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PROJECTORS WITH RESPECT TO SOME SPECIAL SCHUNCK CLASSES

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Abstract. Let π be a set of primes and π' the complement to π in the set of all primes. The paper deals with establishing the projectors with respect to some special Schunck classes: the class $\underline{S}_\pi$ of all solvable π-groups, the class $\underline{N}$ of all finite nilpotent groups and the class $\underline{G}_{\pi'}$ of all π'-groups. A new proof is given for some of W. Gaschütz's results from [7] which show that, in any finite solvable group, the $\underline{S}_\pi$-projectors are the Hall π-subgroups and the $\underline{N}$-projectors are the Carter subgroups. Finally, we prove that, in any finite π-solvable group, the $\underline{G}_{\pi'}$-covering subgroups are exactly the Hall π'-subgroups. Hence, we deduce that, in any finite π-solvable group, the $\underline{G}_{\pi'}$-projectors coincide with the Hall π'-subgroups.

1. Preliminaries

All groups considered in the paper are finite. We denote by π an arbitrary set of primes and by π' the complement to π in the set of all primes.

We remind some useful definitions:

Definition 1.1. a) We call $\underline{X}$ a class of groups if:

(1) $\{1\} \in \underline{X}$;

(2) if $G \in \underline{X}$ and f is an isomorphism of G then $f(G) \in \underline{X}$.

b) A class $\underline{X}$ of groups is a homomorph if $\underline{X}$ is closed under homomorphisms, i.e. if $G \in \underline{X}$ and N is a normal subgroup of G then $G/N \in \underline{X}$.

c) A group G is primitive if there is a maximal subgroup W of G with

$$core_G W = \{1\},$$

where

$$core_G W = \cap\{W^g / g \in G\}.$$

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d) A homomorph $\underline{X}$ is a Schunck class if $\underline{X}$ is primitively closed, i.e. if any group G, all of whose primitive factor groups are in $\underline{X}$, is itself in $\underline{X}$.

Definition 1.2. Let $\underline{X}$ be a class of groups, G a group and $H \leq G$.

a) H is $\underline{X}$-maximal in G if:

(1) $H \in \underline{X}$;

(2) $H \leq K \leq G,\ K \in \underline{X}\ \Rightarrow\ H = K$.

b) H is an $\underline{X}$-projector of G if for any normal subgroup N of G, HN/N is $\underline{X}$-maximal in G/N.

c) H is an $\underline{X}$-covering subgroup of G if:

(1) $H \in \underline{X}$;

(2) $H \leq K \leq G,\ K_0 \trianglelefteq K,\ K/K_0 \in \underline{X}\ \Rightarrow\ K = HK_0$.

Definition 1.3. a) A group G is π-solvable if any chief factor of G is either a solvable π-group or a π'-group. If π is the set of all primes, we obtain the notion of solvable group.

b) A class $\underline{X}$ of groups is π-closed if:

$$G/O_{\pi'}(G) \in \underline{X}\ \Rightarrow\ G \in \underline{X},$$

where $O_{\pi'}(G)$ denotes the largest normal π'-subgroup of G. We shall call π-homomorph, respectively π-Schunck class a π-closed homomorph, respectively a π-closed Schunck class.

Theorem 1.4. ([6]) *Let $\underline{X}$ be a homomorph, G a group and $H \leq K \leq G$. If H is an $\underline{X}$-covering subgroup of G, then H is an $\underline{X}$-covering subgroup of K.*

The connection between the special subgroups introduced above is given in [7] and in [4] and is resumed in the following theorem:

Theorem 1.5. *a) Let $\underline{X}$ be a class of groups, G a group and H a subgroup of G. If H is an $\underline{X}$-covering subgroup or an $\underline{X}$-projector of G then H is $\underline{X}$-maximal in G.*

b) Let $\underline{X}$ be a homomorph, G a group and H a subgroup of G. H is an $\underline{X}$-covering subgroup of G is and only if H is an $\underline{X}$-projector in any subgroup K with $H \leq K \leq G$. Particularly, any $\underline{X}$-covering subgroup of G is an $\underline{X}$-projector of G.

