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BOOK REVIEWS

Nonlinear Evolution Equations and Related Topics –Dedicated to Philippe Bénilan, **W. Arendt, H. Brezis and M. Pierre Eds.**, Birkhäuser, Basel-Boston-Berlin, 2004, ISBN 3-7643-7107-2.

The volume is dedicated to Philippe Bénilan (October 6, 1940 - February 17, 2001), one of the most significant contributors to the theory of nonlinear evolution equations. It contains research papers related to Bénilan's work which cover a wide range of nonlinear and linear equations and appeared in regular issues of the Journal of Evolution Equations, Volumes 3 (2003) and 4 (2004). The main topics are Hamilton-Jacobi equations, the porous medium equation, reaction diffusion systems, integro-differential equations and viscoelasticity, maximal regularity for elliptic and parabolic equations and the Ornstein-Uhlenbeck operator. Thus new developments of nonlinear analysis are presented with applications to physics, mechanics, chemistry, biology and others.

The volume starts with an Introduction presenting Bénilan's main contributions to nonlinear analysis, a list of publications and a list of Ph.D.-Students of P. Bénilan. Furthermore the contains are as follows: F. Hirsch, intrinsic metrics and Lipschitz functions; S. Benachour and P. Laurençot, Decay estimates for "anisotropic" viscous Hamilton-Jacobi equations in R^N; F. Andreu, V. Caselles and J.M. Mazón, The Cauchy problem for linear growth functionals; J.L. Vázquez, Asymptotic behaviour for the porous medium equation posed in the whole space; W. Arendt and M. Warma, Dirichlet and Neumann boundary conditions: what is in between?; S.B. Angenent and D.G. Aronson, The focusing problem for the Eikonal equation; M. Pierre, Weak solutions and supersolutions in L^1 for reaction-diffusion systems; S.-O. Londen,

H. Petzeltová and J. Prüss, Global well-posedness and stability of a partial integro-differential equation with applications to viscoelasticity; P. Bénilan, L.C. Evans and R.F. Gariepy, On some singular limits of homogeneous semigroups; P. Bénilan and N. Igbida, Singular limit of changing sign solutions of the porous medium equation; L. Boccardo, On the regularizing effect of strongly increasing lower order terms; E. Bazhlekova and P. Clément, Global smooth solutions for a quasilinear fractional evolution equation; H. Gajewski and I.V. Skrypnik, On the uniqueness of solutions for nonlinear elliptic-parabolic equations; J. Carrillo, Conservation laws with discontinuous flux functions and boundary condition; V.G. Jakubowski and P. Wittbold, Regularity of solutions of nonlinear Volterra equations; J. Liang, R. Nagel and T.-J. Xiao, Nonautonomous heat equation with generalized Wentzell boundary conditions; H. Heck and M. Hieber, Maximal L^p-regularity for elliptic operators with VMO-coefficients; W.M. Ruess, Linearized stability for nonlinear evolution equations; D. Bothe, Nonlinear evolutions with Carathéodory forcing; H. Amann, Linear parabolic equations with singular potentials; L. Boccardo, L. Orsina and A. Porretta, Some noncoercive parabolic equations with lower order terms in divergence form; E. Feireisl, On the motion of rigid bodies in a viscous incompressible fluid; A. Henrot, Minimization problems for eigenvalues of the Laplacian; A. Haraux, M.A. Jendoubi and O. Kavian, Rate of decay to equilibrium in some semilinear parabolic equations; G. Da Prato, A new regularity result for Ornstein-Uhlenbeck generators and applications; J. Droniou, T. Gallouët and J. Vovelle, Global solution and smoothing effect for a non-local regularization of a hyperbolic equation; M. Gokieli and F. Simondon, Convergence to equilibrium for a parabolic problem with mixed boundary conditions in one space dimension; J. Escher and G. Simonett, Analyticity of solutions to fully nonlinear parabolic evolution equations on symmetric spaces; P. Bénilan and J.I. Díaz, Pointwise gradient estimates of solutions to onedimensional nonlinear parabolic equations; M. Maliki and H. Touré, Uniqueness of entropy solutions for nonlinear degenerate parabolic problems; C.G. Gal, G. Ruiz Goldstein and J.A. Goldstein, Oscillatory boundary conditions for acoustic wave equations; M. Marcus and L. Véron, Existence and uniqueness results for large solutions of general nonlinear elliptic equations; M.G. Crandall and

P.-Y. Wang, Another way to say caloric; P. Bénilan and H. Brezis, Nonlinear problems related to the Thomas-Fermi equation; P. Bénilan and H. Labani, Existence of attractors in $L^{\infty}(\Omega)$ for a class of reaction-diffusion systems; B.P. Andreianov and F. Bouhsiss, Uniqueness for an elliptic-parabolic problem with Neumann boundary condition.

