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BOOK REVIEWS

Advanced Courses in Mathematical Analysis, II, Proceedings of the Second International School, M. V. Velasco and A. Rodríguez-Palacios (Editors), World Scientific Publishers, London - Singapore 2007, xi+213 pp, ISBN:13-978-981-256-652-2 and 10-981-256-652-X.

The volume contains the written versions of the talks and lectures delivered at the Second International Conference on Mathematical Analysis in Andalucía, which was held in Granada from 20 to 24 September, 2004. The first conference took place in Cadiz in September 2003, and its proceeding were published also with World Scientific in 2004.

The aim of the course was to bring together different research groups working in mathematical analysis and to provide the young researchers of these groups with access to the most advanced lines of research.

The present volume contains 11 papers covering a variety of topics from analysis - survey papers, contributed papers and historical surveys as well.

There are three papers of historical nature: F. Bombal, *Alexander Grothendieck's work on functional analysis,* (presenting not only the outstanding contributions of Grothendieck to tensor products and their applications to Banach space theory, but also some aspects of his unusual life and philosophy); L. Narici, *On the Hahn-Banach theorem,* B. Rubio Segovia, *Tribute to Miguel de Guzmán: Reflections on mathematical education centered on the mathematical analysis.* Miguel de Guzmán, one of the leading analysts of Spain, was scheduled to deliver a talk at the conference, but passed away shortly before the holding of the conference.

The paper by R. M. Aron, *Linearity in non-linear situations,* is concerned with the question whether some peculiar classes of functions (e.g, continuous and nowhere differentiable) contain some infinite dimensional linear subspaces - a property called lineability. The paper of Manuel Valdivia, the dean of the main speakers, *On certain spaces of holomorphic functions,* contains some related results (appearing for the first time in print), but concerning spaces of holomorphic functions.

The following papers survey results in the respective topics and present new contributions and open problems: J. Duoadiketxea, *The Hardy-Littlewood maximal function and some of its variants,* Gilles Godefroy, *Linear dynamics* (is concerned with hypercyclic operators, focussing on some recent results obtained by S. Grivaux and F. Bayart), Nigel J. Kalton, *Greedy algorithms and bases from the point of view of Banach space theory* (discusses some recent results about greedy, quasi-greedy and almost-greedy bases in Banach spaces obtained by the author, Konyagin, Temlyakov, Wojtaszczyk, a.o.), Michael M. Neumann, *Spectral properties of Cesáro-like operators* (the fine spectrum of Cesàro-like operators on Hardy spaces and on weighted Bergman space) and Joan Verdera, *Classical potential theory and analytic capacity* (reports on the spectacular solutions given in 1998 by G. David to an old problem of Vitushkin (1967) and by the author in 2003 to a problem by J. Garnett (1972)).

The last paper in the volume, containing mainly previously unpublished results, F. Zó and H. H. Cuenya, *Best approxiamtion ion small regions - A general approach,* proposes a unified approach, via monotone norms and Orlicz spaces, to some problems in local approximation theory by polynomials of Taylor type.

The volume presents interests for mathematicians desiring to get first-hand information on some recent results and trends in various domains of mathematical analysis.

I. V. Şerb

David Bachman, *A Geometric Approach to Differential Forms*, Birkhäuser, Boston - Basel - Berlin, 2007, xii+568 pp, ISBN: 978-3-7643-8146-2.

The integration of differential forms and Stokes' theorem are among the most difficult parts of the multivariable integral calculus. The main difficulty consists in understanding the connection between the algebraic and analytic machinery of differential forms and their geometric support. In concrete applications in $\mathbb{R}^2$ and $\mathbb{R}^3$ one draws pictures and one thinks geometrically, while in higher dimensions abstract calculations must be used. In many cases it is hard to realize how the abstract notions in the calculus of the differential forms look like in our drawings.

The aim of the present book is to reverse the situation - one starts with intuitive geometrical considerations in the spaces $\mathbb{R}^2$ and $\mathbb{R}^3$, and extending them to higher dimensions. As the author mentions in the Preface, the motivation for writing such a book comes from his experience with the abstract algebraic approach to differential forms in the book on differential topology by Guillemin and Pollack and the geometric approach in V. Arnold's book on mathematical methods of classical mechanics.

