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ON SOME INTERPOLATION PROBLEM ON TRIANGLE

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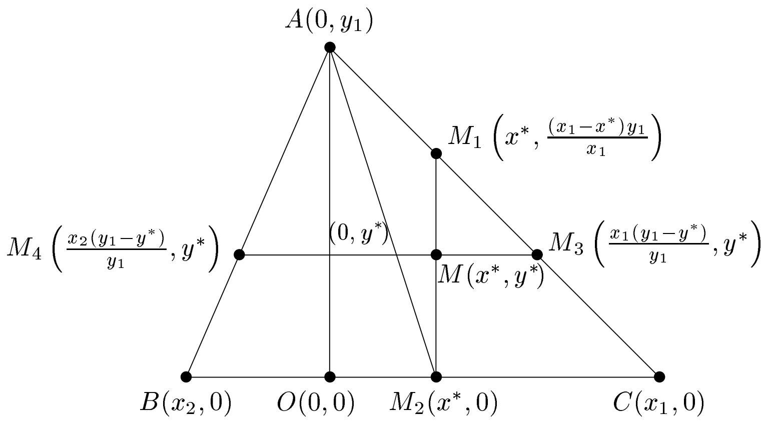
Abstract. Many of the theoretical and practical problems of numerical analysis consist in approximating of some types of functions on some kinds of domains like the triangular or rectangular domains. On the triangular domains the most of the approximations are made on some interior points of the triangle or on some derivatives values of the mentioned points. But also there it exists some types of functions which approximate the values of an entire side of the triangle or an entire interior line of this. The purpose of this paper is to present a this type of above mentioned function.

We shall suppose that we have given an interior point $M(x^*, y^*)$ on the triangle. Using an appropriate coordinates transform we shall suppose that the origin of the Oxy coordinates system and the triangle are situated as shows the next picture:



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Building from M the parallels to AO and BC and denoting by M_1, M_2 and M_3, M_4, respectively, the obtained intersections. In other words, the coordinates of the M_1 will be given by the solution of the system:

$$\begin{cases} AC: \ xy_1 + x_1y - x_1y_1 = 0 \\ x = x^* \end{cases}$$

So we shall find $M_1\left(\frac{(x_1 - x^*)y_1}{x_1}, x^*\right)$. Similar, we shall find the coordinates of the others above mentioned points: $M_2(x^*, 0)$, $M_3\left(\frac{x_1(y_1 - y^*)}{y_1}, y^*\right)$, $M_4\left(\frac{x_2(y_1 - y^*)}{y_1}, y^*\right)$ as being the intersections between $x = x^*$ and OC, $y = y^*$ and AC, and AB.

Let us consider the Lagrange operator L_1^y who interpolate the f function with respect to y in the points $(x^*, 0)$ and $\left(x^*, \frac{(x_1 - x^*)y_1}{x_1}\right)$. Also let us consider the operators L_1^x, L_2^x who interpolate the function with respect to x in the points $(0, y^*)$, (x^*, y^*) and $\left(\frac{x_1(y_1 - y^*)}{y_1}, y^*\right)$ respectively. The expressions of these operators will be given by:

$$(L_1^x f)(x, y) = \frac{(x - x^*)\left(x - \frac{x_1(y - y^*)}{y_1}\right)}{(-x^*)\left(0 - \frac{x_1(y - y^*)}{y_1}\right)} f(0, y^*) +$$

$$+ \frac{(x - 0)\left(x - \frac{x_1(y - y^*)}{y_1}\right)}{(x^* - 0)\left(x^* - \frac{x_1(y - y^*)}{y_1}\right)} f(x^*, y^*) +$$

$$+ \frac{(x - 0)(x - x^*)}{\left(\frac{x_1(y - y^*)}{y_1} - 0\right)\left(\frac{x_1(y - y^*)}{y_1} - x^*\right)} f\left(\frac{x_1(y - y^*)}{y_1}, y^*\right) \Leftrightarrow$$

$$(L_1^x f)(x, y) = \frac{(x - x^*)(y_1x - x_1y + x_1y^*)}{x^*x_1(y - y^*)} f(0, y^*) + \frac{x(y_1x - x_1y + x_1y^*)}{x^*(x^*y_1 - x_1y + x_1y^*)} f(x^*, y^*) +$$

$$\frac{y_1^2 x(x - x^*)}{x_1(y - y^*)(x_1y - x_1y^* - x^*y_1)} f\left(\frac{x_1(y - y^*)}{y_1}, y^*\right)$$

respective

$$(L_2^x)(x, y) = \frac{y_1^2 x(x - x^*)}{x_2(y_1 - y^*)(x_2y_1 - x_2y^* - x^*y_1)} f\left(\frac{x_2(y_1 - y^*)}{y_1}, y^*\right) +$$

$$+\frac{(y_1x - x_2y_1 + x_2y^*)(x - x^*)}{x^*x_2(y_1 - y^*)}f(0, y^*) + \frac{(y_1x - x_2y_1 + x_2y_1^*)x}{(x^*y_1 - x_2y_1 + x_2y^*)}f(x^*, y^*),$$

$$(L_1^y f)(x, y) = \frac{yx_1 - x_1y_1 + x^*y_1}{(x^* - x_1)y_1}f(x^*, 0) + \frac{x_1y}{(x_1 - x^*)y_1}f\left(x^*, \frac{(x_1 - x^*)y_1}{x_1}\right)$$

