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ON GENERATION OF FAMILIES OF SURFACES

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Dedicated to Professor D.D. Stancu on his 75^{th} birthday

Abstract. In this paper we define families of interpolation surfaces of Hermite and Birkhoff type. Particular cases of these surfaces are illustrated graphically.

1. Introduction

The modelling and remodelling of surfaces come up in different activities as civil engineering, industries of airplanes, ships, automobiles, industrial and artistic objects, scientific research and others.

There exists a large number of classical and modern methods for generating surfaces. As modern methods for generating surfaces we mention those of Bézier, Coons, Shepard and others [5], frequently encountered in Computer Aided Design (CAD) and Computer Aided Geometric Design (CAGD).

In our paper we present two procedures for defining surfaces.

2. Surfaces with two support curves and tangent ribbons

In this section we define a family of surfaces each one containing the same two opposite space curves, say (C1) and (C2) and different tangent ribbons (across-boundary derivatives), see Figure 1.

Suppose the curves (C_1) and (C_2) are represented by the equations:

$$(C_1)\begin{cases} y = 0 \\ z = h_0(x) \end{cases} \quad and \quad (C_2)\begin{cases} y = y_1(x) \\ z = h_1(x), \end{cases} \quad x \in [0, a]. \tag{2.1}$$

For the given functions $h_0(x)$, $h_1(x)$, $m_0(x)$ and $y_1(x)$, $x \in [0, a]$, let us find the surface (S), having the equation $z = f(x, y)$, $x \in [0, a]$, $y \in [0, y_1(x)]$, which satisfies

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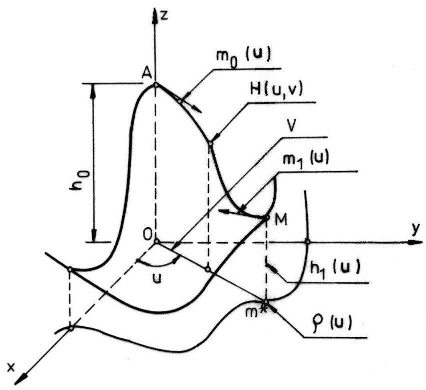

FIGURE 1. A surface with two support curves and tangent ribbons

the following conditions:

$$\begin{aligned} &f(x,0) = h_0(x), \quad f(x,y_1(x)) = h_1(x), \\ &f_y'(x,0) = m_0(x), \quad f_{y^2}''(x,y_1(x)) = m_1(x), \quad x \in [0,a]. \end{aligned} \tag{2.2}$$

The unique solution of the problem (2.2) is the third-degree Hermite interpolation polynomial with respect to the y-variable. By direct calculus one obtains:

$$\begin{aligned} &f\left(x,y;h_0(x),h_1(x),m_0(x),m_1(x)\right) = \\ &\frac{\left[m_0(x)+m_1(x)\right]y_1(x) - 2\left[h_1(x)-h_0(x)\right]}{y_1^3(x)}y^3 + \\ &\frac{3\left[h_1(x)-h_0(x)\right] - y_1(x)\left[2m_0(x)+m_1(x)\right]}{y_1^2(x)}y^2 + m_0(x)y + h_0(x). \end{aligned} \tag{2.3}$$

In cardinal form [5], the polynomial f is:

$$\begin{aligned} &f\left(x,y;h_0(x),h_1(x),m_0(x),m_1(x)\right) = H_3^0\left(y;y_1(x)\right)h_0(x) + \\ &H_3^1\left(y;y_1(x)\right)m_0(x) + H_3^2\left(y;y_1(x)\right)m_1(x) + H_3^3\left(y;y_1(x)\right)h_1(x), \end{aligned} \tag{2.4}$$

where the cardinal (blending) functions $H_3^i(y; y_1(x))$, $i = \overline{0,3}$ are

$$\begin{aligned} H_3^0(y; y_1(x)) &= \frac{[y_1(x) - y]^2 [2y + y_1(x)]}{y_1^3(x)}, \\ H_3^1(y; y_1(x)) &= \frac{[y_1(x) - y]^2 y}{y_1^2(x)}, \\ H_3^2(y; y_1(x)) &= \frac{[y - y_1(x)] y^2}{y_1^2(x)}, \\ H_3^3(y; y_1(x)) &= \frac{y^2 [3y_1(x) - 2y]}{y_1^3(x)}, \quad x \in [0, a],\ y \in [0, y_1(x)]. \end{aligned} \tag{2.5}$$