The book starts with a review of basic results in the multivariable calculus - vectors, functions, multiple integrals partial derivatives and gradients, in the first chapter, and an introduction to parametrization in the second one. In the third chapter the integral of the function $f(x,y) = y^2$ on a semicircle is calculated for two parametrizations with different results. Based on this clever example, the author explains why the differential forms must enter on the stage, a general theory being given in Chapter 4, including computations (addition, multiplication).

The core of the book is formed by Chapters 5. *Differential forms,* 6. *Differentiation of forms,* and 7. *Stokes' theorem,* while Chapter 8 is devoted to applications - Maxwell's equations, foliations and contact structures.

Chapter 9. *Manifolds,* is an introduction to more advanced topics - differential forms on manifolds, culminating with a short presentation of De Rham cohomology.

Non-linear forms defining surface area and arc length are treated in an appendix.

Based on some courses taught by the author at Calyfornia Polythechnic State University, San Luis Obispo and Pitzer College, the book is well written, in a pedestrian and pleasant style, with a lot of illuminating pictures. I consider it a good (and necessary) companion to more advanced books on the subject. In fact, a preliminary knowledge of algebraic theory of differential forms is an advantage in reading this book.

It can be used for a third semester course in calculus, or a sophomore level in Vector Calculus, and parts of it, for advanced undergraduate or graduate courses.

Tiberiu Trif

A. Borel and L. Ji, *Compactifications of Symmetric and Locally Symmetric Spaces*, Birkhäuser (Mathematics: Theory and Applications), 2006, Hardback, 476 pp., ISBN-10: 0-8176-3247-6, ISBN-13: 978-0-8176-3247-2.

Symmetric and locally symmetric spaces occur, as the authors of this monograph emphasize from the very beginning, in many branches of modern mathematics. In many situations, they are noncompact. As it usually happens, it is a lot easier to deal with *compact* spaces, therefore there were elaborated several ways to compactify them. This is, to my knowledge, the first serious attempt to present in a unified manner, the most important compactification methods known so far.

The authors point out that there are, essentially, three types of compactifications:

1. compact spaces that contain a symmetric space as an open dense subset;
2. compact smooth analytic manifolds containing a disjoint union of finitely many (but al least two!) symmetric spaces as an open dense subspace and
3. compact spaces containing a locally symmetric space as an open dense subset.

Clearly, the first and the last compactifications are the usual ones (in the sense of the point set topology), but the compactifications of type 2 are also important in many applications.

The book is, roughly, structured according to this classification of compactifications. Thus, after a short introductions, including, also, historical material, the first part of the book is devoted to compactifications of type 1 (more specifically, the compactifications of Riemannian symmetric spaces), the second part is concern with the smooth compactifications of semisimple symmetric spaces, while the last part is dedicated to compactifications of locally symmetric spaces.

In all the cases, as I've already pointed out, there are discussed particular compactification methods, but there are also made effort to unify different approaches.

This is a highly technical book, addressed only to researchers or advanced graduate students, as the prerequisite are rather demanding: semisimple Lie groups, algebraic geometry, algebraic groups, etc. Much of the material is taken from authors publications, some of it is here for the first time in a monograph. Both author are well known experts in the field. Actually, the first author (Armand Borel) who, unfortunately, passed away before the publication of the book, was one of the finest mathematician of the twentieth century and this is, in a way, his scientific testament.

The book is, definitely, a very valuable addition to the literature on compactification theory for symmetric and locally symmetric spaces, and it will soon become an indispensable reference for anyone working in the field.

Paul Blaga

Albrecht Pietsch, ***History of Banach Spaces and Linear Operators,*** Birkhäuser Verlag AG, Boston - Basel - Berlin, 2007, xxiii + 855 pp, ISBN: 10: 0-8176-4367-2 and 13:978-0-8176-4367-6.