We denote by T the ABC triangle, by T_1 the AOC triangle and by T_2 the ABO triangle. We define the operators:

$$G_1 : T_1 \to R, \quad G_1 f = L_1^x \oplus L_1^y f,$$

respective

$$G_2 : T_2 \to R, \quad G_2 f = L_2^x \oplus L_1^y f.$$

Remark 1. We can easily verify that G_1 interpolate the f function on the frontier of T_1 and on the interior of AM_2 and G_2 interpolate the function on the frontier of T_2 and on the interior of AM_2, that means:

1) $G_1 f = f$ on $\partial T_1 \cup AM_2$;

2) $G_2 f = f$ on $\partial T_2 \cup AM_2$.

Remark 2. We can also verify that $dex(G_1) = 3$ and $dex(G_2) = 3$.

We shall build the F function who will interpolate the f function on the frontier of the T triangle, on the height AO and on the interior line AM_2 as follows:

$$F : T \to R, \quad F(x, y) = \begin{cases} G_1(x, y), & (x, y) \in T_1 \\ G_2(x, y), & (x, y) \in T_2 \setminus [AO] \end{cases}$$

We shall consider that, starting from the expression of G_1, we can give the next approximation formula:

$$f = G_1 f + R_{G_1} f.$$

Regarding the remainder of the above mentioned approximation formula, the next theorem show us how can be expressed this using the well known Peano's theorem.

Theorem 1. If $G_1 \in B_{2,2}(0, 0)$ then:

$$(R_{G_1} f)(x, y) = \int_0^{x_1} \varphi_{04}(x, y, t) f^{(0,4)}(0, t)dt + \int_0^{y_1} \varphi_{13}(x, y, t) f^{(1,3)}(0, t)dt +$$

$$+\iint\limits_{T_1} \varphi_{22}(x,y,s,t) f^{(2,2)}(s,t)dsdt$$

where

$$\varphi_{04}(x,y,t) = R_{G_1}^{xy}\left[\frac{(y-t)_+^3}{6}\right], \quad \varphi_{13}(x,y,t) = R_{G_1}^{xy}\left[x\frac{(y-t)_+^2}{2}\right],$$

$$\varphi_{22}(x,y,s,t) = R_{G_1}^{xy}[(x-s)_+(y-t)_+].$$

We can also give, starting from G_2, the next approximation formula:

$$f = G_2 f + R_{G_2} f$$

and if we take care of Theorem 1 we have that for $G_2 \in B_{2,2}(0,0)$ the remainder has the next approximation formula:

$$(R_{G_2}f)(x,y) = \int_0^{x_2} \varphi_{04}(x,y,t) f^{(0,4)}(0,t)dt + \int_0^{y_1} \varphi_{13}(x,y,t) f^{(1,3)}(0,t)dt +$$

$$+\iint\limits_{T_2} \varphi_{22}(x,y,s,t) f^{(2,2)}(s,t)dsdt$$

where $\varphi_{04}(x,y,t), \varphi_{13}(x,y,t), \varphi_{22}(x,y,s,t)$ have the same mentioned expressions.

Let us consider the approximation formula on T:

$$f = Tf + R_T f$$

Regarding the remainder $R_T f$ we can define it as follows:

$$R_T(f) = \begin{cases} R_{T_1} f, & (x,y) \in T_1 \\ R_{T_2} f, & (x,y) \in T_2 \setminus [AO] \end{cases}$$

References

[1] K. Bohner, Gh. Coman, *Blending interpolation scheme son triangle with error bounds*, Springer Verlag, Berlin-Heidelberg-New York, 1977, 14-37.

[2] K. Bohner, Gh. Coman, *Smooth interpolations schemes in triangles with error bounds*, Mathematica (Cluj), **18**(41), 1, 1976, 15-27.

[3] K. Bohner, Gh. Coman, *On some approximation schemes on triangles,* Mathematica (Cluj), **24**(45), 1980, 231-235.

[4] Gh. Coman, I. Gânsca, L. Ţâmbulea, *Some practical application of blending interpolation*, Proceedings of the Colloqium on Approximation and Optimization, Cluj-Napoca, October 25-27, 1984.

[5] Gh. Coman, *Analiză numerică*, Ed. Libris, Cluj-Napoca, 1995.

[6] D. D. Stancu, Gh. Coman, P. Blaga, *Analiză numerică şi teoria aproximării*, vol. II, Presa Universitară Clujeană, Cluj-Napoca, 2002.

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