The volume will interest all mathematicians working in nonlinear analysis and its applications. It is a nice tribute to one of the most original mathematicians with a deep and decisive impact on the theory of Evolution Equations.

Radu Precup

William Arveson, ***A Short Course on Spectral Theory***, Graduate Texts in Mathematics, Vol. 209, Springer, New York, Berlin, Heidelberg, 2002, x+135 pp., ISBN 0-387-95300-0.

The fundamental problem of operator theory is the calculation of spectra of operators on infinite dimensional spaces, especially on Hilbert spaces. The theory has deep applications to partial differential and integral operators, to mathematical foundation of quantum mechanics, noncommutative K-theory and the classification of simple C^*-algebras.

The aim of the present book, based on a fifteen-week course taught for several times by the author at the University of Berkeley, is to make the reader acquainted with the basic results in spectral theory, needed for the study of more advanced topics listed above. The prerequisites are elementary functional analysis and measure theory.

In the first chapter, *Spectral theory and Banach algebras,* the theory is developed in the natural framework of Banach algebras and includes spectral radius, regular representation, the spectral permanence theorem, and an introduction to analytic functional calculus. The abstract notions are illustrated on concrete examples of operators.

Ch. 2, *Operators on Hilbert space,* is concerned with spectral theory for operators on Hilbert space and their C^*-algebras, normal operators, compact operators, spectral measures. For the sake of clarity the treatment is restricted to separable Hilbert spaces. A good companion in reading this part could be another book by the same author: *An invitation to C^*-algebras,* Springer Verlag 1998.

Ch. 3, *Asymptotics: Compact perturbations and Fredholm theory,* contains the Calkin algebra, Riesz theory for compact operators, Fredholm operators and Fredholm index.

In the last chapter, Ch. 4, *Methods and applications,* a variety of operator theoretic methods are applied to determine the spectra of Toeplitz operators, the results being definitive only for Toeplitz operators with continuous symbol. An elementary theory of Hardy spaces H^2 is also developed. The book ends with the study of states on C^*-algebras and a proof of Gelfand-Naimark representation theorem.

The book is a clear, short and thorough introduction to spectral theory, accessible to first or second year graduate students. As the author points out in the Preface: "this material is the essential beginning for any serious student in modern analysis".

S. Cobzaş

François Bouchut, ***Nonlinear Stability of Finite Volume Methods for Hyperbolic Conservation Laws and Well-Balanced Schemes for Sources***, Birkhäuser Verlag, Basel-Boston-Berlin, 2004, 135 pp., ISBN 3-7643-6665-6.

This very good monograph is devoted to finite volume methods for hyperbolic systems of conservation laws. All examples included in the book are of gas dynamics type. The author presents systematically sufficient conditions for a scheme to preserve an invariant domain or to satisfy discrete entropy inequalities.

The book consists of two parts. The first part is concerned with the notion of approximate Riemann solver and the relaxation method. Certain practical formulas are obtained in a new variant of HLLC solver for the gas dynamics system, taking into account contact discontinuities, entropy conditions, and including vacuum.

The second part of the book is devoted to the numerical treatment of source terms that can appear additionally in hyperbolic conservation laws, with the extension of the notions of invariant domains, entropy inequalities, and approximate Riemann solvers. The author compares several methods that have been developed in the literature especially for the Saint Venant problem, concerning the positivity and the ability to treat resonant data. In particular, the hydrostatic reconstruction method is presented in details.

The book is clearly written, with rigorous proofs, in a pleasant and accessible style. It is warmly recommended as a useful guide for all engineers and researchers interested in the nonlinear stability of finite volume methods for hyperbolic systems of conservation laws.

