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BOOK REVIEWS

Free Boundary Value Problems – Theory and Applications, **Pierluigi Colli, Claudio Verdi, Augusto Visintin,** Editors, Birkhäuser Verlag, ISNM Vol. 147, 2004, ISBN 3-7643-2193-8.

The volume collects the proceedings of the conference on free boundary problems, Trento (Italy), June 2002. The contributions concern problems which are either directly related to free boundaries, or may be so in perspective. Special emphasis was put on interdisciplinarity and on issues of applicative relevance. Talks included twenty plenary addresses, and seven sessions were devoted to selected topics: free boundary problems in polymer science, image processing, grain boundary motion, numerical aspects of free boundary problems, free boundary problems in biomathematics, modelling in crystal growth and transition in anisotropic materials. I remark some contributions about numerical aspects:

Structural Optimization by the Level-Set Method by G. Allaire et al. describes a new numerical method based on a combination of the classical shape derivative and of the level-set method for front propagation.

Finite Element Methods for Surface Diffusion by E. Bnsch et al. presents a novel variational formulation for the parametric case of this 4th order highly nonlinear geometric driven motion of a surface. The authors also develop a finite element method and propose a Schur complement approach to solve the resulting linear system.

Upscaling of Well Singularities in the Flow Transport through Heterogeneous Porous Media by Z. Chen and X. Yue presents a method based on the recently introduced over-sampling multiscale finite element method and on the introduction of new base functions that locally resolve the well singularities.

On Plasma Expansion in Vacuum by P. Degond et al. formally and numerically justifies why electron emission produces a reaction pressure which slows down the plasma expansion .

A Posteriori Error Control of Free Boundary Problems by R. H. Nochetto assesses the derivation of a posteriori error estimators (including interface error estimators): computable quantities depending on the discrete solutions and the data, which provide upper and lower bounds for the error.

Shape Deformations and Analytic Continuation in Free Boundary Problems by F. Reitich presents an analysis of stability and bifurcation of steady states and travelling waves for a class of free boundary problems. These results lead to an understanding of the mechanisms behind the observed performance of a class of numerical algorithms based on shape-perturbation theory.

A Multi-mesh Finite Element Method for 3D Phase Field Simulations by A. Schmidt presents a general framework for the adaptive solution of coupled systems and its application to phase field simulations, making 3D simulations possible even on desktop computers.

Of course, all the 26 contributions included, which reflect and study the free boundary problems with applications in industry, make this volume interesting for a large spectrum of readers and offer opportunities of collaboration among mathematicians, physicists, engineers, material scientists, biologists and other researchers.

Damian Trif

George Grätzer, ***General Lattice Theory***, Second Edition, Birkhäuser Verlag, 2003, ISBN 3-7643-6996-5.

From its first edition George Grätzer's *General Lattice Theory* was a fundamental work in the lattice theory. It can be used as a course of lattice theory for students as well as a source of research problems for specialists. The last edition is enriched by 8 appendices and a new (and updated) bibliography. In this form, the

book covers so well the study of lattices that it is almost impossible for someone who works in this field not to find here something useful for his research. Besides the rich valuable information concerning the developments of the last two decades, the first appendix (**Retrospective**) is the history of the last 20years of lattice theory. The other 7 appendices are surveys on various topics of lattice theory. They are recommended by the value of their authors.The original chapters of the book are: **I. First Concepts;II. Distributive Lattices; III. Congruences and Ideals; IV.Modular and Semimodular Lattices; V. Varieties of Lattices; VI.Free Products; Concluding Remarks**, and the appendices are **A. Retrospective; B. Distributive Lattices and Duality** by B.Davey and H. Priestley; **C. Congruence Lattice** by G.Grätzer and E. T. Schmidt; **D. Continuous Geometry** by F.Wehrung; **E. Projective Lattice Geometries** by M. Greferathand S. Schmidt; **F. Varieties of Lattices** by P. Jipsen and H. Rose; **G. Free Latices** by R. Freese; **H. Formal Concept Analysis** by B. Ganter and R. Wille. The book ends with the new bibliography which contains 530 title.