The equation:

$$z = f(x, y; y_1(x), h_0(x), h_1(x), m_0(x), m_1(x)), \quad x \in [0, a],\ y \in [0, y_1(x)],$$

where f is given by (2.3) or (2.4), represents a family of surfaces which depend on $y_1(x)$, $h_0(x)$, $h_1(x)$, $m_0(x)$, $m_1(x)$. The functions $m_0(x)$ and $m_1(x)$ determine the shape of each surface.

Remark 2.1.

a) A surface (S) and its symmetric with respect to xOz plane has the equation:

$$z = f(x, |y|, y_1(x), h_0(x), h_1(x), m_0(x), m_1(x)), \tag{2.6}$$

where $y \in [0, a]$, $|y| \leq y_1(x)$.

b) A surface (S) and its symmetric with respect to yOz plane is represented by the equation:

$$z = f(|x|, y, y_1(|x|), h_0(|x|), h_1(|x|), m_0(|x|), m_1(|x|)), \tag{2.7}$$

where $|x| \leq a$, $y \in [0, y_1(x)]$.

c) The equation of the surface (S) and its symmetric with respect to xOz and yOz planes is:

$$z = f(|x|, |y|; y_1(|x|), h_0(|x|)), m_0(|x|), m_1(|x|), \tag{2.8}$$

$|x| \leq a$, $|y| \leq y_1(x)$.

Figure 2 shows the surface from the family (2.8), where f is given by (2.3) corresponding to the following data:

$$h_0(x) = \frac{4}{75}(x-15)(x-20) - \frac{4}{125}x(x-20) + \frac{4}{125}x(x-15),$$
$$h_1(x) = 1 + \frac{1}{2}\sin\frac{2\pi}{15}\left(x+\frac{3}{2}\right),$$
$$m_0(x) = \frac{1}{6}\cos\frac{\pi}{5}\left(x+1\right),$$
$$m_1(x) = -\frac{1}{4}, \quad a = 20, \quad y_1(x) = 20.$$

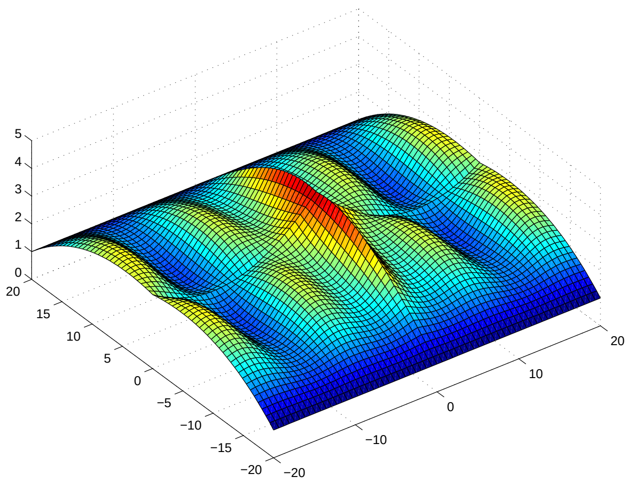

FIGURE 2. A surfaces from the family (2.8)

3. Surfaces having a point and a curve as supports

Let us consider the point $A(0,0,h_0)$ and a curve (C) represented, in cylindrical coordinate system, by the following equations:

$$(C)\begin{cases} x = \rho_1(u)\cos u, \\ y = \rho_1(u)\sin(u), \\ z = h_1(u), \qquad u \in [0, 2\pi]. \end{cases} \tag{3.1}$$

(see figure 3).

