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BOOK REVIEWS

Pietro Aiena, ***Fredholm and Local Spectral Theory, with Applications to Multipliers***, Kluwer Academic Publishers, Dordrecht-Boston-London 2004, xiv + 444 pp, ISBN: 1-4020-1830-4.

Let T be an operator acting on a complex Banach space X. The local resolvent of T at a point $x \in X$ is the set $\rho_T(x)$ of all $\lambda \in \mathbb{C}$ for which there exist an open neighborhood $U_\lambda \subset \mathbb{C}$ of λ and an analytic function $f : U_\lambda \to X$ such that (1) $(\mu I - T)f(\mu) = x$, for all $\mu \in U_\lambda$. Obviously that the analytic function $f_x(\mu) = (\mu I - T)^{-1}x$ satisfies this relation on the resolvent set $\rho(T)$ of the operator T, but it could exist other analytic functions satisfying (1), even on neighborhoods of some points in the spectrum $\sigma(T)$ of T. The set $\rho_T(x)$ is open and contains $\rho(T)$. The local spectrum of T at x is $\sigma_T(x) = \mathbb{C} \setminus \rho_T(x)$, and $\sigma_T(x) \subset \sigma(T)$.

The local spectral theory is systematically treated in a recent book by K. B. Laursen and M. M. Neumann, *An Introduction to Local Spectral Theory,* Oxford University Press 2000, containing also some elements of Fredholm theory, mainly those which can be approached by the methods of local spectral theory.

The emphasis of the present book is first on Fredholm theory, developed in connection with Kato decomposition and a property called the single-valued extension property (SVEP). This property, which means that the only analytic function f satisfying the relation $(\mu I - T)f(\mu) = 0$ for any μ in a neighborhood of an arbitrary point $\lambda \in \mathbb{C}$ is the null function $f \equiv 0$, has deep connections with Fredholm theory. It was considered first by Dunford in 1952 and is systematically studied in the treatise on operator theory by Dunford and Schwartz, and in other books on spectral theory of operators. The author presents also the abstract Fredholm theory in semi-prime

Banach algebras. The main applications of Fredholm theory considered in the book are to multipliers on Banach algebras.

Another direction of investigation studied in the book is that of perturbation theory for classes of operators which occur in Fredholm theory, completing the existing monographs on this topic (as, e.g., Kato's classical treatise) with more recent results.

The author tried to make the book as self-contained as possible, by proving some elementary facts about the notions considered. Of course that, as a research monograph, it requires from the reader an acquaintance with basic complex function theory and functional analysis, including classical Fredholm theory.

The book can be used for graduate courses in operator theory or by professional mathematicians working in the subject or interested in its applications to other areas of investigation.

S. Cobzaş

Fundamental Directions in Mathematical Fluid Mechanics, **G. P. Galdi, J. G. Heywood, R. Rannacher** (Editors), Birkhäuser Verlag, 2000.

The present volume consists of six articles, written by very good experts in the field, each article treating an important topic in the theory of the Navier-Stokes equations, at the research level. As it is well known, the most famous problem in this area is to go beyond the presently known global existence of weak solutions, to the global existence of smooth solutions, for which uniqueness and continuous dependence on the data can be proved. In fact, Galdi's article, *An introduction to the Navier-Stokes initial-boundary value problem*, gives an overview of the state of research regarding this subject.

Then the book moves on to a discussion of new developments of the finite element Galerkin method. The article by Rannacher, *Finite element methods for the incompressible Navier-Stokes equations*, treats both the theory and implementation of the finite element methods, with an emphasis on a priori and a posteriori error estimation and adaptive mesh refinement.

The article by Gervasio, Quarteroni and Saleri *Spectral approximation of Navier-Stokes equations* is devoted on spectral Galerkin methods and their extension to domains with complicated geometries, by employing the techniques of domain decomposition.

The article by Heywood and Nagata *Simple proofs of bifurcation theorems* introduces in a rigorous way bifurcation theory in a general setting that is convenient for application to the Navier-Stokes equations.

The two articles by Heywood and Padula, *On the steady transport equation* and *On the existence and uniqueness theory for the steady compressible viscous flow*, yield a simplified approach to the theory of steady compressible viscous flow. The extension of Navier-Stokes theory to compressible viscous flows, studied in these papers, opens up a beautiful point of view of theoretical and numerical problems.

The book is very well written and enjoyable. It is addressed to researchers, advanced students, and all mathematicians interested to the research level on some of the most important topics in the field of fluid mechanics.