C. Pelea

Alfred Göpfert, Hassan Riahi, Christiane Tammer, and Constantin Zălinescu, ***Variational Methods in Partially Ordered Spaces***, Springer-Verlag, New York, 2003 (CMS Books in Mathematics/Ouvrages de Mathématiques de la SMC, 17), xiv+350 pp, ISBN 0-387-00452-1.

This book is intended to provide a systematic and self-contained presentation of recent significant developments in vector optimization and adjacent fields, in connection with the own research of the four authors. The targeted audience includes researchers and graduate students in pure and applied mathematics, economics, engineering, geography, and town planning, who want to study modern variational methods in general partially ordered linear spaces and their concrete applications.

Since vector optimization is nowadays an attractive and quickly growing field, it is understood that authors have not intended to present an exhaustive treatment, and coercive choices were imposed in order to pack all of the topics surveyed into the following format:

Chapter 1. Examples: Section 1.1 (Göpfert-Tammer-Zălinescu): *Cones in vector spaces*; Sections 1.2–1.6 (Göpfert-Tammer): *Equilibrium problems, Location problems in town planning, Multicriteria control problems, Multicriteria fractional programming problems, Stochastic efficiency in a set*;

Chapter 2. Functional analysis over cones: Sections 2.1–2.3 (Göpfert-Tammer-Zălinescu): *Order structures, Functional analysis and convexity, Separation theorems for not necessarily convex sets*; Sections 2.4–2.7 (Zălinescu): *Convexity notions for sets and multifunctions, Continuity notions for multifunctions, Continuity properties of multifunctions under convexity assumptions, Tangent cones and differentiability of multifunctions.*

Chapter 3. Optimization in partially ordered spaces: Sections 3.1 (Göpfert-Tammer-Zălinescu): *Solution concepts*; Sections 3.2–3.6 (Zălinescu): *Existence results for efficient points, Continuity properties with respect to a scalarization parameter, Well-posedness of vector optimization problems, Continuity properties, Sensitivity of vector optimization problems*; Section 3.7 (Göpfert-Tammer): *Duality*; Sections 3.8–3.9 (Riahi): *Vector equilibrium problems and related topics, Applications to vector variational inequalities*; Section 3.10 (Göpfert-Tammer-Zălinescu): *Minimal-point theorems in product spaces and corresponding variational principles*; Section 3.11 (Göpfert-Tammer): *Optimality conditions*

Chapter 4. Applications: Section 4.1 (Göpfert-Tammer): *Approximation problems*; Section 4.2: *Solution Procedures*; Subsections 4.2.1–4.2.3 (Göpfert-Tammer): *A proximal-point algorithm for real-valued control approximation problems, Computer programs for the application of the proximal-point algorithm, An interactive*

algorithm for the vector control approximation problem; Subsections 4.2.4–4.2.5 (Riahi): *Proximal algorithms for vector equilibrium problems, Relaxation and penalization*; Sections 4.3–4.6 (Göpfert-Tammer): *Location problems, Multicriteria fractional programming, Multicriteria control problems, Stochastic efficiency in a set.*

The bibliography counts almost four hundreds items and allows the reader to easily find up-to-date literature on the field. The book also contains two lists providing an overview of illustrative figures, abbreviations and notations, and an index of selective terminology used throughout. In contrast to the wide spectrum of surveyed topics, the book reads easily, an unifying approach being visible throughout the whole text.

On one hand, specialists will certainly enjoy this monograph, especially because of the following features: many important results from functional analysis and partially ordered space theory are stated in a very general setting without undue abstraction; a large variety of relevant notions currently used in the literature are presented in a systematic way, the relationship between them being illustrated by diagrams; the authors use advanced techniques from different modern fields.

On the other hand, since the book is written in an rigorous, understandable, and teachable way, it may certainly serve to support courses on vector optimization, applied functional analysis, set-valued analysis, etc., targeted at the graduate level. For designing accompanying exercises, instructors will find in the text a good number of qualitative examples, which can be used to illustrate the results or to justify the stated assumptions. In particular, the emphasis on location theory applications will be especially appealing to graduate students of geography and researchers dealing with town planning. The final part of the book could also serve as a know-how support for practitioners who need to design multiple criteria decision software.

As a whole, this book can be strongly recommended as an excellent reference of general interest in vector optimization.

Nicolae Popovici