This book is a welcome and waited addition to the existent books on the history of functional analysis. In fact there were two such books - A. F. Monna, *Functional analysis in historical perspective,* Oetshoek, Utrecht, 1973, and J Dieudonné, *History of functional analysis,* North-Holland, Amsterdam, 1981 - both of them covering the period up to 1950, so that a book dealing with the modern developments in Banach space theory was strongly required, a difficult task taken and brilliantly accomplished by the author of the present book. Due to broadening of the subject and the explosion of results, writing a book about functional analysis is almost impossible, so the author restricted to Banach spaces and bounded linear operators acting on them, fields in which he was actively involved over the last 50 years - he received his M.Sc. in 1958, exactly when a new era started in Banach space theory. In fact the author divides the development of Banach spaces in seven periods: 1900-1920 - **the prenatal period** (Fredholm, Hilbert, Riesz-Fischer theorem); 1920 -**the birth**, marked by Banach's thesis; 1920 - 1932 - **the youth** (the principles of uniform boundedness, closed graph and open mapping, Hahn-Banach theorem); 1932 - **the maturity** marked by the publication of Banach's monograph *Théorie des opérations linéaires*; 1932 - 1958 - **post-Banach period** (interrupted by Holocaust and World War II); 1958 - **classical books** (Dunford - Schwartz, vol. I, Hille-Phillips, Taylor), **midlife crisis** and **big bang** (Grothendieck's resumé, Mazur's school in Warszawa, Dvoretzky's theorem), and **the modern period** from 1958 on.

A crucial event in the last years was the publication of the *Handbook of the geometry of Banach spaces,* edited by W. B. Johnson and J. Lindenstrauss, Elsevier, Amsterdam, vol. I (2001), vol. II (2003), concerned almost exclusively with the present-day situation in Banach space theory. The author considers that his book may be regarded as a historical companion of these volumes.

The exposition is divided into seven chapters: 1. *The birth of Banach spaces,* 2. *Historical roots and basic results,* 3. *Topological concepts - weak topologies,* 4. *Classical Banach spaces,* 5. *Basic results from the post-Banach period* (analysis in Banach spaces, spectral theory, convexity and extreme points, geometry of the unit ball, bases, tensor products and approximation properties), 6. *Modern Banach* *space theory* (geometry of Banach spaces, s-numbers and operator ideals (author's specialty), eigenvalue distributions, interpolation theory, function spaces, probability on Banach spaces), 7. *Miscellaneous topics* (modern techniques - probabilistic and combinatorial, counterexamples, Banach spaces and axiomatic set theory).

The last chapter of the book, 8. *Mathematics is made by mathematicians,* starts with a tribute to mathematicians victims of the terror - killed in wars or murdered by totalitarian regimes - especially from the former Soviet Union and Poland. Then one presents some important schools as well as short biographies of some prominent contributors to the development of the Banach space theory or who have written influential books. As the author mentions in the Preface his main concern is not *Who proved a theorem?* nor to rank mathematicians, but rather to answer the question *Why and how was a theorem proved?* For this reason precise definitions and statements are formulated and, in some cases, even proofs are included.

The book is very well organized - a huge bibliography (approximatively 2600 items and 4600 quotations), an index name, a chronological index, a notion index.

The book is written in a very pleasant style, combining erudition and precision with anecdotes about mathematicians, witty remarks and pertinent comments by the author, making the reading instructive and entertaining as well.

Professor Pietsch made a great service to the mathematical community by writing this book.

S. Cobzaş

S. Dragomir and G. Tomassini, *Differential Geometry and Analysis on CR Manifolds*, Birkhäuser (Progress in Mathematics, 246), 2006, Hardback, 487 pages, ISBN-10: 0-8176-4388-5, ISBN-13: 978-0-8176-4388-1.

A CR-manifold M^m is, essentially, a real manifold endowed with a special rank n complex subbundle of its complexified tangent bundle. n and $k = m - 2n$ are called, respectively, the CR-dimension and CR-codimension of the CR-manifold M. A very important class of examples (and the original motivation) is that of real hypersurfaces of complex manifolds. Obviously, such a hypersurface has an *odd* real dimension, therefore it cannot carry a complex structure, but it inherits, nevertheless, a CR-structure, of codimension 1, from the ambient complex manifold. More generally, all the codimension 1 CR-manifolds are called of hypersurface type.

This monograph, written by two of the active researchers in the field and including much original material, treats some of the moderns aspects of the very interesting field, lying at the intersection between complex analysis, differential geometry and partial differential equations.

Much of the book is devoted to CR-manifolds of hypersurface type, which are most interesting from the geometrical point of view, since they carry significant geometrical structure (connections and metric). Thus, the first chapter of the book is devoted to the so-called pseudo-Hermitian geometry, namely the geometry of CR-manifolds endowed with a canonical connection, introduced by Tanaka and Webster in that late seventieth. The second chapter deals with another essential geometric object associated to a CR-manifold, the Fefferman metric, a Lorentzian metric (which doesn't live on the manifold itself, but it is intimately related to it). The remaining part of the book investigates different objects and problems that can be formulated starting from the basic geometric structure, by analogy with the classical Riemannian geometry: the CR Yamabe problem, pseudoharmonic maps, pseudo-Einsteinian manifolds, psudo-Hermitian immersions, spectral geometry, Yang-Mills fields on CR-manifolds, quasiconformal mappings.

