Products of functions with bounded Hess^+ complement

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We denote by $\operatorname{Hess}^+(f)$ the set of all points $p \in \mathbb{R}^n$ such that the Hessian matrix $H_p(f)$ of the C^2-smooth function $f : \mathbb{R}^n \longrightarrow \mathbb{R}$ is positive definite.

In this paper we provide a class of norm-coercive polynomial functions with large Hess^+ regions, as their Hess^+ complements happen to be bounded. A detailed analysis concerning the Hess^+ region of a particular polynomial function along with some basic properties of its level curves, such as regularity, connectedness and convexity, is also done. For such functions we also prove several properties, such as connectedness and even convexity, of their level sets for sufficiently large levels. Apart from the mentioned source of such examples we provide some sufficient conditions on two functions $f, g : \mathbb{R}^2 \longrightarrow \mathbb{R}$ with bounded Hess^+ complements whose product fg keeps having bounded Hess^+ complement as well.

References

[1] Carmo, M., Differential geometry of curves and surfaces, Prentice Hall, 1976.

[2] Montiel S., Ros, A., Curves and Surfaces, American Mathematics Society, Graduate Studied in Mathematics, Vol. 69, 2005.

[3] Pintea, C., Tofan, A., Convex decompositions and the valence of some functions, J. Nonlinear Var. Anal., 4 (2) (2020), 225-239.

[4] Rybnikov, K. On convexity of hypersurfaces in the hyperbolic space. Geom Dedicata 136, 123-131 (2008).

[5] Zălinescu, C., Convex analysis in general vector spaces, World Scientific, 2002.