Mirela Kohr

Leszek Gasiński and Nikolaos S. Papageorgiou, *Nonsmooth critical point theory and nonlinear boundary value problems*, Chapman & Hall/CRC, 2004.

One often encounters practical situations where the associated energy functional to a nonlinear elliptic problem is not smooth. Several methods have been elaborated in the last decades in order to handle such kind of problems, see the theories of Chang (1981), Szulkin (1986), Degiovanni and Marzocchi (1994), Frigon (1998), Motreanu and Panagiotopoulos (1999).

The aim of the monograph of L. Gasiński and N. S. Papageorgiou is to present a comprehensive exposition of the aforementioned (non-smooth) critical point theories, as well as to provide us with various applications and concrete examples.

The book is as self-contained as possible and it is made more interesting by the perspectives in various sections, where the authors mention the historical background and development of the material and provide the reader with detailed explanations and updated references.

The first chapter is dedicated to the background material used throughout the book, as basic elements from Sobolev spaces, Set-Valued analysis, Non-smooth analysis (Clarke's calculus of locally Lipschitz functions, weak slope), Nonlinear Operators.

In the second chapter the authors present the existing nonsmooth critical point theories. This part is very well written; the reader obtains a complete picture about these theories. In the first two sections the locally Lipschitz functionals (Chang's theory) as well as constrained locally Lipschitz functionals (the non-smooth version of Struwe's theory) are treated. In the third section the critical point theory of locally Lipschitz functions is developed which are perturbed by a convex, proper and lower semicontinuous functional. This part unifies the theories of Chang and Szulkin. We point out that Motreanu and Panagioutopoulos (1999) were the first authors, and not Kourogenis, Papadrianos and Papageorgiou (2002) as it is mentioned in the book (page 204, paragraph 2.3), who considered this class of functionals. In the fourth section the classical local linking theorem is extended to locally Lipschitz functions, while the last two sections are devoted to the theory of weak slopes, in the sense of Degiovanni-Marzocchi (for continuous functionals), and Frigon (for multivalued functionals). In all the cases, deformation and minimax results are obtained (with Palais-Smale, or Cerami compactness conditions).

The rest of the book deals with applications. Chapter 3 is devoted to the study of nonlinear boundary value problems for ordinary differential equations. Several kind of problems are treated: Dirichlet problems, periodic problems, Hamiltonian inclusions, problems with nonlinear boundary conditions. A great variety of methods and techniques are used, as upper-lower solutions, fixed-point and degree theory arguments, nonsmooth analysis, set-valued analysis.

The biggest part of this book is Chapter 4 (more than 250 pages), which is devoted to the study of nonlinear elliptic equations. The theoretical material, presented in the second chapter, is consistently applied in order to establish existence and multiplicity results for several type of resonance problems (like semilinear, nonlinear, variational-hemivariational inequalities and strongly resonant problems); Neumann problems (homogeneous and non-homogeneous type); problems with an area-type term; problems which involve discontinuous nonlinearities.

In my opinion, the book is very readable, and it can serve as a start point for researchers and students in order to carry out further investigations in the nonsmooth critical point theory as well as in its applications in mechanics, mathematical physics and engineering.

A. Kristály

E. I. Gordon, A. G. Kusraev and S. S. Kutateladze, *Infinitesimal Analysis*, Mathematics and Its Applications, Vol. 544, Kluwer A. P. , Dordrecht-Boston-London, 2002, xiii + 422 pp, ISBN: 1-4020-0738-8.

Infinitesimals or infinitely small quantities, and infinitely large quantities were used for two millennia by scientists and philosophers, starting with Archimedes. The infinitesimals were basic tools in the foundation of mathematical analysis by Leibniz and Newton, and were used by their followers as well, e.g. Euler, until the 19th century when Bolzano, Cauchy and Weierstrass founded the analysis on the notion of limit and $\epsilon - \delta$ technique. After that the use of infinitesimals was considered as lacking of rigor, until the sixties of the 20th century when A. Robinson created nonstandard analysis and put firm basis for the use of infinitely small and infinitely large quantities in mathematics. A brief survey on the historical evolution of ideas in mathematical analysis is presented in the first chapter of the book *Excursus in the history of calculus.*

The term infinitesim al analysis is used to designate a technique of studying general mathematical objects by discriminating between standard and nonstandard ones. The present book is the third in the series "Nonstandard Methods of Analysis" published at Novosibirsk by Sobolev Institute Press under the guidance of Professor Kutateladze. The previous two books were *Boolean Valued Analysis*, Kluwer 1999, by the same authors, and a collection of papers Nonstandard Analysis and Vector Lattices, Kluwer 2000. All these books were written in Russian and then translated (in a revised form) into English and published by Kluwer, as well as another book of the authors *Nonstandard Methods of Analsyis*, Kluwer 1994, which gave the name to the series.

