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# BOOK REVIEWS

T. Banakh, T. Radul, M. Zarichnyi, *Absorbing Sets in Infinite-Dimensional Manifolds* Mathematical Studies Monograph Series, Volume 1, VNTL Publishers, Ukraine, 1996, 232 pp.

The book is devoted to the theory of absorbing sets and its applications, most of them consisting in beautiful and short characterizations of many remarkable spaces. The term *space*, stands for *separable, metrizable topological space*.

The first chapter of the book contains an exposition of the basic theory of absorbing sets.

A space $X$ is said to be $\mathcal{C}$-*absorbing* (with respect to a given class $\mathcal{C}$ of spaces) if:

a) $X$ is strongly $\mathcal{C}$-*universal ANR* (absolute neighborhood retract) satisfying *SDAP* (strong discrete approximation property)

b) $X \in \sigma\mathcal{C}$ (i.e. $X$ is a countable union of spaces from $\mathcal{C}$)

c) $X$ is a $Z_\sigma$ space (i.e. $X$ is a countable union of $Z$-spaces in $\mathcal{C}$.

[recall that a set $A \subseteq X$ is a $Z$-space if for every open cover $\mathcal{U}$ of $X$, there exists a continuous map $f : X \to X$ such that $f(X) \cap A = \emptyset$ and $f, id_X$ are $\mathcal{U}$-close, i.e. for each $x \in X$ with $f(x) \neq x$, there exists $U \in \mathcal{U}$ such that $x \in U, f(x) \in U$].

The second chapter, "Construction of absorbing sets", contains examples of absorbing sets with respect to several classes of sets, sometimes defined by dimensional conditions.

The third chapter contains some even more technical results concerning strong universality for pairs and for spaces.

The last two chapters include applications to infinite products, topological groups, convex sets, spaces of probability measures.

The results included in the 232 pages of the book integrate the work of the authors with work of many other mathematicians such as K. Borsuk, C. Bessaga, A. Pełczińsky, H. Toruńczyk, M. Bestvina, J. Mogilski, T. Dobrowolski, O. Keller.

The book contains many exercises and notes and comments at the end of each chapter.

V. Anisiu

G.P. Galdi, J.G. Heywood and R. Rannecher Eds., *Fundamental Directions in Mathematical Fluid Mechanics*, Birkhäuser, Boston-Basel-Berlin 2000, 293 pp., ISBN 3-7643-6414-9.

This volume consists of six research articles, written by excellent experts in fluid mechanics. Each of these articles treats an important topic in the theory of Navier-Stokes equations in a rigorous mathematical manner. A very important problem in this area is to go beyond the presently known global existence of weak solutions, to the global existence of smooth solutions, for which uniqueness result and continuous dependence on the data can be provided. For this reason, Galdi's article *An introduction to the Navier-Stokes initial-boundary value problem* gives an overview of this topic. The article of Gervasio, Quarteroni and Saleri *Spectral approximation of Navier-Stokes equations* is devoted to extension of spectral Galerkin methods to domains with complicated geometries by using the techniques of domain decomposition. It is well known that the rigorous explanation of bifurcation phenomena in fluid mechanics has been a main topic in the theory of the Navier-Stokes equations. The article of Heywood and Nagata *Simple proofs of bifurcation theorems* introduces bifurcation theory in a general setting that is convenient for application to the Navier-Stokes equations. The two articles of Heywood and Padula *On the steady transport equation* and *On the existence and uniqueness theory for the steady compressible viscous flow* give a simplified approach to the theory of steady compressible viscous flows. Finally the article of Rannacker *Finite element methods for the incompressible Navier-Stokes equations* combines the theory and implementation of the finite element method, with an emphasis on a priori and a posteriori error estimation and adaptive mesh refinement.

The book is an important addition to the literature and it will be a very good investment for interested researchers.

M. Kohr

Andreescu, T., Gelca, R., *Mathematical Olympiad Challenges,* Birkhauser, Boston - Basel - Berlin, 2000, 260 pp + xv, ISBN 0-8176-4155-6.

