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BOOK REVIEWS

Matoušek, Jiří, *Lectures on Discrete Geometry*, Springer (Graduate Texts in Mathematics, 212), 2002, 481 pp., Softcover, ISBN 0-387-95334-4.

Discrete geometry is not quite a newcomer on the stage of mathematics. Isolated results belonging to this field can be found already in the works of Descartes, Euler, Dirichlet and, in more modern times, Voronoi, Delaunay, Minkowski, Brunn, Helly, and many others. It deals, mainly, with finite sets of simple geometrical objects such as points, lines, circles or their higher dimensional analogues and it studies things like reciprocal positions of these objects, counts the intersection points or the zones determined by higher dimensional objects (for instance arrangements of hyperplanes, in particular of straight lines) and other problems of the same kind.

In the last few decades the discrete geometry has seen a more rapid development, in connection to some related fields, such as computational geometry or computational geometrical optimization. In spite of the increasing interest in the field, there are still only few reliable textbooks on the market. A notable contribution is, no doubt, the book of Pach and Agarwal (*Combinatorial Geometry*, Wiley, 1995).

The book under review covers, in my opinion, a gap in the pedagogical literature, providing an expository treatment of a wide range of topics in discrete geometry, without assuming too many prerequisites from the reader. We choose just a couple of subjects examined into the book, taken from a list provided by the author himself:

- foundational results from affine and convex geometry, including the Minkowski theorem on lattice points, a couple of words about Voronoi diagrams and Delaunay triangulations a.o.
- combinatorial complexity of geometric configurations (line-point incidences, complexity of arrangements, Davenport-Schinzel sequences, probabilistic methods);
- intersection patterns and transversal of convex sets;
- geometric Ramsey theory, related to the existence, in any sufficiently large configuration, of a subconfiguration which is, in a specific sense, regular;
- polyhedral combinatorics and high-dimensional complexity;
- representation of finite metric spaces by coordinate.

The list is far from being complete. Of course, the field is quite vast, so a lot of subjects had to be left aside. Still, the book is very comprehensive and starts from a low level (only some linear algebra, elementary calculus, probability and combinatorics are assumed), so it will be an ideal to be used both as a textbook and for self-study. The expected audience includes graduate students and researchers in discrete and computational geometry, optimization and computer science. The author is a well-known expert, rather in computational geometry than in discrete geometry and, sometimes, his personal tastes are easily recognized. In facts, in some sense, this book can be used as a "mathematical companion" to a textbook on computational geometry where, usually, the authors focuses on the description and analysis of algorithms rather than on the mathematics which is behind these algorithms.

The book is completed with a lot of examples and exercises, not to mention the impressive number of line diagrams, which cannot miss in such a kind of book.

Paul A. Blaga

I. John Cagnol, Michael P. Polis, Jean-Paul Zolesio (Eds.), *Shape Optimization and Optimal Design,* Lecture Notes in Pure and applied Mathematics, vol. 216, Marcel Dekker, New York-Basel, 2001, ISBN: 0-8247-0556-4.

II. Giuseppe Da Prato, Jean-Paul Zolesio (Eds.), *Partial Differential Equation Methods in Control Analysis,* Lecture Notes in Pure and Applied Mathematics, vol. 188, Marcel Dekker, New York-Basel-Hong Kong, 1997, ISBN: 0-8247-9837-6.

The first volume mentioned comprises papers from the sessions "Distributed Parameter Systems" and "Optimization Methods and Engineering Design" held within the 19th conference System Modeling and Optimization in Cambridge, England.

The second volume presents papers from the Conference on Control and Shape Optimization held at Scuola Normale Superiore di Pisa, Italy. Both the conferences were organized by the International Federation for Information Processing (IFIP).

The papers present the latest developments and major advances in the fields of active and passive control for systems governed by partial differential equations- in particular in shape analysis and optimal shape design.

Traditionally, optimal shape design has been treated as a branch of the calculus of variations, more specifically of optimal control. The subject interfaces with at least four fields: optimization, optimal control, PDEs and their numerical solutions.

The main question that optimal shape design tries to answer is: "What is the best shape for a physical system?".

Many problems that arise in technical and industrial applications can be formulated as the minimization of functionals with respect to a geometrical domain which must belong to an admissible family. Optimal shape design is used in various fields, like those mentioned in the books: fluid mechanics, linear elasticity, thermo-elasticity, soil mechanics, electricity, aircraft industry, material sciences, biodynamics.

The authors of the articles are well known for important results in this field of research.