The purpose of the present book is to make the methods of nonstandard analysis more accessible to a larger audience. To this end the second chapter, *Naive foundation of infinitesimal analysis*, contains an intuitive and illustrative introduction to the subject, but sufficient for effective applications, without appealing to any logical formalism.

The cantorian set theory is presented in Ch. 3, *Set-theoretic formalisms of infinitesimal analysis.* Beside the Zermelo-Frenkel system, Nelson internal set theory and the external set theories of Hrbaçek and Kawai are included.

The rest of the book is dedicated to applications of nonstandard analysis to various branches of mathematics – topology in Ch. 4, *Monads in general topology,* and subdifferential calculus and non-smooth analysis in Ch. 5, *Infinitesimals and subdifferentials.* Remark that another book of the authors on the same topic *Subdifferentials: Theory and Applications*, Kluwer 1995, makes extensive use of nonstandard methods. Ch. 6, *Technique of hyperaproximation* deals with nonstandard hulls of normed spaces defined by Luxemburg, and Loeb measures. The technique of hyperapproximation for the Fourier transform on a locally compact abelian group is considered in Ch. 7, *Infinitesimals in harmonic analysis.*

The last chapter of the book, Ch. 8, *Exercises and problems*, contains some exercises along with some open problems of varying difficulty.

The authors have included in the book a lot of philosophical and historical comments. The bibliography at the end of the book contains 542 items.

The book is aimed first to researchers in various branches of mathematics desiring to be acquainted with the powerful tools of nonstandard analysis. Teachers will find in the book a lot of interesting things: – methodological, historical and philosophical.

S. Cobzaş

Martin Väth, *Integration Theory*, World Scientific, New Jersey - Singapore - London, 2002, viii + 27 pp, ISBN: 981-238-115-5.

This book on measure and integration proposes a very general approach to the subject, allowing the simultaneous treatment of both scalar and vector cases. The framework is that of a measure space (S, Σ, μ) and of functions on S taking values in a space $Y = [0, \infty,\ [-\infty, \infty]$, or a Banach space with an ideal element ∞. This approach, presented in the first chapter of the book, Ch. 1, *Abstract Integration*, is based on some results such as the exhaustion theorem, the covering theorem and a theorem on approximation of measurable functions, appearing for the first time in this general form. The Carathéodori method of constructing a measure from an outer measure along with some extension theorems are also included, with applications to Tonelli and Fubini theorems.

Radon measures are treated in the second chapter which contains also some basic results from topology, including Urysohn and Tychonov theorems. Luzin measurability theorem is proved. The highlight of the chapter is Riesz representation theorem for positive linear functionals on the space of continuous functions with compact support defined on a locally compact Hausdorff space.

The existence, uniqueness and basic properties of Haar invariant measure on a locally compact group are considered in the third chapter.

These first three chapters form Part 1, *Basic Integration Theory*, of the book. The first chapter of Part 2, *Advanced Topics*, is concerned with Lebesgue-Bochner function spaces $L_p(S, \Sigma, \mu)$ and their duals, treated as particular cases of ideal spaces. Orlicz spaces are discussed in exercises.

The fundamental properties of convolutions, a basic tool in harmonic analysis and in approximation theory, are discussed in the fifth chapter. As application one proves an extension of a famous result of H. Steinhaus: if M is a subset of positive measure of a Hausdorff locally compact group S with a left invariant Haar measure, then $M^{-1}M$ is a neighborhood of e. H. Steinhaus (1920) proved the result for $S = \mathbb{R}^n$.

Chapter 6 contains a fine discussion on the connections of some results in measure theory with mathematical logic and set theory. Some famous paradoxes, such as Hausdorff's and Banach-Tarski, are presented along with their consequences for the problem of the existence of finitely additive measures on $\mathbb{R}^n$.

The fundamental results on Lebesgue integration on $\mathbb{R}^n$ – absolutely continuous functions, a.e. differentiability, change of variable formula – are presented in Chapter 7. The last chapter of the book, Chapter 8, is concerned with some useful formulas in Lebesgue integration theory, as ,e.g., the differentiation under integral sign, the change of the order of integration, the Cavalieri principle.

All the notions and results presented in the book are accompanied by comments and examples warning the reader about some delicate points of the subject, or on errors that could be done (or were done). The exercises at the end of each chapter complete the main text with related results and examples.