This beautiful book of T. Andreescu and R. Gelca is a comprehensive collection of high level and non-standard problems for mathematical olympiads and competitions. The both authors were educated at the strong Romanian mathematics school and therefore their present work contains a part of this past experience. At present, Titu Andreescu is the Executive Director of the American Mathematics Competitions. Under his guidance the US Olympic Team has obtained very high scores (first place with perfect score in 1994), and was situated on the third place at the least World Mathematical Olympiad in Korea. USA will be the host of the 2001 Mathematical Olympiad and T. Andreescu is the director of the organizing committee. I have to note that the book begins with the following words of the famous Romanian mathematician Grigore Moisil : "Matematică, matematică, matematică, atâta matematică? Nu, mai multă! ("Mathematics, mathematics, mathematics, so much mathematics? No, even more! "). In fact these words give us a good expression of the spirit of this book : create and solve more and more problems since this is the best way to learn and to understand mathematics.

The included problems are clustered into three self-contained chapters : Geometry and Trigonometry, Algebra and Analysis, Number Theory and Combinatorics, each of them containing ten sections. For instance, the topics of sections in Geometry and Trigonometry are the following : A property of equilateral triangles, Dissections of polygonal surfaces, Regular polygons, Cyclic quadrilaterals, Power of a point, Geometric constructions and transformations, Problems with physical flower, Tetrahedra inscribed in parallelepipeds, Telescopic sums and products in trigonometry, Trigonometric substitutions. A background material, some representative examples, and beautiful diagrams are included to complete each section. Most of the proposed problems were successfully tested in classrooms as well as in national and international mathematical competitions. From this point of view the Romanian experience is very well represented. The second part of the book contains the completed and detailed solutions of the proposed problems. These are presented in a very didactic way, encouraging the readers to move away from routine exercises and memorized algorithms toward creative solutions and non-standard problem-solving techniques. The name of author and source of most of the proposed problems are mentioned in this part. At the end of the book a glossary of used definitions and fundamental properties is included.

In the authors‘ preface one can read that this work ”... is written as a textbook to be used in advanced problem solving courses, or as a reference source for people interested in tackling challenging mathematical problems”. I have to conclude that in this respect all the purposes of the book are successfully fulfilled.

Dorin Andrica

T.M. Atanackovic, A. Guran, *Theory of Elasticity for Scientists and Engineers*, Birkäuser, Boston-Basel-Berlin, 2000, 374 pp., ISBN 0-8176-4072-X.

The present book is intended to be an introduction to theory of elasticity. It is a new and comprehensive text, as well as a good reference work providing an excellent introduction to theory of elasticity and its applications. The book contains ten chapters. The first chapter has an introductory character, containing the theory of stress. The second chapter begins with the description of deformation at a point. Further, the nonlinear strain tensor is obtained and the geometrical meaning of its components are studied. On the other hand, the strain tensor in the case of small deformations is derived by linearization and its properties are examined. We remark that these preliminary chapters treat the basic concepts of stress and strain, using only Cartesian vector and tensor notation. Chapter three treats the relation between stress and strain. This chapter introduces constitutive equations for an elastic body and the thermoelastic stress-strain relation. Chapter four is devoted to some boundary value problems of elasticity theory. The authors give a summary of equations of linear elasticity theory and use the scalar and vector potential theory to solve several problems in this field (the Lamé potential, the Galerkin vector, the Love function, the method of Papkovich and Neuber, etc.). The aim of chapter five is to give a presentation of a large number of some important problems of elasticity theory for which solutions are available (torsion, bending and rotation of a prismatic rod). This chapter is a very good reference source for researchers in the field. Chapter six is concerned with the plane strain, plane stress, and generalized plane stress problems and presents several methods to solve these problems (the complex variable method, Fourier transform method). Chapter seven is devoted to the energy method in elasticity theory. The aim of chapter eight is to derive the von Kármán theory of plates and in chapter nine is treated the contact and elastic impact problems for elastic bodies. The last chapter of the book refers to the stability of elastic bodies. Some examples illustrating stability analysis are included. At the end of each chapter a selected number of problems are given.

To conclude, I think this book is very important for introducing readers in mechanical engineering, mechanics and applied mathematics to a modern view of theory of elasticity. The book is clearly written, contains a wealth of information, introducing the reader to a modern and active area of investigations.

I recommend this book to all specialists in this area.