Some of the aspects treated are:

- shape sensitivity analysis (that is the sensitivity of the solutions with respect to the shape of the domain) for the Navier-Stokes equation, Maxwell's equation, for some problems with singularities (I)
- the study of the material derivative, the shape derivative on a fractured manifold (I), the shape derivative for the Laplace-Beltrami equation (II), the shape hessian for a nondifferentiable variational free boundary problem (II),the shape gradients for mixed finite element formulation (II),the eulerian derivative for non-cylindrical functionals (I)
- numerical aspects (using finite element approximation and other methods, some of them original) for
  - shape problems in linear elasticity (I)
  - parallel solution of contact problems (I)
  - modeling of oxygen sensors (I)
  - control of a periodic flow around a cylinder (I)
  - shape identification problems associated with the stationary heat conduction in 2D(II)
- boundary controllability of thermo-elastic plates (I)
- regularity properties for the weak solutions to certain parabolic equations(II)
- homogenization and continuous dependence for Dirichlet problems, asymptotic analysis on singular perturbations (II), asymptotic analysis of aircraft wing model in subsonic flow (I)
- mapping method in problems governed by hemivariational inequalities (I)
- feedback laws for the optimal control of parabolic variational inequalities

Many more subjects are treated in the 41 papers by 50 authors, which allow the reader to get a good idea about the latest research directions in this very active field of applied mathematics.

Daniela Inoan

Unsöld, Albrecht, Baschek, Bodo, *Der neue Kosmos*, Springer, 2002, 575 pp., Hardcover, ISBN 3-540-44177-7.

The book under review is the 7^{th} edition of Unsöld's textbook "Neue Kosmos", whose first edition was published in 1967. Starting with its third edition the book was updated jointly with Bodo Baschek. He continued to up date and add to contents of the book new after Unsöld pass away. This new edition, that came three years after the sixth edition of the book, contains new results about the Solar System and the Universe as a whole, obtained in these last years.

The book has has four parts: classical astronomy and Solar System, practical astronomy, stellar structure and cosmology and cosmogony. The first part is devoted to the foundations of the astronomy (spherical astronomy, time, celestial mechanics) and a description of the motion of celestial bodies (planets, Sun, Moon). In the last chapters from these part are described the Solar System bodies (planets, satellites, asteroids, comets and meteorites) from the physical point of view. The second part of the book contains a brief introduction in the problems of the practical astronomy. Firstly, the are given the basic notions about radiation and its interaction with the matter and after that there are described the astronomical tools and techniques. The third part of the book is devoted to the physics of the stars. Here are presented the main topics related to the stellar structure and evolution. There are described different types of stars including the Sun. The stellar systems, stellar clusters and galaxies, are described in the last part of the book. Another task of the book is to introduce the problems related to cosmology and cosmogony. Each part begins with a short historical note and at the end of the book there are two appendices devoted to the astronomical units, respectively a list of the constellations. There are also selected problems that could be used during the learning and teaching process.

The book also includes 278 images and line diagrams, including 20 colour plates. The sources of the images are given in an appendix.

The book is highly recommended to students in astronomy and astrophysics, being ideal as a textbook. Let me also mention the impressive graphical qualities of the books, something that, unfortunately, is increasingly rare nowadays.

Cristina Blaga

V. Benci, G. Cerami, M. Degiovanni, D. Fortunato, F. Giannoni, A.M. Micheletti (Eds.), *Variational and Topological Methods in the Study of Nonlinear Phenomena,* Progress in Nonlinear Differential Equations and Their Applications, Vol. 49, Birkhäuser Verlag, Basel-Boston-Berlin 2002, vii + 131 pp., ISBN 3-7643-4278-1 and 0-8176-4278-1.

The articles in this volume are an outgrowth of the international conference *Variational and Topological Methods in the Study of Nonlinear Phenomena*, held in Pisa in January/February 2000, under the framework of the research project *Differential Equations and the Calculus of Variations.* The specific aim of the conference was to celebrate the 60th birthday of Antonio Marino, one of the leaders of the research group, with significant contributions to this area.

The volume contains 9 papers: 1. M. Clapp, *Morse indices and mountain pass orbits of symmetric functionals;* 2. M.J. Esteban and E. Séré, *On some linear and nonlinear eigenvalue problems in relativistic quantum chemistry;* 3. A. Ioffe and E. Schwartzman, *Convexity at infinity and Palais-Smale conditions;* 4. W. Marzantowicz, *Periodic solutions of nonlinear problems with positive oriented periodic coefficients;* 5. M. Mrozek and P. Pilarczyk, *The Conley index and rigorous numerics of attracting periodic orbits;* 6. R. Ortega, *Dynamics of a forced oscillator having an obstacle;* 7. M. del Pinto, P. Felmer and M. Musso, *Spike patterns in the supper-critical Bahri-Coron problem;* 8. P. Sintzoff and M. Willem, *A semilinear elliptic equation on* $\mathbf{R}^{\mathbf{N}}$ *with unbounded coefficients;* 9. R.E.L. Turner, *Traveling waves in natural systems.*

As it can be seen by this enumeration, the contributions highlight recent advances in nonlinear functional analysis, with applications to nonlinear partial or ordinary differential equations, having as unifying theme the use of variational and topological methods. There are worth to mention the applications to biology and chemistry included in the volume.

The volume will be an excellent reference text for researchers and graduate students working in these areas.

S. Cobzaş

Weaver, N., *Mathematical Quantization*, Chapman & Hall / CRC (Studies in Advanced Mathematics), 2001, 278 pp., Hardcover, ISBN 1-58488-001-5.