c) Let $\underline{X}$ be a Schunck class, G a finite solvable group, S and $\underline{X}$-projector of G and $S \leq H \leq G$. Then S is an $\underline{X}$-projector in H.

d) Let $\underline{X}$ *be a Schunck class,* G *a finite solvable group and* S *a subgroup of* G*. The following conditions are equivalent:*

(1) S *is an* $\underline{X}$*-projector of* G*;*

(2) S *is an* $\underline{X}$*-covering subgroup of* G*.*

The following properties of projectors are also of special interest for this paper:

Theorem 1.6. ([7]) *Let* $\underline{X}$ *be a homomorph,* G *a group and* $H \leq G$*.*

a) H *is an* $\underline{X}$*-projector of* G *is and only if:*

(1) H *is* $\underline{X}$*-maximal in* G*;*

(2) for any minimal normal subgroup M *of* G*,* HM/M *is an* $\underline{X}$*-projector in* G/M*.*

b) If H *is an* $\underline{X}$*-projector of* G *and* N *is a normal subgroup of* G*, then* HN/N *is an* $\underline{X}$*-projector of* G/N*. This property holds for* $\underline{X}$*-covering subgroups too.*

Theorem 1.7. ([7]) *Let* $\underline{X}$ *be a Schunck class,* G *a solvable group and* B *a normal abelian subgroup of* G*. If:*

(1) SB/B *is an* $\underline{X}$*-projector of* G/B*;*

(2) S *is* $\underline{X}$*-maximal in* SB*,*

then S *is an* $\underline{X}$*-projector of* G*.*

In [2], [3] and [4], we established for finite π-solvable groups the following result:

Theorem 1.8. ([2]) *Let* $\underline{X}$ *be a* π*-homomorph.*

a) $\underline{X}$ *is a Schunck class if and only if any* π*-solvable group has* $\underline{X}$*-covering subgroups.*

b) Any two $\underline{X}$*-covering subgroups of a* π*-solvable group* G *are conjugate in* G*.*

Theorem 1.9. ([3], [4]) *Let* $\underline{X}$ *be a* π*-homomorph. Then* $\underline{X}$ *is a Schunck class if and only if any* π*-solvable group has* $\underline{X}$*-projectors.*

Corollary 1.10. *Let* $\underline{X}$ *be a* π*-homomorph. The following conditions are equivalent:*

(1) $\underline{X}$ *is a Schunck class;*

(2) any π*-solvable group has* $\underline{X}$*-covering subgroups;*

(3) any π*-solvable group has* $\underline{X}$*-projectors.*

Theorem 1.11. ([3]) *If $\underline{X}$ is a π-Schunck class, then any two $\underline{X}$-projectors of a π-solvable group G are conjugate in G.*

Particularly, for π the set of all primes, theorems 1.8-1.11 give well-known results from [7] and [10] referring to finite solvable groups.

2. **Projectors with respect to the class $\underline{S}_\pi$**

Denote by $\underline{S}_\pi$ the class of all solvable π-groups. We give a new proof for the following result given by W. Gaschütz in [7]: In any finite solvable group, the $\underline{S}_\pi$-projectors are exactly the Hall π-subgroups.

A positive integer n is said to be a π-number if for any prime divisor p of n we have $p \in \pi$.

Definition 2.1. a) A finite group G is a π-group if $|G|$ is a π-number.

b) A subgroup S of a group G is a π-subgroup if S is a π-group.

c) A subgroup S of a group G is an Hall π-subgroup if:

(1) S is a π-subgroup;

(2) $(|S|, |G : S|) = 1$, i.e. $|G : S|$ is a π'-number.

We shall use the following properties of the Hall subgroups ([9]):

Proposition 2.2. If G is a group, S is an Hall π-subgroup of G and H is a subgroup of G such that $S \leq H \leq G$, then S is an Hall π-subgroup of H.

Proposition 2.3. If G is a group, S is an Hall π-subgroup of G and N is a normal subgroup of G, then SN/N is an Hall π-subgroup of G/N.