Mirela Kohr

Calin, O., Chang, D.-C., ***Geometric Mechanics on Riemannian Manifolds. Applications to Partial Differential Equations***, Birkhäuser (Applied and Numerical Harmonic Analysis), 2004, Hardback, 278 pp., ISBN 0-8176-4354-0.

Unlike other parts of mathematics, in the theory of partial differential equations it is quite difficult to find results which apply to any equation or, at least, to a large class of equations. Instead, in this theory one studies *individual* equations, such as, for instance, the heat equation, the Laplace or the Poisson equations. It turns out that many equations are related, in a way or another, to mechanics and, as such, the geometrical approaches to mechanics can be used to investigate them, as alternative to the classical methods, such that that of the integral transformations. It is the aim of this monograph to give an introduction to this geometric approach to some important partial differential equations.

The book starts with an outline of the fundamentals of differential geometry, examines the Laplace operators on Riemannian manifolds and proceeds to discuss the main approaches to mechanics on Riemannian manifolds (Lagrangian, Hamiltonian and Hamilton-Jacobi). As important examples, there are discussed the harmonic maps and, in particular, the minimal hypersurfaces. It follows a analysis of radially symmetric spaces and a discussion of the fundamental solutions for heat operators with potentials and of elliptic operators on this kind of spaces.

The book ends with a chapter devoted to special classes of curves that the authors call "mechanical curves" and which appear as solution to different mechanical problems (for instance curves that minimize a potential, cycloids, astroids a.o.).

The differential operators which are treated in the book are among the most important, not only in the theory of partial differential equation, but they appear naturally in geometry, mechanics or theoretical physics (especially quantum mechanics). Thus, the book should be of interest for anyone working in these fields, from advanced undergraduate students to experts.

The book is written in a very pedagogical manner and does not assume many prerequisites, therefore it is quite appropriate to be used for special courses or for self-study. I have to mention that all chapter ends with a number of well-chosen exercises that will improve the understanding of the material and, also, that there are a lot of worked examples that will serve the same purpose.

Paul Blaga

Andrew J. Kurdila, Michael Zabarankin, ***Convex Functional Analysis***, Systems & Control: Foundations & Applications, Birkhäuser Verlag, Boston-Basel-Berlin 2005, xiv+228 pp, ISBN-10:3-7643-2198-9.

The aim of the present book is to make the students in applied mathematics and engineering acquainted with the basic principles and tools of functional and convex analysis, as required by modern treatments of some problems in variational calculus, mechanics and control. The emphasis is not on foundation and proofs, but rather on examples and applications. For this reason some results are given with full proofs, while for others one gives only references for detailed presentation. A good idea on the content of the book is done by the headings of its chapters: 1. *Classical abstract spaces in functional analysis*; 2. *Linear functionals and linear operators* (including a complete proof of the open mapping theorem and of Riesz's representation theorem for the dual of $C[a,b]$); 3. *Common function spaces in applications* (L^p and Sobolev spaces of scalar or Banach-valued functions); 4. *Differential calculus in normed vector spaces* (containing many examples of differential operators); 5. *Minimization of functionals* (including the Lagrange multipliers rule in constrained differential optimization, deduced via Ljusternik's theorem); 6. *Convex functionals* (including a section on ordered vector spaces and convex programming in such spaces); 7. *Lower semi-continuous functionals.*

The bibliography contains a list of basic textbooks and monographs covering the topics the book is dealing with.

The book is useful for students in engineering interested in a quick and accessible presentation of basic tools of functional analysis needed for applications, as well as for students in applied mathematics interested in possible applications of these disciplines.

S. Cobzaş

Jon P. Davis, ***Methods of Applied Mathematics with a MATLAB Overview***, Birkhäuser Verlag, Boston-Basel-Berlin 2004, ISBN 0-8176-4331-1.

This book is devoted to the application of Fourier Analysis. The author mixed in a remarkable way theoretical results and applications illustrating the results. Flexibility of presentation (increasing and decreasing level of rigor, accessibility) is a key feature.

The first chapter is an introductory one.

An introduction to Fourier series based mainly on inner product spaces is given in Chapter 2.

The third chapter treats elementary boundary value problems. Besides applications of the Fourier series, it presents standard boundary value problem models and their discrete analogous problems.