After almost a century from the creation of quantum mechanics there is still no general agreement on what we should mean by general "quantum theory", as well as by "quantization". Besides, there still is a gap between physicists which are, in the end, mainly interested in the phenomenological aspects of quantum theory, and mathematicians, interested in rigor and building more sophisticated theories. Nevertheless, there is a large amount of mathematics (especially functional analysis) that everybody agrees that should be a basic ingredient of any "quantization procedure" and can safely go under the name of "mathematical quantization".

The book of Weaver intends to expose, in a coherent manner, both the foundational material and some of the contemporary achievements, related especially to noncommutative geometry and quantum groups theory. The basic philosophy of the book is that quantization means replacing sets by Hilbert spaces and the author finds quantum analogues of the main ingredients of classical mathematical physics (topological spaces, distances, measures, a.o.). Of course, the idea itself is not new. After all, even at the initial stage of development of quantum mechanics quantization basically meant just replacing classical observables (functions defined on the configuration space) by operators on Hilbert spaces. However, Weaver is the first to make an extended use of this idea to build the quantum analogues of the classical notions mentioned above, replacing, in particular, the spaces by C^*-algebras and spaces with measure by von Neumann algebras.

The book starts with a brief mathematical review of classical mechanics and continues with Hilbert spaces and linear operators on them. Now come into play the first "quantum" notion: the quantum plane. There follows two chapters on C^*-algebras and von Neumann algebras which are, in the sequel, applied to quantum field theory. The rest of the book is devoted to foundational material in noncommutative geometry (Hilbert modules), Lipschitz algebras and quantum groups.

The intention is to lay the mathematical foundations for physical applications, therefore, no prior knowledge of physics is assumed (although it is, of course, very helpful). In fact, except the chapter devoted to quantum field theory, no applications to physics are discussed, still, someone which is familiar to quantum physics will recognize easily many physical notions and results "in disguise".

The book is a comprehensive exposition of the modern mathematics necessary for quantum theory and the author manages to describe a surprisingly large amount of material in an attractive and clear manner. Of course, it cannot replace the detailed texts in more special topics, anyway, the reader, graduate student or researcher, can get an idea on the state of the art of the theories regarding quantization. Let me also emphasize that, as it is easily understood, only a limited quantity of quantization tools are exposed here. The book can be thought off, in a way, as an introduction into noncommutative geometry (or, rather, into the prerequisites of noncommutative geometry).

To conclude, the book is very well written and provides a lucid and clear exposition of some of the most important tools of quantization theory.

Paul A. Blaga

C. Zălinescu, *Convex Analysis in General Vector Spaces*, World Scientific, New Jersey-London-Singapore-Hong Kong 2002, xx+367 pp, ISBN: 981-238-067-1.

This well written book is devoted to convex analysis in infinite dimensional spaces. What makes it different from other existing books on convex analysis and optimization is the fact that the results are presented in their most generality, known at this time, as well as the inclusion of new and recent material. The author is a well known specialist in the field and the book incorporates many of his original results. In order to obtain this generality, the author has included in the first chapter of the book, Ch. 1, *Preliminary results on functional analysis*, a detailed study of convex series (cs) closed, lower cs-closed (lcs-closed), ideally convex, lower ideally convex (li-convex) sets and multivalued mappings, allowing him to prove very general open mapping and closed graph theorems of Ursescu-Robinson type. The chapter contains also a fine study of separation of convex sets and a presentation, with complete proofs, of Ekeland's variational principle and of Borwein-Preis smooth variational principle.

The second chapter of the book, Ch. 2, *Convex analysis in locally convex spaces*, beside classical results, contains also the study of some more general classes of functions, corresponding to the sets studied in the first chapter: cs-closed, cs-complete, lcs-closed, ideally convex, bcs-complete and li-convex functions. The conjugate function, duality formulae, the subdifferential and the ϵ-subdifferential calculus, are also included. The developed machinery is applied to convex programming, perturbed problems, convex optimization with constraint and to minimax theorems.

The last chapter of the book, Ch. 3, *Some results and applications of convex analysis in normed spaces*, contains some deep results in convex analysis that are specific to this framework, which have important applications to optimization and to other areas. We mention the Brønsted-Rockafellar theorem with applications to the proofs of Bishop-Phelps theorem and of Rockafellar's maximal monotonicity of the subdifferential of a convex function. Zagrodny mean value theorem for abstract subdifferentials yields a short proof of the converse of the above result: every cyclically maximal monotone multivalued mappings is subdifferential of a convex functions. The important classes of uniformly convex and uniformly smooth functions are studied, as well as the interplay of their properties and of the differentiability of convex functions, with the geometry of underlying normed space. The last section of this chapter, based on some recent results of S. Simons, is concerned with monotone multivalued mappings.

There are a lot of exercises spread through the book. Some of them contain technical parts of some proofs or examples, while the others are concerned with results which did not fit in the main stream of the exposition.

The book is fairly selfcontained, the prerequisites for the reading being familiarity with basic functional analysis, including topological vector spaces and locally convex spaces.

The book is very well organized, with comprehensive indexes of notation and of notions, and a rich bibliography.

It can be used as a textbook for advanced graduate courses, or as a reference text by specialists.

S. Cobzaş