The result is a fine book on measure theory and integration, based on a general approach to the subject and discussing many difficult topics in the area. It can be recommended for advanced courses in measure theory, but it is suitable also for self-study by graduate students.

V. Anisiu

Vladimir A. Zorich, ***Mathematical Analysis***, Springer Verlag, Berlin-Heidelberg 2004, Vol. I: xviii + 574 pages, ISBN: 3-540-40386-8; Vol. II: xv + 681 pages, ISBN: 3-540-40633-6.

This is the translation of the fourth edition of a well known course on mathematical analysis, taught for several years by the author at the Moscow State University (MSU) and at other universities. Together with V.I. Arnold and S.P. Novikov, the author is one of the organizers of advanced experimental courses at MSU, this experience being reflected in the book too. Written in the good tradition of Russian mathematical textbooks, the present one combines intuition and accessibility with modern mathematical rigor.

The book is divided into two volumes. The main part of the first volume is concerned with the calculus of functions of one variable, developed in the first 6 chapters: 1. *Some logical and mathematical concepts and notation*; 2. *The real number system* (introduced axiomatically); 3. *Limits* (including a treatment of limits with respect to a filter base that are used in several places throughout the book, as e.g. in integration theory); 4. *Continuous functions*; 5. *Differential calculus* (including the calculus of primitives, complex numbers and power series of complex numbers which are used to define e^z); 6. *Integration* (meaning Riemann integration and improper Riemann integrals). Beside the basic theoretical material, these chapters contain many worked examples of applications of the methods of mathematical analysis to other branches of mathematics (as, for instance, a proof of the fundamental theorem of algebra), or from natural and physical sciences (the barometric formula, the motion of a body with variable mass, the falling of a body in atmosphere, radioactive decay, etc).

The last two chapters of the first volume deal with functions of several variables – 7. *Functions of several variables* (continuity questions), and 8. *Differential calculus in several variables.* This last chapter contains some deep results, as the implicit function and inverse function theorems, the tangent space to a k-dimensional surface in $\mathbb{R}^n$ and constrained extrema.

The volume ends with some midterm examination problems, as well as some final examination problems for the first semester (one variable theory) and the second semester (integration and multivariate calculus).

The second volume contains more advanced topics and basically correspond to the second year curriculum in the mathematics departments at MSU. It can be read independently of the first volume, because the first two chapters, 9. *Continuous mappings - General theory* and 10. *Differential calculus from a general viewpoint*, contain in a compressed and generalized form the results on continuity and differentiability from the first volume: basic properties and constructions for metric and topological spaces, continuous mappings, differential calculus for mappings between normed spaces, higher-order differentials, Taylor's formula, and a general implicit function theorem. Multiple Riemann integration and improper multiple Riemann integrals are treated in Chapter 11, *Multiple integrals.* Chapter 12, *Surfaces and differential forms in* $\mathbb{R}^n$, is concerned with surfaces, orientation, area surface and elementary properties of differential forms, preparing the ground for the next two chapters, 13, *Line and surface integrals,* which contains the proofs of the fundamental integral formulas of Green Ostrogradski-Gauss and Stokes, and 14, *Elements of vector analysis.* Chapter 15, *Integration and differential forms on manifolds*, can be viewed as a synthesis at a higher level of abstractization of the topics treated in chapters 11-14. Uniform and pointwise convergence of sequences of functions are treated in Chapter 16, which contains also proofs of the Arzela-Ascoli compactness theorem and of Stone approximation theorem. The integrals depending on a parameter (including improper and multiple integrals) are treated in Chapter 16, with applications to Euler's functions Beta and Gamma. Convolutions and generalized functions are also briefly discussed in this chapter.

The last two chapters of the book are 18, *Fourier series and Fourier transform* and 19, *Asymptotic expansions.*

As the first one, this volume ends also with some midterm and final examination questions.

The bibliography is grouped in four categories: 1. Classical books; 2. Textbooks; 3. Classroom material; 4. Further reading. For the convenience of the readers, some English titles were added for this edition.

There are a lot of exercises and problems, of varying difficulty, spread through the book, needed for a better understanding of the subject, as well as historical notes about the great names who contributed along the centuries to the building of the edifice of mathematical analysis.

This comprehensive course on mathematical analysis provides the readers, first of all students specializing in mathematics, with rigorous proofs of the fundamental theorems, but also with its applications in mathematics itself and outside it. It is correlated with subsequent disciplines relying on its methods and results, as differential equations, differential geometry, functions of a complex variable and functional analysis.

T. Trif