Existence of solutions to initial value problems for nonlinear fractional differential equations on the semi-axis

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Abstract

The purpose of our talk is to present some results concerning the global existence of solutions to initial value problems for nonlinear fractional differential equations on the semi-axis. More precisely, we deal with the initial value problem

$$(1) \qquad \begin{cases} D_{0+}^{\alpha} x(t) = f(t, x(t)), & t \in (0, \infty) \\[2mm] \lim_{t \to 0+} t^{1-\alpha} x(t) = x_0, \end{cases}$$

where $0 < \alpha < 1$, D_{0+}^{α} denotes the Riemann-Liouville fractional derivative of order α, and $f : (0, \infty) \times \mathbb{R} \to \mathbb{R}$ is a continuous function. Unlike the previous papers dealing with the problem of existence of solutions to (1), we solve it by constructing a special locally convex space which is metrizable and complete. Then Schauder's fixed point theorem enables us to provide sufficient conditions on f, ensuring that (1) possesses at least one solution. We point out that our growth conditions imposed to f are weaker than other similar conditions already used in the literature.