The Hall subgroups were given in [8]. Ph. Hall studied them for finite solvable groups. In [5], S.A. Čunihin extended this study for finite π-solvable groups.

Theorem 2.4. (Ph. Hall, S.A. Čunihin, [9]) a) Any finite π-solvable group G has Hall π-subgroups and Hall π'-subgroups.

b) If G is a finite π-solvable group, then:

(i) any two Hall π-subgroups of G are conjugate in G;

(ii) any two Hall π'-subgroups of G are conjugate in G.

Particularly, any finite solvable group G has Hall π-subgroups and they are conjugate in G.

In preparation to the main theorem of this section, we prove the following results:

Theorem 2.5. $\underline{S}_\pi$ *is a homomorph.*

Proof. Let $G \in \underline{S}_\pi$ and N be a normal subgroup of G. G being solvable, G/N is also solvable. G being a π-group, from $|G/N|$ divides $|G|$ we obtain that G/N is a $\sqrt{\pi}$-group too. □

Theorem 2.6. *If G is a finite solvable group and S is an Hall π-subgroup of G, then S is $\underline{S}_\pi$-maximal in G.*

Proof. By induction on $|G|$. We verify the two conditions from 1.2.a).

(1) $S \in \underline{S}_\pi$. Indeed, S is a π-group and S is solvable being a subgroup of a solvable group.

(2) Let $S \leq H \leq G$ and $H \in \underline{S}_\pi$. We prove that $S = H$. By 2.2, S is an Hall π-subgroup of H. We consider two cases:

a) $H = G$. Then $G \in \underline{S}_\pi$ and G is its own Hall π-subgroup. But S is also an Hall π-subgroup of G. Applying 2.4.b), S and G are conjugate in G, hence $S = G = H$.

b) $H \neq G$. By the induction, S is $\underline{S}_\pi$-maximal in H. But $H \in \underline{S}_\pi$. Hence $S = H$. □

The main theorem in this section is from [7] and we give here a new proof.

Theorem 2.7. *Let G be a finite solvable group and S a subgroup of G. S is an $\underline{S}_\pi$-projector of G if and only if S is an Hall π-subgroup of G.*

Proof. By induction on $|G|$.

Let S be an $\underline{S}_\pi$-projector of G. We prove like in [7] that S is an Hall π-subgroup of G. Clearly S is a π-subgroup of G. We show that $|G : S|$ is a π'-number. Let M be a minimal normal subgroup of G. G being solvable, we have $|M| = p^k$, where p is a prime. Put $S' = SM$. By 1.6, S'/M is an $\underline{S}_\pi$-projector of G/M. Hence, by the induction, S'/M is an Hall π-subgroup of G/M. Two cases are considered:

a) $p \in \pi$. Then $S' \in \underline{S}_\pi$. But, by 1.5.a), S is $\underline{S}_\pi$-maximal in G. So $S = S'$. Then

$$|G : S| = |G : S'| = |G/M : S'/M| \text{ is a } \pi'\text{-number.}$$

b) $p \notin \pi$. Then $M \cap S = \{1\}$. So $|G : S|$ is a π'-number, because:

$$|G : S| = |G : S'||S' : S| = |G/M : S'/M||SM : S|,$$

where $|G/M : S'M|$ is a π'-number and $|SM : S| = |M : M \cap S| = |M| = p^k$, where $p \notin \pi$, hence $p \in \pi'$.

The converse has an original proof based on 1.6.a) and 2.6. Let S be an Hall π-subgroup of G. We shall prove that S is an $\underline{S}_\pi$-projector of G. We use 1.6.a).

(1) S is $\underline{S}_\pi$-maximal in G, by 2.6.

