Proximal Minimization

Let $f : \mathcal{H} \to \overline{\mathbb{R}}$ be a proper lower semicontinous function, where $\mathcal{H}$ is a real Hilbert space. Then $Prox_f x$ is the unique point in $\mathcal{H}$ that satifies

$$\min_{y\in\mathcal{H}} \left(f(y) + \frac{1}{2}\|x-y\|^2 \right) = f(Prox_f x) + \frac{1}{2}\|x - Prox_f x\|^2.$$

The operator $Prox_f : \mathcal{H} \to \mathcal{H}$ is the **proximity operator** of proximal mapping.

Proximity operators can be used to devise efficient algorithms to solve minimization problems. Such algorithms are called proximal algorithms, and representative examples of them are presented in the following.

Minimizing a lower semicontinuous function $f : \mathcal{H} \to \overline{\mathbb{R}}$ defined on a Hilbert space amounts to finding a zero of its subdifferential operator ∂f, which is a maximally monotone operator with the resolvant $J_{\partial f} = Prox_f$. Connected to it we present the proximal-point algorithm. Related to the sum of two functions we discuss the Doulgas-Rachford, forward-backward and Tsengs splitting algorithms. Illustrative practical examples are discussed as well.