BOOK REVIEWS

Function Spaces, Krzysztof Jarosz (Editor), Contemporary Mathematics, Vol. 435, v+394 pp, American Mathematical Society, Providence, Rhode Island 2007, (ISSN: 0271-4132; v. 435), ISBN: 978-0-8218-4061-0.

Starting with 1990 a Conference on Function Spaces was held each fourth year at the Southern Illinois University Edwardsville. The volumes of the first two conferences were published with Marcel Dekker in *Lecture Notes in Pure and Applied Mathematics*, while the Proceedings of the last three conferences were published by AMS in the series *Contemporary Mathematics*, as volumes 232, 328 and 435 (the present one).

The Fifth Conference which took place from May 16 to May 20, 2006, was attended by 120 participants from 25 countries. The lectures covered a broad range of topics related to the general notion of "function space" - Banach algebras, C^*-algebras, spaces and algebras of continuous, differentiable or analytic functions (scalar and vector as well), geometry of Banach spaces. The main purpose of the Conference was to bring together mathematicians, working in the same domains or in related ones, to share opinions and ideas about the topics they are interested in. For this reason, the lectures have a general informal character, being addressed to non-experts, the survey papers and the papers containing new results as well.

The present volume contains 33 papers covering topics as Young-Fenchel transform and some characteristics of Banach spaces (Ya. I. Alber), Hardy spaces and operators acting on them (O. Balsco, D. P. Blecher, L. E. Labushagne, N. Arcozzi, R. Rochberg, E. Sawyer), spaces of bad (e.g., nowhere differentiable) functions (R. M. Aron et al), cohomology of Banach algebras and the geometry of Banach spaces (A. Blanco, N. Grobnaek), the Kadison-Singer theorem (P. G. Casazza, D. Edidin), strongly proximinal subspaces (S. Dutta, D. Narayana), uniform algebras (G. Bulancea, J. F. Feinstein, M. J. Heath, S. Lambert, A, Luttman, T. Tonev), various questions on Orlicz spaces (A. Kaminska, Y. Raynaud, A. Yu. Karlovich, M. Gonzales, B. Sari, M. Wojtowicz), the moment problem (D. Atanasiu, F. H. Szafraniec), algebras of continuous functions - Stone-Wierstrass type theorems, surjections which preserve spectrum (D. Honma, J. Kauppi), composition operators on spaces of analytic functions ((J. S. Manhas), quasi-similar operators with the same essential spectrum (T. L. Miller, V. G. Miller, M. M. Neumann), joint spectrum (A. Soltysiak), spectral isometries (M. Mathieu, C. Ruddy), complex Banach manifolds (I. Patyi), algebraic equations in C^*-algebras (T. Miura, D. Honma), Takesaki duality ((K. Watanabe), algebras of analytic functions and polynomials on Banach spaces (A. Zagorodnyuk).

Surveying or presenting new results in various areas of analysis related to function spaces, the present volume appeals to a large audience, first of all people working in this domain, but also researchers in related areas who want to be informed about results and methods in this field.

S. Cobzaş

Peter M. Gruber, ***Convex and Discrete Geometry,*** Springer-Verlag, Berlin-Heidelberg, 2007, Grundlehren der mathematischen Wissenschaften, Volume 336, xiii+578 pp, ISBN 978-3-540-71132-2.

The aim of the present book is to give an overview of basic methods and results of convex analysis and discrete geometry and their applications. The general idea of the book is that there are a plenty of beautiful and deep classical results and challenging problems in the domain, which are still in the focus of current research. Some of the problems as, for instance, the isoperimetric problems, the Platonic solids, the volumes of pyramids, have their roots in antiquity, while the modern research in convex geometry concerns local theory of Banach spaces, best and random approximation, surfaces and curvature measures, tilings and packings. Their solution requires tools and methods from various fields of mathematics, as Fourier analysis, probability theory, combinatorics, topology, and, in turn, the results from convex and discrete geometry are very useful in many domain of mathematics.

The book is divided into four parts: *Convex Functions, Convex Bodies, Convex Polytopes* and *Geometry of Numbers.*

The first part presents the basic properties of convex functions of one variable (Chapter 1) and of several variables (Chapter 2): continuity properties and differentiability properties, the highlight being the proof of Alexandrov's theorem on a.e. second-order differentiability of convex functions. Among applications we mention: the use of convex functions in proving various inequalities, the characterization of gamma function by Bohr and Mollerup, and a sufficient condition in the calculus of variation due to Courant and Hilbert.