(2) Let M be a minimal normal subgroup of G. We prove that SM/M is an $\underline{S}_\pi$-projector of G/M. Indeed, by 2.3, SM/M is an Hall π-subgroup of G/M. Hence, by the induction, we obtain that SM/M is an $\underline{S}_\pi$-projector of G/M. □

Corollary 2.8. $\underline{S}_\pi$ *is a Schunck class.*

Proof. From 1.9, 2.7 and 2.4. □

Corollary 2.9. *Let G be a finite solvable group and S a subgroup of G. The following conditions are equivalent:*

(1) S is an Hall π-subgroup of G;

(2) S is an $\underline{S}_\pi$-projector of G;

(3) S is an $\underline{S}_\pi$-covering subgroup of G.

Proof. From 2.7, 2.8 and 1.5.d). □

3. **Projectors with respect to the class $\underline{N}$**

Denote by $\underline{N}$ the class of all finite nilpotent groups. We shall give a new proof for the following W. Gaschütz's result from [7]: In any finite solvable group, the $\underline{N}$-projectors are exactly the Carte subgroups.

Definition 3.1. Let G be a group and S a subgroup of G. S is a Carter subgroup of G if:

(1) S is nilpotent;

(2) $N_G(S) = S$.

The following properties given in [1] are important for our considerations:

Proposition 3.2. If G is a group, S is a Carter subgroup of G and H is a subgroup of G such that $S \leq H \leq G$, then S is a Carter subgroup of H.

Proposition 3.3. If G is a group, S is a Carter subgroup of G and N is a normal subgroup of G, then SN/N is a Carter subgroup of G/N.

Theorem 3.4. (R. Carter [1]) *Let G be a finite solvable group. Then:*

a) G has Carter subgroups;

b) any two Carter subgroups of G are conjugate in G.

Theorem 3.5. *$\underline{N}$ is a Schunck class.*

Proof. Obviously $\underline{N}$ is a homomorph. Further, $\underline{N}$ is primitively closed. Indeed, if G is a group such that all primitive factor groups $G/N \in \underline{N}$, we can prove that $G \in \underline{N}$. For this, we use Wieland's criterium for finite groups to be nilpotent: we show that any maximal subgroup W of G is normal in G. Denote by $N = core_G W$. Then G/N is primitive, hence $G/N \in \underline{N}$. But W/N is maximal in G/N. Applying now Wieland's criterium for the nilpotent group G/N, we obtain that W/N is normal in G/N, hence W is normal in G. □

Theorem 3.6. *If G is a finite solvable group and S is a Carter subgroup of G, then S is $\underline{N}$-maximal in G.*

Proof. By induction on $|G|$.

(1) $S \in \underline{N}$, because S is a Carter subgroup.

(2) Let $S \leq H \leq G$ with $H \in \underline{N}$. We prove that $S = H$. Indeed, we have by 3.2 that S is a Carter subgroup of H. Consider two cases:

a) $H = G$. Then $G \in \underline{N}$ and so G is its own Carter subgroup. Applying 3.4.b), S and G are conjugate in G. It follows that $S = G = H$.

b) $H \neq G$. By the induction, S is $\underline{N}$-maximal in H. But $H \in \underline{N}$. Hence $S = H$. □

The main result is:

Theorem 3.7. *Let G be a finite solvable group and S a subgroup of G. S is an $\underline{N}$-projector of G if and only if S is a Carter subgroup of G.*

Proof. Let S be an $\underline{N}$-projector of G. We prove that S is a Carter subgroup of G like in [7]. Clearly S is nilpotent. In order to show that $N_G(S) = S$, let us suppose that $S \neq N_G(S)$. Then S is maximal in H, where $H \leq N_G(S)$. So S is a normal subgroup in H. Now H solvable and S maximal in H imply $|H : S| = p^k$, with p prime. So H/S is a finite p-group, hence H/S is nilpotent. By 1.5.c), S is an $\underline{N}$-projector in H. From S normal in H, follows by 1.6.b) that SS/S is an $\underline{N}$-projector in H/S, hence SS/S is $\underline{N}$-maximal in $H/S \in \underline{N}$. So $S = H$, in contradiction with the choice of H.