Higher-dimensional, non rectangular problems is the topic of the fourth chapter. These includes Sturm-Liouville Theory, series solutions, Bessel equations and nonhomogeneous boundary value problems.

Chapter 5 is an introduction to functions of complex variable. Here ones discuss basic results and their applications to problems of fluid flow and transform inversion.

The sixth chapter introduces Laplace transform and their applications to ordinary differential equations, circuit analysis and input-output analysis of linear systems.

Continuous Fourier transform is the topic of seventh chapter. Also applications of Fourier transform to ordinary differential equations, integral equations, partial differential equations are included here.

Chapter 8 is on discrete variable transforms. It treats discrete variable models, z-transform, discrete and fast Fourier transform and their properties. Computational aspects of fast Fourier transform are also pointed.

The last chapter "Additional Topics" introduces methods that are specialization of those treated previously such as two-sided and Walsh transform, wavelets analysis and integral transform.

The book contains extensive examples, presented in an intuitive way with high quality figure (some of them quite spectacular), useful MATLAB codes. MATLAB exercises and routines are well integrated within the text, and a concise introduction into MATLAB is given in an appendix. The emphasis is on program's numerical and graphical capabilities and its applications, not on its syntax. A large variety of problems graded from difficulty point of view. Applications are modern and up to date. Reach and comprehensive references are attached to each chapter.

Intended audience: especially students in pure and applied mathematics, physics and computer science, but also useful to applied mathematicians, engineers and computer scientists interested in applications of Fourier analysis.

Radu Trîmbiţaş

Cabral, Hildeberto, Diacu, Florin ***Classical and Celestial Mechanics (The Recife Lectures)***, Princeton University Press, 2002, Hardcover, 385 pages, ISBN 0-691-05022-8.

The University of Pernambuco (Brazil) invited, between 1991 and 1999, several international experts to lecture in Recife (Brazil) on different topics in classical or celestial mechanics. The editors managed to convince some of the lecturer to prepare an elaborate version of their lecture and gathered everything into a book: this one.

Due to the nature of the book, it doesn't have a unitary character (and it is not suppose to have!). The subjects treated include: "classical" celestial mechanics (the motion of the Moon (Dieter Schmidt) and the two-body problem (Alain Albouy)), the theory of equilibria and applications to celestial mechanics (central configurations and relative equilibria for the N-body problem (Dieter Schmidt), normal forms of Hamiltonian systems and stability of equilibria (Hildeberto Cabral)), geometrical methods in classical mechanics (Mark Levi, and Jair Koiller et al.), topological methods in celestial mechanics (Poincaré's compactification (Ernesto Pérez-Chavela)), singularities of the N-body problem (Florin Diacu) and bifurcations from families of periodic solutions (Jack Hale and Plácido Táboas).

Modern classical and celestial mechanics (and, especially, their mathematical tools) represent a vast field which is impossible to be described completely in a single monograph or textbook, not to mention the fact that there is no individual researcher who can call himself an expert in all the particular subjects. The summer schools are ideal opportunities for the discussion of the latest developments from a field or to describe a classical field from anew perspective. Unfortunately, the lectures from the summer schools are only rarely published and, besides, it is only very rarely possible to gather in the same place a large number of very good experts in a field. The editors of this book did a far better job. They managed to gather the lectures (most of them enlarged and polished) from several "schools" (or series of lectures, if you prefer) and, as such, they give the reader the possibility to interact with the science of some of the worldwide best experts in classical and celestial mechanics. This unique book (which, as argued above, is more a collection of graduate "minitextbooks" than a proceedings of a particular school) would be extremely useful especially for graduate students interested in the field, but, due to the wealth of the subjects treated, the researchers will also find, I am absolutely sure, many new results or new perspective on the classical material.

I would like to say, to finish, that the book benefits of a foreword written by Donald Saari.

Cristina Blaga

Bolsinov, A.V., ***Integrable Hamiltonian Systems: Geometry, Topology, Classification***, Chapman and Hall/CRC, 2004, 730 pp., Hardback, ISBN 0-415-29805-9.

The field of integrable systems is a very rich field that gave rise to several important developments in mathematics in the last decades. It has strong relations with domain as: symplectic and Poisson geometry, quantum groups, algebraic geometry and even quantum field theory.