Much of the material of the book is for the first time present in the monograph literature and, which is, perhaps, even more important, the book has a distinct geometric flavor, unlike many of the recent books on this topic, which were written mainly by analysts and treated especially problems from the realm of complex analysis of several variables or partial differential equations. We have, finally, a monograph trying to do justice to both parts, insisting, however (as I said) more on geometry than on analysis. Let me also say that the description of the geometrical part is very technical and can only be done mostly with the tools of the analyst.

The book is very well written and, although this is, clearly, a research monograph, can be read with real benefit also by advanced graduate students and PhD students with interests in all the three fields mentioned above: differential geometry, complex functions of several variables and PDE.

I couldn't help noticing the impressive literature list (449 titles), giving an idea of the documentation work lying behind this excellent book.

Paul Blaga

Pavel Drábek and Jaroslav Milota, ***Methods of Nonlinear Analysis - Applications to Differential Equations,*** Birkhäuser Advanced Texts, Birkhäuser Verlag AG, Basel - Boston - Berlin, 2007, xii+568 pp, ISBN: 978-3-7643-8146-2.

This introductory course contains the basic results and methods in nonlinear analysis, with applications to boundary value problems for ordinary and partial differential equations. To avoid technicalities and make the text accessible to beginners, some of the assertions and examples are not treated in the most general known form, but rather in typical situations containing the essential features of the problem. In fact, the book is written at two levels - the basic material contained in the seven chapters of the book and the appendices, containing more advanced topics. The basic material can be read independently, while the appendices, following some sections in the main text and written in a smaller font size, depend on the basic material.

The first chapter 1. *Preliminaries,* contains some results from linear algebra and normed linear spaces, including a presentation of basic of L^p- and Sobolev spaces. Some results are given with proofs. Chapter 2. *Properties of linear and nonlinear operators*, is concerned with the basic principles of functional analysis in normed spaces and some properties of linear compact operators (Schauder compactness theorem and Riesz spectral theory). The part on nonlinear operators deals with Banach contraction principle, Browder fixed point theorem for nonexpansive mappings in Hilbert space, Edelstein fixed point theorem. Chapter 3. *Abstract and differential calculus,* contains the basic results on Riemann integral for Banach space valued functions, Bochner integral and Dunford functional calculus, differential calculus in normed spaces, and Newton method in an appendix.

The inverse function theorem (local and global), the implicit function theorem, local structure of differentiable mappings and bifurcation, the rank theorem, Morse theorem, are treated in the fourth chapter *Local properties of differentiable mappings.* Some more refined results, as differentiable manifolds and vector fields, differential forms and Poincaré theorem, integration on manifolds and Stokes theorem, Brouwer degree with applications to Borsuk-Ulam antipodal theorem and Jordan separation theorem, are included in the appendices to this chapter.

Chapter 5. *Topological and montonicity methods,* presents Brouwer and Schauder fixed point theorems, topological degree, and some results on monotone operators. The appendices contain some fixed point theorems (involving measures of noncompactness) for noncompact operators, Rabinowitz global bifurcation theorem (Izé's proof), topological degree for generalized monotone operators (following Browder and Skrypnik), Krein-Rutman theorem.

Chapter 6. *Variational methods,* is devoted to the basic methods of the variational calculus - local and global extrema, Lagrange multipliers, the Mountain Pass and Saddle Point Theorems, Ritz method and, in appendices, Krasnoselski potential bifurcation theorem, Ekeland variational principle with applications, Lusternik-Schirelman category method, Rabinowitz linking theorem.

The last chapter, 7. *Boundary value problems for partial differential equations,* contains applications of the results and methods developed in the previous chapters to boundary value problems for partial differential equations - classical and weak solutions as well.

There are a lot of examples and exercises spread through the book as well as a lot of explanatory footnotes.

The book ends with some tabular synthesis material - summaries of methods and of typical applications, a comparison of bifurcation results, a list of symbols, an index and a bibliography of 137 titles.

The book is well written, in an accessible and clear style and well organized. It can be used for graduate courses in nonlinear analysis or for self-study.

Damian Trif