The converse has an original proof, based on 1.7 and 3.6. Let S be a Carter subgroup of G. We prove that S is an $\underline{N}$-projector of G by using 1.7. Let B be a minimal normal subgroup of G. G being solvable, B is abelian. By 3.5, $\underline{N}$ is a Schunck class. By 3.6 we have that S is $\underline{N}$-maximal in G and so S is $\underline{N}$-maximal in SB. Further, because, by 3.3, SB/B is a Carter subgroup of G/B, we can use the induction for G/B and so SB/B is an $\underline{N}$-projector of G/B. We apply now 1.7 and so S is an $\underline{N}$-projector of G. □

Corollary 3.8. *Let G be a finite solvable group and S a subgroup of G. The following conditions are equivalent:*

(1) S is a Carter subgroup of G;

(2) S is an $\underline{N}$-projector of G;

(3) S is an $\underline{N}$-covering subgroup of G.

Proof. From 3.7 and 1.5.d). □

4. Projectors with respect to the class $\underline{G}_{\pi'}$

In this section we establish the $\underline{G}_{\pi'}$-projectors of a finite π-solvable group, proving that they coincide with the Hall π'-subgroups.

Remind that π is an arbitrary set of primes and π' is the complement to π in the set of all primes. Denote by $\underline{W}_{\pi}$ the class of all finite π-solvable groups and by $\underline{G}_{\pi'}$ the class of all π'-groups. Obviously

$$\underline{G}_{\pi'} \subseteq \underline{W}_{\pi}.$$

All groups considered in this section are finite π-solvable groups.

Theorem 4.1. *The class $\underline{G}_{\pi'}$ is a π-homomorph.*

Proof. Let $G \in \underline{G}_{\pi'}$ and N a normal subgroup of G. Then $|G/N|/|G|$, hence $G/N \in \underline{G}_{\pi'}$. So $\underline{G}_{\pi'}$ is a homomorph. $\underline{G}_{\pi'}$ is π-closed. Indeed, if $G/O_{\pi'}(G) \in \underline{G}_{\pi'}$, then

$$|G| = |G/O_{\pi'}(G)||O_{\pi'}(G)|$$

is a π'-number and so $G \in \underline{G}_{\pi'}$. □

Theorem 4.2. *Let G be a finite π-solvable group and H a subgroup of G. H is a $\underline{G}_{\pi'}$-covering subgroup of G is and only if H is an Hall π'-subgroup of G.*

Proof. Let H be a $\underline{G}_{\pi'}$-covering subgroup of G. We prove by induction on $|G|$ that H is an Hall π'-subgroup of G.

(1) H is a π'-subgroup of G, because $H \in \underline{G}_{\pi'}$.

(2) $(|H|, |G:H|) = 1$. Indeed, let M be a minimal normal subgroup of G. H being a $\underline{G}_{\pi'}$-covering subgroup of G, HM/M is by 1.6.b) a $\underline{G}_{\pi'}$-covering subgroup of G/M, hence, by the induction, HM/M is an Hall π'-subgroup of G/M. Being a minimal normal subgroup of the π-solvable group G, M is either a solvable π-group or a π'-group. We consider now two cases:

a) $G/M \in \underline{G}_{\pi'}$. Then, HM/M being $\underline{G}_{\pi'}$-maximal in G/M (see 1.5.a)), we have $HM/M = G/M$ and so $HM = G$. Let us suppose now that M is a solvable π-group. Then

$$|G:H| = |HM:H| = |M:H\cap M|/|M|$$

is a π-number. Suppose that M is a π'-group. We know that $G/M = HM/M \in \underline{G}_{\pi'}$. Then $|G| = |G/M||M|$ is a π'-number. So $G \in \underline{G}_{\pi'}$. But H is $\underline{G}_{\pi'}$-maximal in G. Hence $H = G$ is its own Hall π'-subgroup.

b) $G/M \notin \underline{G}_{\pi'}$. Then $HM/M \neq G/M$, hence $HM \neq G$. By 1.4, H is a $\underline{G}_{\pi'}$-covering subgroup in HM and it follows, by the induction, that H is a Hall π'-subgroup of HM. Then $|HM:H|$ is a π-number and $|G:HM| = |G/M:HM/M|$ is also a π-number. It follows that

$$|G:H| = |G:HM||HM:H|$$

is a π-number.