M. Kohr

William G. Litvinov, *Optimization in Elliptic Problems with Applications to Mechanics of Deformable Bodies and Fluid Mechanics,* Operator Theory Advances and Applications, Vol. 119, Birkhäuser Verlag, Basel-Boston-Berlin 2000, 522 pages, ISBN 3-7643-6199-9.

The author offers the reader a thorough introduction to contemporary research in optimization theory for elliptic systems with its numerous applications, and a textbook at the undergraduate and graduate levels for courses in pure and applied mathematics.

The mathematical models of the modern technology and production contain elliptic equations and systems. Optimization of the processes from these models is reduced to optimization problems for elliptic equations and systems. The numerical solution of such problems is associated with some questions. Some of them are the following:

- The setting of the optimization problem ensures the existence of a solution on a set of admissible controls, which is a subset of some infinite-dimensional vector space.
- Reduction of the infinite-dimensional problem to a sequence of finite dimensional problems such that the solutions of the finite-dimensional problems converge to the solution of the infinite-dimensional problem.
- Numerical solution of the finite dimensional problems.

The book is devoted to these questions. Attention is focused on the settings of the problems, on the proof of existence theorems, and on the method of approximate solution of optimization problems. For elliptic equations and systems the author investigates optimization problems in which the coefficients of equations, the shape of domains, and right-hand sides of equations are considered to be controls. The results are applied to various optimization problems of mechanics of deformable bodies, plates, shells, composite materials, and structures made of them, as well as to the optimization problems of mechanics of viscous fluids.

The book is written in an accessible and self-contained manner. It will be of interest to research mathematicians and science engineers working in solid and fluid mechanics, and in optimization theory of partial differential equations.

P. Szilágyi

Vladimir Maz'ya, Serguei Nazarov, Boris Plamenevskij, *Asymptotic Theory of Elliptic Boundary Value Problems in Singularly Perturbed Domains,* Volume I. 435 pages, ISBN 3-7643-6397-5, Volume II. 323 pages, ISBN 3-7643-6398-3, Operator Theory Advances and Applications, Vols. 111 and 112, Birkhäuser Verlag, Basel-Boston-Berlin, 2000.

The book is devoted to the development and applications of asymptotic methods to boundary value problems for elliptic equations in singularly perturbed domains $\Omega$, which can have corners, edges, small holes, small slits, thin ligaments etc. The boundary value problems are considered first in domains $\Omega(\epsilon)$ that depend on a small parameter $\epsilon$, where $\Omega(0) = \Omega$. So the boundary of the domain $\Omega(0)$ is not smooth and contains a number of singular points, contours or surfaces. A transition from $\Omega(0)$ to $\Omega(\epsilon)$ results in the fact that isolated points convert into small cavities, contours convert into thin tubes and surfaces into flat holes, or the boundary of the domain near a conical point or an edge becomes smooth and so on. These perturbations of the domain are said to be singular, in the contrast to regular perturbations, when the boundaries of the domains $\Omega(0)$ and $\Omega(\epsilon)$ are closed smooth surfaces. The authors investigate the behaviour of solutions $u_\epsilon$ of the boundary value problems, eigenvalues of the corresponding operator, and the behaviour of different functionals as $\epsilon \to 0$.

A lot of attention is paid to particular problems of mathematical physics. Most of the problems considered in the two volumes emerged from problems in hydrodynamics and aerodynamics, the theory of elasticity, fracture mechanics, electrostatics and others. A substantial body of results has been accumulated on the applications of asymptotic methods to physical problems. This knowledge has been particularly useful in a broad range of engineering problems.

The authors offer in the 20 chapters of the book a complete theory of boundary value problems in domains, which have corners, edges or other singularities. Most of the material presented in the book is based on results of the authors, which have been partly published in scientific journals. This book can be considered as unique in the mathematical literature, because it presents for the first time a profound and complete mathematical analysis of the asymptotic theory of elliptic boundary value.

Finally we remark, that the book originally was published by Akademic Verlag GmbH, Germany, under the title "Asymptotic Theorie Elliptischer Radwertaufgaben in singulär gestörten Gebieten" in 1991.

P. Szilágyi

*Ring Theory and Algebraic Geometry*, Editors: Ágnel Granja, Hose Ágnel Hermida, Alain Verschoren, Lecture Notes in Pure and Applied Mathematics, vol. 221, Marcel Dekker (2001), xv+339pp, ISBN 0-8247-0559-9.