This new book on the subject approaches a fundamental problem: that of the classification of integrable systems. Many dynamical systems (integrable or not) are described by means of a system of differential equations. Even if these systems are different from one dynamical system to another, their solutions do have, sometimes, similarities, they "behave" analogously, in a certain sense. It is the aim of the classification theory to spot such similarities and to exploit them.

In this book three kind of equivalence relations between dynamical systems are examined:

- *conjugacy*, which, loosely, means that the systems can be transformed on into the other through a change of variables;
- *orbital equivalence*, which means that between the manifolds on which the dynamical systems are defined there is a diffeomorphism that turns the trajectories of a systems into the the trajectories of the other (although the parameters along the trajectories are not necessarily preserved);
- *Liouville equivalence* (only for integrable systems): two integrable systems are said to be equivalent in the sense of Liouville if their phase spaces are foliated in the same manner into Liouville tori.

Certainly, it is hardly possible to solve the classifications problem for arbitrary Hamiltonian dynamical systems, even if they are integrable. Therefore, this book focuses on a particular, but very important class of systems: nondegenerate integrable Hamiltonian systems with two degrees of freedom. The first half of the book (the first 9 chapters) are devoted to foundational material on symplectic and Poisson geometry, followed by a discussion of the three equivalence relation and the solution of the Liouville and orbital classification problem for the aforementioned class of dynamical systems. The solution is based on a new approach to the qualitative theory of dynamical systems invented by A.T. Fomenko and developed by him and its collaborators in a series of papers.

The remaining of the book is devoted to applications of the classification theory to specific integrable Hamiltonian systems coming from mechanics and geometry. Two systems are considered more important and are treated in details: the integrable cases of the equation of motion for rigid bodies and the integrable geodesic flows of Riemannian metrics on two-dimensional surfaces.

The book is largely based on the works of the two authors (two well-known experts in the field) and of their collaborators and it is addressed, mainly, to researchers in dynamical systems, geometry and mechanics, managing, successfully, to fill a gap in the existing literature (in fact much of the material was never published into a book). However, the exposition is cursive and understandable, there is enough foundational material and there are enough worked examples, which makes it appropriate also for graduate students, both for self-study or as a textbook for an advanced course.

Paul Blaga

Giovanni P. Galdi, John G. Heywood, Rolf Rannacher (Editors), *Contributions to Current Challenges in Mathematical Fluid Mechanics*, Birkhäuser Verlag, Basel-Boston-Berlin, 2004, 151 pp., ISBN 3-7643-7104-8.

This volume consists of five very good research articles, each of them being dedicated to an important topic in the mathematical theory of the Navier-Stokes equations, for compressible and incompressible fluids. The results presented in this volume are all new and represent a key contribution to this topic, with particular interest to turbulence modelling, regularity of solutions to the initial-value problem, flow in region with an unbounded boundary and compressible flow.

The first article of this volume, due to Andrei Biryuk, deals with the Cauchy problem for a multidimensional Burgers type equation with periodic boundary conditions. The author obtains upper and lower bounds for derivatives of solutions for this equation, which are expressed in terms of powers of the viscosity. In addition, it is discussed how these bounds relate to the Kolmogorov-Obukhov spectral law. Moreover, these estimates are used to obtain bounds for derivatives of solutions to the Navier-Stokes system.

The second article, due to Dongho Chae and Jihoon Lee, is concerned with the problem of global well-posedness stability in the scale invariant Besov spaces for the modified $3D$ Navier-Stokes equations with the dissipation term, $-\triangle u$ replaced by $(-\triangle)^{\alpha}u$ for $0 \leq \alpha < 5/4$. The authors prove the unique existence of a global-in -time solution in $B^{5/2-2\alpha}_{2,1}$ for initial data having small $\dot{B}^{5/2-2\alpha}_{2,1}$ norm for $\alpha \in [1/2, 5/4)$.

In the next article the authors A. Dunca, V. John and W.J. Layton deal with the space averaged Navier-Stokes equations, which are the basic equations for the large eddy simulation of turbulent flows. In deriving these equations it is understood that differentiation and averaging operations can be interchanged. This procedure introduces a *commutation error* term that is typically ignored. However, in this paper the authors furnish a characterization of this term to be neglected. In fact, the authors show that the commutation error is asymptotically negligible in $L^p(\mathbb{R}^d)$ if and only if the fluid and the boundary exert exactly zero force on each other. In addition, the authors study the influence of the commutation error on the energy balance of the filtered equations.