Conversely, let H be an Hall π'-subgroup of G. We shall prove that H is an $\underline{G}_{\pi'}$-covering subgroup of G.

(1) $H \in \underline{G}_{\pi'}$ is clear.

(2) $H \leq L \leq G,\ L_0 \trianglelefteq L,\ L/L_0 \in \underline{G}_{\pi'}$ imply $L = HL_0$. We prove this by induction on $|G|$. Two cases are considered:

(i) $L \neq G$. By 2.2, H is an Hall π'-subgroup of L. Applying the induction, from $H \leq L = L$, $L_0 \trianglelefteq L$, $L/L_0 \in \underline{G}_{\pi'}$ follows $L = HL_0$.

(ii) $L = G$. Again two cases are considered. If $L_0 = 1$, then $G = L \cong L/L_0 \in \underline{G}_{\pi'}$ and G is its own Hall π'-subgroup. By 2.4.b), H and G are conjugate in G. Then

$$L = G = H = HL_0.$$

If $L_0 \neq 1$, we have that there is a minimal normal subgroup M of G such that $M \leq L_0$. From 2.3, HM/M is an Hall π'-subgroup of G/M. We apply the induction for G/M. Then, from

$$HM/M \leq G/M = G/M,\ L_0/M \trianglelefteq G/M,\ (G/M)/(L_0/M) \cong G/L_0 = L/L_0 \in \underline{G}_{\pi'},$$

it follows that

$$G/M = (HM/M)(L_0/M).$$

Hence $L = G = HML_0 = HL_0$. □

Corollary 4.3. *The class $\underline{G}_{\pi'}$ is a π-Schunck class.*

Proof. Follows from 1.10, 4.1, 4.2 and 2.4.a). □

Corollary 4.4. *Let G be a finite π-solvable group and $H \leq G$. If H is an Hall π'-subgroup of G, then H is a $\underline{G}_{\pi'}$-projector in G.*

Proof. H being a Hall π'-subgroup of G, H is by 4.2 a $\underline{G}_{\pi'}$-covering subgroup of G. Then, H is by 1.5.b) a $\underline{G}_{\pi'}$-projector in G. □

The main theorem in this section is the following:

Theorem 4.5. *Let G be a finite π-solvable group and H a subgroup of G. H is a $\underline{G}_{\pi'}$-projector of G if and only if H is an Hall π'-subgroup of G.*

Proof. If H is an Hall π'-subgroup of G, then H is by 4.4 a $\underline{G}_{\pi'}$-projector of G.

Conversely, let H be a $\underline{G}_{\pi'}$-projector of G. We prove by induction on $|G|$ that H is an Hall π'-subgroup of G.

(1) H is a π'-subgroup of G because, H being $\underline{G}_{\pi'}$-maximal in G (see 1.5.a)), we have $H \in \underline{G}_{\pi'}$.

(2) $|G : H|$ is a π-number. Indeed, let M be a minimal normal subgroup of G. HM/M is by 1.6.b) a $\underline{G}_{\pi'}$-projector in G/M, hence by the induction HM/M is an Hall π'-subgroup of G/M. It follows that

$$|G : HM| = |G/M : HM/M|$$

is a π-number. M being a minimal normal subgroup of the π-solvable group G, M is either a solvable π-group or a π'-group.

a) If M is a solvable π-group, then $|G : H| = |G : HM||HM : H|$, where $|HM : H| = |M : H \cap M|/|M|$ is a π-number. It follows that $|G : H|$ is a π-number.

b) If M is a π'-group, using that HM/M is a π'-group we notice that HM is a π'-group. But H being a $\underline{G}_{\pi'}$-projector in G, H is by 1.5.a) $\underline{G}_{\pi'}$-maximal in G.

Hence from $H \leq HM \leq G$ and $HM \in \underline{G}_{\pi'}$ follows that $H = HM$. Then

$$|G : H| = |G : HM| = |G/M : HM/M|$$

is a π-number. □

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