The second part is devoted to the study of convex bodies, simple to define, but "which possess a surprisingly rich structure", according to a quotation from Ball's book on convex geometry, Cambridge U.P., 1997. The author present the basic properties of convex bodies in the Euclidean space E^d - combinatorial properties (the theorems of Carátheodory, Helly and Radon), boundary structure, extremal points (including Krein-Milman theorem), mixed volumes and Brun-Minkowski inequality, symmetrization, intrinsic metrics, approximation of convex bodies, simplices and Choquet's theorem, Baire category methods in convexity (many, in the sense of Baire category, convex bodies have good rotundity and smoothness properties). Some nice applications to this results are included - Hartogs' theorem on power series in $\mathbb{C}^d$, Lyapunov's convexity theorem, Pontryagin's maximum principle, Birkhoff's theorem on doubly stochastic matrices.

Although convex polytopes, as particular case of convex bodies, are freely used in the second part, their systematic study is done in the third part. Here, after the formal definitions and some elementary properties, one studies the combinatorial theory of polytopes (Euler's formula or, more correctly, Descartes-Euler - "the first important event in algebraic topology", according to a quotation from the fundamental treatise on topology by Alexandrov and Hopf), volumes of polytopes and Hilbert's third problem, the theorems of Alexandrov, Minkowski and Lindelöf, lattice polytopes, Newton polytopes. This part ends with an introduction to linear optimization, including simplex algorithm and a presentation of Khachiyan's polynomial ellipsoid algorithm. Applications are given to irreducibility criteria for polynomials and the Minding-Bernstein theorem on the number of zeros of systems of polynomial equations.

The last part of the book, *Geometry of Numbers,* is concerned with the interplay between the group theoretic notion of lattice in E^d and the geometric concept of convex set - the lattices represent periodicity, while the convex sets the geometry. This field was baptized "Geometry of numbers" by Hermann Minkowski who made breakthrough contributions to the area, some of of them being included in the book, as Minkowski's fundamental theorem and Minkowski-Hlawka theorem giving upper, respectively lower, bounds for the density of lattice packings. There are strong connections with the geometric theory of positive quadratic forms. Among the topics included in this part we do mention: the study of the density of tiling and packing with convex bodies, including the solution by Hales (Annals of Mathematics, 2005) of Kepler's famous conjecture on ball packing, optimum quantization, Koebe's representation theorem for planar graphs. The applications deal with Diophantine approximation, error correcting codes, numerical integration, and an algorithmic approach to Riemann mapping theorem.

There a lot of historical detours in the book as well as pertinent comments of the author about various questions. Some results are given two or three proofs, each shedding a new light on the problem and having its own beauty and originality. A large bibliography of 1052 titles, each of them being referred to in the text, tries to cover all the facets of the subject, from its origins to the present day state.

The author is a well-known specialist in the area with important contributions. Beside numerous research papers, he is the co-editor of two outstanding volumes - *Convexity and its Applications,* Birkhäuser 1983, and *Handbook of Convex Geometry,* A,B, North-Holland 1993, (both with J. M. Wills), as well as the co-author of a book, *Geometry of Numbers,* North-Holland, 1987 (with C. G. Lekkerkerker).

Since the problems in convex and discrete geometry are easy to formulate (and understand) but hardly to solve, the included material and the clear and pleasant style of presentation, make the book accessible to a large audience, including graduate students, teachers and researchers in various areas of mathematics.

S. Cobzaş

J. Kollár, *Lectures on Resolution of Singularities*, Princeton University Press (Annals of Mathematics Studies, 166), 2007, Paperback, 208 pages, ISBN-10: 0-691-12923-1, ISBN-13: 978-0-12923-5.

Resolution of singularities is one of the most venerable topics in algebraic geometry. We may say that it was, in a way, born before the algebraic geometry, as we know it today, existed.

The essence of the theory is easy to explain. If we consider an arbitrary algebraic variety, it usually has singular points and the variety is difficult to study because of these points. It is, however, possible to parameterize any variety by a smooth variety (without singular points) and many properties of the parameterizing variety ar similar to the original one. The process of finding a parameterizing smooth variety of an arbitrary variety is called *the resolution of singularities* for the given variety.