1. For the beginning we determine a curve (C^*) which passes through the point A and a fixed point of the curve (C), which will be denoted by

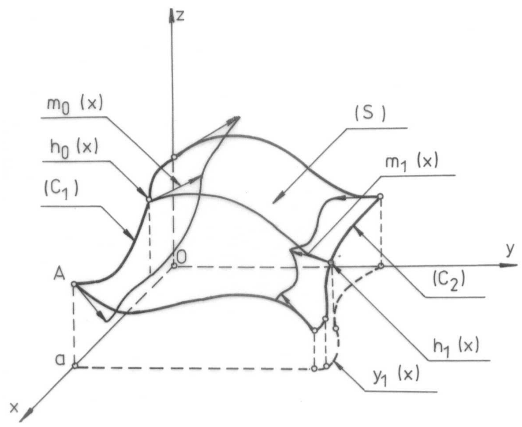

FIGURE 3. A surface having a point and a curve as supports

$B(\rho_1(u)\cos u, \rho_1(u)\sin u, h_1(u))$, u - fixed for the moment and having in A and b the slopes $m_0(u)$ and $m_1(u)$ respectively.

The curve (C^*) is uniquely represented by a third degree Hermite interpolation polynomial in v-variable, similar to (2.3):

$$(C^*)\begin{cases} x = v\cos u \\ y = v\sin u \\ z = h\left(v; h_0, h_1(u), m_0(u), m_1(u)\rho_1(u)\right), \end{cases}$$

$v \in [0, \rho_1(u)]$, $u \in [0, 2\pi]$, u fixed, where

$$\begin{aligned} &h\left(v; h_0, h_1(u), m_0(u), m_1(u), \rho_1(u)\right) = \\ &\frac{[m_0(u) + m_1(u)]\,\rho_1(u) 2\,[h_1(u) - h_0]}{\rho_1^3(u)} v^3 \\ &+ \frac{3\,[h_1(u) - h_0] - \rho_1(u)\,[2m_0(u) + m_1(u)]}{\rho_1^2(u)} v^2 + m_0(u) + h_0, \end{aligned} \tag{3.2}$$

or in canonical form

$$\begin{aligned} &h\left(v; h_0, h_1(u), m_0(u), m_1(u)\rho_1(u)\right) = H_3^0\left(v; \rho_1(u)\right) h_0 + \\ &+ H_3^1\left(v; \rho_1(u)\right) m_0(u) + H_3^2\left(v; \rho_1(u)\right) m_1(u) + H_3^3\left(v; \rho_1(u)\right) h_1(u) \end{aligned} \tag{3.3}$$

with

$$H_3^0(v;\rho_1(u)) = \frac{[\rho_1(u)-v]^2\,[2v+\rho_1(u)]}{\rho_1^3(u)},$$
$$H_3^1(v;\rho_1(u)) = \frac{[\rho_1(u)-v]^2\,v}{\rho_1^2(u)},$$
$$H_3^2(v;\rho_1(u)) = \frac{[v-\rho_1(u)]\,v^2}{\rho_1^2(u)}, \tag{3.4}$$
$$H_3^3(v;\rho_1(u)) = \frac{v^2\,[3\rho_1(u)-v]}{\rho_1^3(u)},$$

$v \in [0,\rho_1(u)],\ u \in [0,2\pi],\ u$ fixed.

The surface (Γ) generated by the curve (C^*), for u-variable is represented by the equations:

$$(\Gamma)\begin{cases} x = v\cos u, \\ y = v\sin u, \\ z = h\left(v; h_0, h_1(u), m_0(u), m_1(u), \rho_1(u)\right), \end{cases} \tag{3.5}$$

where $u \in [0,2\pi]$, $v \in [0,\rho_1(u)]$, and h is given by (3.2) or (3.3).

