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An equilibrium problem approach for the existence of anti-periodic solutions for nonlinear implicit evolution equations

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Abstract. In this work, we study the existence of anti-periodic solutions for nonlinear implicit differential equations associated to a time dependent pseudomonotone (or quasimonotone) operator in the sense of Brézis. The method adopted in this work is new and differs from the most used technics in literature, it is based on recent results on the theory of equilibrium problems. By this approach, we provide some new results which improve and unify most of the recent results obtained in this direction.

Key words: Equilibrium problem; Evolution equation; Maximal monotone operator; Pseudomonotone operator; Quasimonotone operator.
AMS subject classification: 35A01,35A23,49J35,49J40,90C33.

1. Introduction

Let V be a real Hilbert space and V^* denotes its topological dual. In V, we consider the following anti-periodic problem of implicit nonlinear differential equations

$$(1) \quad \begin{cases} \frac{d}{dt}(Bx(t)) + \mathcal{A}(t)x(t) = f(t), \ \text{ for a.e } \ t \in (0,T) \\ Bx(0) = -Bx(T) \quad . \end{cases}$$

Where $B : V \to V^*$ is a linear, bounded, symmetric and positive operator, $\mathcal{A}(t) : V \to V^*$ is a nonlinear time-dependent operator, and $f : [0,T] \to V^*$ is a functional.
Due to their natural arising in the mathematical modeling of a variety of physical processes, anti-periodic solutions have been studied extensively by many authors. When B is the identity operator, the study of anti-periodic solutions for nonlinear evolution equations was initiated by Okochi [2]

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in Hilbert spaces.
Many problems for nonlinear evolution equations related to problem (1) have been investigated by applying the theory of monotone operators see [1]. Usually, these equations are governed by the sum of a discontinuous monotone operator and a Nemytskij operator satisfying suitable smoothness and growth conditions. In contrast the problem we study here is governed by a time-dependent pseudomonotone (or quasimonotone) operator in the sense of Brézis, and the approach adopted is new, it is based on the recent development on the theory of equilibrium problems. The idea is to introduce an auxiliary problem and study the existence of its solutions, then from these obtained results for the auxiliary problem, we derive the existence of anti-periodic solutions for the initial problem (1).
The results obtained in this work are new and lead to improve and unify most of the recent results obtained in this direction.

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