The volume under review presents papers presented at the "Fift International Conference on Algebra and Algebraic Geometry (SAGA V)", held at the University of León, Spain.

The aim of this book is to exhibit some interaction between algebra and algebraic geometry and it contains 20 research papers and surveys. The contributors are important specialists in some actual domains in mathematics: modules and lattices, algebras and representation theories, affine and projective algebraic varieties, simplicial and cellular complexes, cones, polytipes, arithmetics, etc.

*Brzeziński, Caenepeel, Militaru* and *Zhu* study condition when induction functors and their adjuncts are separable; *Bueso, Gómez-Torecillas* and *Lobillo* characterize the solvable polinomial algebras and present an algorithm to compute the Gelfand-Kirillov dimension for f.g. modules over these algebras; *Campillo* and *Pisón* "show how mathematics in toric geometry can be understood of appropriate classes of commutative semigroups"; *Cuadra* and *Van Oystaeyen* present properties for some invariants of coalgebras (the Picard group and the Brauer group); *Escoriza and Torrecillas* give the concept of multiplication object in monoidal categories; *Facchini* describes some connections between "semi-local endomorphism ring" and "Krull-Schmidt theorems"; *Hartillo-Hermoso* presents an algoritm which compute a global Bernstein polynomial; *Morey* and *Vasconcelos* study the divisors of Rees algebras of ideals. Mention that the others contributors of this volume are: *Cabezas, Camacho, Gómez,* *Jiménez-Merhan, Pastor, Reyes, Rodrigeuz, Calderón-Martin, Martin-Gonzáles, Corriegos, Sáncez-Giralda, Castro-Jiménes, Moreno-Frias, Gago-Vargas, Gonzáles, Idelhadj, Yahya, S. Gonzáles, Matinez, Malliavin, Núnez, Pisaborro, Smet, Verchoren, Ucha-Enriquez, Verchoren, Vidal.*

The authors are well-knows experts from quite different schools.

The book permits an easy access to the present state of knowledge. Students and researchers interested in Ring Theory and in Algebraic Geometry will take a full benefit and they find here a good source of inspiration.

Simion Breaz

Constantin Udrişte, *Geometric Dynamics*, Mathematics and Its Applications, Vol. 513, Kluwer Academic Publishers, Dordrecht-Boston-London 2000, xvi+395 pp., ISBN: 0-7923-5277-7.

A field line is a curve $\alpha : I \to D$ of class $C^1$, satisfying the differential equation $\alpha'(t) = X(\alpha(t))$ (or the equivalent integral equation $\alpha(t) = \alpha(t_0) + \int_{t_0}^{t} X(\alpha(s))ds$) where $D$ is an open connected subset of $R^n$ and $X$ is a vector field of class $C^1$ on $D$. Geometric dynamics is a tool for developing a mathematical representation of real world phenomena, based on the notion of field line. The author systematically exemplifies the theoretical mathematical concepts on examples from the applied sciences: theoretical mechanics, physics, thermodynamics, biology, chemistry etc. The basic idea of the author is to emphasize that a field line is a geodesic of a suitable geometrical structure on a given space (the so called Lorentz-Udrişte world-force law). That means that creating wider classes of Riemann-Jacobi, Riemann-Jacobi-Lagrange, or Finsler-Jacobi manifolds, one obtains that all trajectories of a given vector field are geodesics.

The book is divided into 11 chapters headed as follows: 1. *Vector fields,* 2. *Particular vector fields,* 3. *Field lines*, 4. *Stability of equilibrium points,* 5. *Potential differential systems of order one and catastrophe theory*, 6. *Field hypersurfaces,* 7. *Bifurcation theory,* 8. *Submanifolds orthogonal to field lines,* 9. *Dynamics induced by a vector field,* 10. *Magnetic dynamical systems and Sabba Ştefănescu conjectures,*

11. *Bifurcation in the mechanics of hypoelastic granular material* (this last chapter is written by Lucia Drăguşin).

The characteristic feature of the book is the strong interplay between mathematics and its applications to other areas, which makes it of interest to a large audience, including first years graduates, teachers, and researchers whose work involves mathematics, mechanics, physics, engineering, biology, and economics.

Stefan Cobzaş