The first resolution was given by Newton, for curves in the complex plane. The resolution of algebraic surfaces was given at the beginning of the twentieth century, by different authors, while Zariski, in 1944, solved the problem for 3-folds. It was only in 1964 that Hironaka, in a 218 pages paper, managed to settled the general case (for varieties over a field of characteristic zero).

As one can readily guess, the proof of Hironaka is extremely complicated and, until recently, there wasn't any manageable proof available. In the last decade, however, it was given a new, different and much easier proof, accessible even for graduate students. It is the aim of the book, written by one of the most respected experts in algebraic geometry, and based on a course given in 2004/2005 at the Princeton University, to provide an introduction to the resolution of singularities and, in particular, to expose this new proof.

The first chapter of the book is devoted to the resolution of curves and there are given as many as thirteen (!) different proofs of the existence of resolutions. Many of the proof, as the author himself emphasize, are so elementary that they can be given in a first course of algebraic geometry.

The second chapter is concerned with the resolution of surfaces. More elaborate methods are needed here and, again, most of them are specific to this particular case and cannot be easily extended to the general case.

The third (and last) chapter deals with the general case. The new proof is presented and then there are discusses a lot of examples. It is to be noticed that this new proof is given on thirty pages. It is not short, of course, but if we compare it to the original one, we can appreciate the improvement.

The book is written in a very pedagogical manner, with many examples. Many proofs are given in an algorithmic manner. It is, probably, the first really comprehensive textbook in the resolution of singularities, one of the most important topics of algebraic geometry, as mentioned earlier. Of course, many advanced topics are not touched and the proofs refer only to the characteristic zero case, but this make the proof even more useful for graduate students, which are, usually, ont prepared to attack directly the general case. Otherwise, I think it provides a fairly complete picture of resolutions for varieties. Beside the proof of the general theorem, I particularly liked the discussion of the low dimensional cases, many of them of importance for the early history of algebraic geometry.

The prerequisites for this book include, in my opinion, a first course in algebraic geometry and in algebra. As I mentioned earlier, some of the proofs can be even discussed *within* a first course in algebraic geometry. The book will be an invaluable tool not only for graduate student, but also for algebraic geometers. Mathematicians working in different fields will also enjoy the clarity of the exposition and the wealth of ideas included. This will become, I'm sure, as it happened to most books in this series, one of the classics of modern mathematics.

Paul Blaga

Mathematical aspects of Nonlinear Dispersive Equations, Jean Bourgain, Carlos E. Kenig & S. Klainerman (Editors), Annals of Mathematics Studies No. 163, Princeton University Press, Princeton and Oxford 2007, vii + 300 pp., ISBN 13: 978-0-691-12955-6 and 10: 0-691-12955-X.

These are the written versions of a number of lectures delivered at the CMI/IAS Workshop on mathematical aspects of nonlinear PDEs, in the spring of 2004 at the Institute for Advanced Study in Princeton. The workshop is a conclusion of a year-long program at IAS about this topic, leading to significant progress and to the broadening of the subject. At least two important breakthroughs were obtained - the first one is the understanding of the blowup mechanism for critical focusing Schrödinger equation, and the other is a proof of global existence and scattering for the 3D quintic equation for general smooth data. In both cases, hard analysis, in addition to the more geometric approach, turned to play a key role in energy estimates.

The volume contains 12 papers (called chapters), some of them of expository nature (as, e.g., that by W. Schlag on dispersive estimates for Schrödinger operators), describing the state of the art and research directions, while the others are contributed papers, both kinds being fully original accounts. The papers concentrate on new developments on Scrödinger operators, nonlinear Scgrödinger and wave equations, hyperbolic conservation laws, Euler and Navier-Stokes equations.

Among the contributors we mention Jean Bourgain (two papers, one with W.-M. Wang), A. Bressan, H. K. Jensen, H. Brezis, M. Marcus, P. Gérard, N. Tzvetkov, P. Constantin, A. D. Ionescu, B. Nikolaenko, Terence Tao.

The volume contains valuable contributions to the area of nonlinear PDEs, making it indispensable for all researchers interested in partial differential equations and their applications.

Radu Precup