The forth article of Toshiaki Hishida deals with the nonstationary Stokes and Navier-Stokes flows in aperture domains $\Omega \subset \mathbf{R}^n$, $n \geq 3$. The authors proves $L^q - L^r$ estimates of the Stokes semigroup. Then the author applies these estimates to the Navier-Stokes initial value problem, and obtains the global existence of a unique strong solution, which satisfies the vanishing flux condition through the aperture and some sharp decay properties as $t \to \infty$, when the initial velocity is sufficiently small in the space L^n.

The last article of T. Leonavičicene and K. Pileckas is concerned with steady compressible Navier-Stokes equations with zero velocity conditions at infinity in a three dimensional exterior domain. They consider the case of small perturbations of large potential forces. To solve this problem the authors apply a decomposition scheme and decompose the nonlinear problem into three linear problems of the following types: Neumann-type, modified Stokes problem and transport equation. Then they solve the resulted linear problems in weighted function spaces with detached asymptotics. Finally they prove certain results related to existence, uniqueness and asymptotics for the linearized problem and for the nonlinear problem.

Each paper from this volume is clearly written, with rigorous proofs, in a pleasant and accessible style. This voume is warmly recommended to all researchers interested in modern topics of the mathematical theory of fluid mechanics.

Mirela Kohr

Ram P. Kanwal, ***Generalized functions. Theory and applications***, 3rd revised ed. Boston, MA: Birkhäuser, 2004, xvii+476 pp., ISBN 0-8176-4343-5.

This book on generalized functions is suitable for physicist, engineers and applied mathematicians. The author presents the notion of generalized functions , their properties and their applications for solving ordinary differential equations and partial differential equations.

Chapters 1 to 8 contain a concise definition of distributions and their standard properties are proved. Chapters 9 to 15 deal with applications to ordinary differential equations, partial differential equations, boundary value problems, wave propagation, linear systems, probability and random processes, economics, microlocal theory.

The author demonstrate through various examples that familiarity with the generalizes functions is very helpful for students in mathematics, physical sciences and technology. The proposed exercises are very good for better understanding of notions and properties presented in the chapters. The book contains new topics and important features:

- Examination of the Poisson Summation Formula and the concepts of differential forms and the delta distribution on wave fronts.
- Enhaced presentation of the Schrödinger, Klein-Gordon, Helmholtz, heat and wave equations.
- Exposition driven by additional examples and exercises.

Marcel-Adrian Şerban

Stephen Lynch, ***Dynamical Systems with Applications Using MATLAB***, Birkhäuser Verlag, Boston-Basel-Berlin 2004, xviii+462 pp, ISBN 0-8176-4321-4.

The book is a good introduction to dynamical systems theory. In the first part real and complex discrete dynamical systems are considered, with examples taken from population dynamics, economics, biology, nonlinear optics, neural networks and electromagnetic waves. In the second part of the text, differential equations are used to model examples taken from mechanical systems, chemical kinetics, electric circuits, interacting species and economics. The theory and applications are presented with the aid of the MATLAB package. Throughout the book, MATLAB is viewed as a tool for solving systems or producing exciting graphics. The author suggests that the reader should save the relevant example programs. These programs can then be edited accordingly when attempting the exercises at the end of each chapter. The text is aimed at graduate students and working scientists in various branches of applied mathematics, natural sciences and engineering. The material is intelligible to readers with a general mathematical background. Fine details and theorems with proof are kept at a minimum. This book is informed by the research interests of the author which are nonlinear ordinary differential equations, nonlinear optics and fractals. Some chapters include recently published research articles and provide a useful resource for open problems in nonlinear dynamical systems. An efficient tutorial guide to MATLAB is included. The knowledge of a computer language would be beneficial but not essential. The MATLAB programs are kept as simple as possible and the author's experience has shown that this method of teaching using MATLAB works well with computer laboratory class of small sizes.

I recommend "Dynamical Systems with Applications using MATLAB" as a good handbook for a diverse readership, for graduates and professionals in mathematics, physics, science and engineering.

Damian Trif