k+1} \\ x_k \end{bmatrix}$ and by M_k the above matrix, we have

$$z_k = M_k \cdot z_{k-1}, \quad k = 1, 2, \ldots \tag{6}$$

In order to get the best approximation, x_n, according with a given error, we have to compute the matrices product

$$M_1 \cdot M_2 \cdot \ldots \cdot M_n$$

which can be performed in $\log_2 n$ steps on a binary tree connectivity. So, the execution time is not of order $O(n)$, but of order $O(\log_2 n)$.

Remark. In the case of the secant method, we gave a parallel approach at the data level.

3. **Dekker's method**

By combining the secant method with the bisection method (see [1]), Dekker's method is obtained.

Supposing the same hypothesis for the function f and the existence of a real solution, x^*, of the equation $f(x) = 0$ in the interval $[a, b]$, by Dekker's method, two provisional approximated values of the solution are computed, at every step $k = 1, 2, \ldots$:

- the first is given by the secant method:

$$s = b_k - \frac{b_k - b_{k-1}}{f(b_k) - f(b_{k-1})} \cdot f(b_k),$$

  where b_k and b_{k-1} are the current guesses
- the second one is given by the bisection method:

$$m = \frac{a_k + b_k}{2},$$

  where a_k is a point such as $f(a_k)$ and $f(b_k)$ have opposite signs, so the interval $[a_k, b_k]$ contains the solution. Furthermore, $|f(b_k)|$ should be less than or equal to $|f(a_k)|$, so that b_k is a better guess for the unknown than a_k.

Remark 1. $b_{-1} = a_0$.

Remark 2. If the result of the secant method, s, lies between b_k and m, then it becomes the next iterate ($b_{k+1} = s$), else the midpoint is used ($b_{k+1} = m$).

Then, the value of a_{k+1} is chosen such that $f(a_{k+1})$ and $f(b_{k+1})$ have opposite signs. If $f(a_k)$ and $f(b_{k+1})$ have opposite signs, then $a_{k+1} = a_k$, otherwise $a_{k+1} = b_k$.

Finally, if $|f(a_{k+1})| < |f(b_{k+1})|$, then a_{k+1} is a better guess for the solution than b_{k+1}, and hence the values of a_{k+1} and b_{k+1} are exchanged.

3.1. A parallel approach

In this case, due to the fact that at every step $k = 1, 2, \ldots,$ the current approximation is used in computing two values (s and m), another parallel approach may be more appropriate. If we consider two processors "slaves" and one processor "master", we have the following "master-slave" type of execution at every step k:

{Processor Master}
Repeat
Send Message-to-Slave (k, a_k, b_k, ind);
Get Message-from-Slave (s, m);
Makes-Comparison;
Until End-Condition;
{Processor-Slave}
Get Message-from-Master (k, a_k, b_k, ind);
If $ind = 1$ then Compute (s);
Send Message-to-Master (s);
else Compute (m);
Send Message-to-Master (m);
End

Remark 1. The variable "*ind*" stands for the identification number of each slave processor.

Remark 2. In the case of Dekker's method, we gave a parallel approach at the execution level.

Conclusions

In this paper we have presented different types of parallel approaches for two numerical methods for solving nonlinear equations. These methods were chosen because their expressions are appropriate for our purpose. Of course, other methods may be considered, with the same remarks. E.g., the bisection method requires an execution type of parallelism, Steffensen's method can be parallelized at the data level, and so on.

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