A surface from the family (3.5) corresponding to the data

$$m_0(u) = -\frac{1}{5},\quad m_1(u) = \frac{1}{2},\quad h_0 = 10,$$
$$h_1(u) = 5 + \sin 5u - |\sin 5u|\,,\quad \rho_1(u) = 15.$$

is represented in Figure 4.

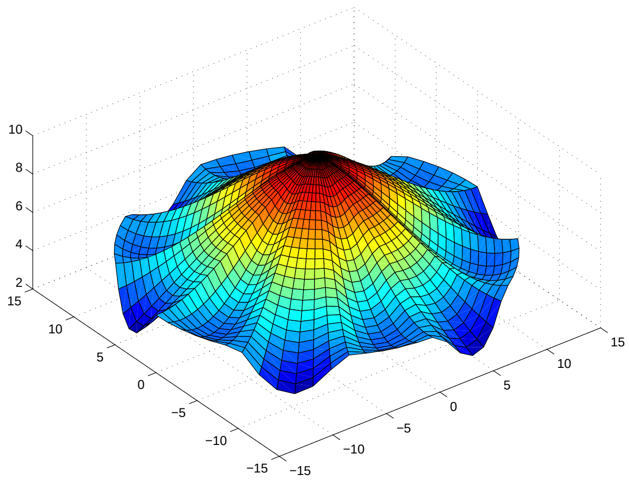

FIGURE 4. A surface from family (3.5)

2. Next, we consider that the generating curve (C^*) passes through the points A and B, in A has the slope $m_0(u)$ and for $v = \rho_0(u)$, u-fixed, it has an inflexion point. Such a curve is unique represented with the aid of a third degree Birkhoff interpolation polynomial [4].

The surface (Σ) generated by the curve (C^*) when u is variable, $u \in [0, 2\pi]$ has the following parametric equations:

$$\begin{aligned} x &= v \cos u \\ y &= v \sin u \\ z &= B\left(v; h_0, h_1(u), m_0(u), \rho_0(u), \rho_1(u)\right), \end{aligned} \tag{3.6}$$

where

$$B\left(v; h_0, h_1(u), m_0(u), \rho_0(u), \rho_1(u)\right) = \frac{h_0 - h_1(u) + m_0(u)\rho_1(u)}{\rho_1^2(u)\left[3\rho_0(u) - \rho_1(u)\right]} v^2 \left[\left[v - 3\rho_0(u)\right] + m_0(u)v + h_0,\right.$$

$u \in [0, 2\pi]$, $v \in [0, \rho_1(u)]$.

We note that the equations (3.6) represents a family of surfaces. The shape of each member of this family depends on h_0, $h_1(u)$, $m_0(u)$, $\rho_0(u)$ and $\rho_1(u)$. The surface from family (3.6) corresponding to the particular case

$$\begin{aligned} &h_0 = 8, \quad h_1(u) = 1 + \frac{1}{4}\sin 12u, \quad m_0(u) = -1, \\ &\rho_0(u) = 15, \quad \rho_1(u) = 20 \end{aligned}$$

is given in Figure 5.

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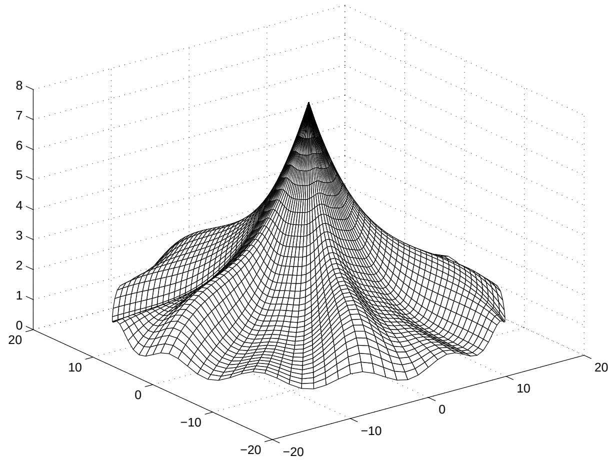

FIGURE 5. A surface from the family (3.6)

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